CODATA Recommended Values of the Fundamental Physical Constants: 2006*

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This paper gives the 2006 self-consistent set of values of the basic constants and conversion factors of physics and chemistry recommended by the Committee on Data for Science and Technology (CODATA) for international use. Further, it describes in detail the adjustment of the values of the constants, including the selection of the final set of input data based on the results of least-squares analyses. The 2006 adjustment takes into account the data considered in the 2002 adjustment as well as the data that became available between 31 December 2002, the closing date of that adjustment, and 31 December 2006, the closing date of the new adjustment. The new data have led to a significant reduction in the uncertainties of many recommended values. The 2006 set replaces the previously recommended 2002 CODATA set and may also be found on the World Wide Web at physics.nist.gov/constants.

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Glossary

AMDC	Atomic Mass Data Center, Centre de Spectrométrie Nucléaire et de Spectrométrie de Masse (CSNSM), Orsay, France
AME2003	2003 atomic mass evaluation of the AMDC
$A_\mathrm{r}(X)$	Relative atomic mass of X: $A_\mathrm{r}(X) = m(X)/m_\mathrm{u}$
A_{90}	Conventional unit of electric current: $A_{90} = V_{90}/\Omega_{90}$
Å*	Ångström-star: $\lambda(\mathrm{WK}\alpha_1) = 0.209\,010\,0$ Å*
a_e	Electron magnetic moment anomaly: $a_\mathrm{e} = (\lvert g_\mathrm{e}\rvert - 2)/2$
a_μ	Muon magnetic moment anomaly: $a_\mu = (\lvert g_\mu\rvert - 2)/2$
BIPM	International Bureau of Weights and Measures, Sèvres, France
BNL	Brookhaven National Laboratory, Upton, New York, USA
CERN	European Organization for Nuclear Research, Geneva, Switzerland
CIPM	International Committee for Weights and Measures
CODATA	Committee on Data for Science and Technology of the International Council for Science
CPT	Combined charge conjugation, parity inversion, and time reversal
c	Speed of light in vacuum
cw	Continuous wave
d	Deuteron (nucleus of deuterium D, or ^{2}H)
d_{220}	{220} lattice spacing of an ideal crystal of naturally occurring silicon
$d_{220}(X)$	{220} lattice spacing of crystal X of naturally occurring silicon
E_b	Binding energy
e	Symbol for either member of the electron-positron pair; when necessary, e$^-$ or e$^+$ is used to indicate the electron or positron
e	Elementary charge: absolute value of the charge of the electron
F	Faraday constant: $F = N_\mathrm{A}e$
FCDC	Fundamental Constants Data Center, NIST, USA
FSU	Friedrich-Schiller University, Jena, Germany
$\mathcal{F}_{90}$	$\mathcal{F}_{90} = (F/A_{90})$ A
G	Newtonian constant of gravitation
g	Local acceleration of free fall
g_d	Deuteron g-factor: $g_\mathrm{d} = \mu_\mathrm{d}/\mu_\mathrm{N}$
g_e	Electron g-factor: $g_\mathrm{e} = 2\mu_\mathrm{e}/\mu_\mathrm{B}$
g_p	Proton g-factor: $g_\mathrm{p} = 2\mu_\mathrm{p}/\mu_\mathrm{N}$
g'_p	Shielded proton g-factor: $g'_\mathrm{p} = 2\mu'_\mathrm{p}/\mu_\mathrm{N}$
g_t	Triton g-factor: $g_\mathrm{t} = 2\mu_\mathrm{t}/\mu_\mathrm{N}$
$g_X(Y)$	g-factor of particle X in the ground (1S) state of hydrogenic atom Y
g_μ	Muon g-factor: $g_\mu = 2\mu_\mu/(e\hbar/2m_\mu)$
GSI	Gesellschaft für Schwerionenforschung, Darmstadt, Germany

HD	HD molecule (bound state of hydrogen and deuterium atoms)
HT	HT molecule (bound state of hydrogen and tritium atoms)
h	Helion (nucleus of ^{3}He)
h	Planck constant; $\hbar = h/2\pi$
Harvard; HarvU	Harvard University, Cambridge, Massachusetts, USA
ILL	Institut Max von Laue-Paul Langevin, Grenoble, France
IMGC	Istituto di Metrologia "T. Colonetti," Torino, Italy
INRIM	Istituto Nazionale di Ricerca Metrologica, Torino, Italy
IRMM	Institute for Reference Materials and Measurements, Geel, Belgium
JINR	Joint Institute for Nuclear Research, Dubna, Russian Federation
KRISS	Korea Research Institute of Standards and Science, Taedok Science Town, Republic of Korea
KR/VN	KRISS-VNIIM collaboration
K_{J}	Josephson constant: $K_{\mathrm{J}} = 2e/h$
$K_{\mathrm{J-90}}$	Conventional value of the Josephson constant K_{J}: $K_{\mathrm{J-90}} = 483\,597.9$ GHz V^{-1}
k	Boltzmann constant: $k = R/N_{\mathrm{A}}$
LAMPF	Clinton P. Anderson Meson Physics Facility at Los Alamos National Laboratory, Los Alamos, New Mexico, USA
LKB	Laboratoire Kastler-Brossel, Paris, France
LK/SY	LKB and SYRTE collaboration
LNE	Laboratoire national de métrologie et d'essais, Trappes, France
MIT	Massachusetts Institute of Technology, Cambridge, Massachusetts, USA
MPQ	Max-Planck-Institut für Quantenoptik, Garching, Germany
MSL	Measurement Standards Laboratory, Lower Hutt, New Zealand
$M(X)$	Molar mass of X: $M(X) = A_{\mathrm{r}}(X)M_{\mathrm{u}}$
Mu	Muonium (μ^+e^- atom)
M_{u}	Molar mass constant: $M_{\mathrm{u}} = 10^{-3}$ kg mol^{-1}
m_{u}	Unified atomic mass constant: $m_{\mathrm{u}} = m(^{12}\mathrm{C})/12$
m_X, $m(X)$	Mass of X (for the electron e, proton p, and other elementary particles, the first symbol is used, *i.e.,* m_{e}, m_{p}, *etc.*)
N_{A}	Avogadro constant
N/P/I	NMIJ-PTB-IRMM combined result
NIM	National Institute of Metrology, Beijing, China (People's Republic of)
NIST	National Institute of Standards and Technology, Gaithersburg, Maryland and Boulder, Colorado, USA
NMI	National Metrology Institute, Lindfield, Australia
NMIJ	National Metrology Institute of Japan, Tsukuba, Japan
NMR	Nuclear magnetic resonance
NPL	National Physical Laboratory, Teddington, UK
NRLM	National Research Laboratory of Metrology, Tsukuba, Japan
n	Neutron
PRC	People's Republic of China
PTB	Physikalisch-Technische Bundesanstalt, Braunschweig and Berlin, Germany
p	Proton
$\overline{\mathrm{p}}\,{}^A\mathrm{He}^+$	Antiprotonic helium ($^A\mathrm{He}^+ + \overline{\mathrm{p}}$ atom, $A = 3$ or 4)
QED	Quantum electrodynamics
$Q(\chi^2\|\nu)$	Probability that an observed value of chi-square for ν degrees of freedom would exceed χ^2
R	Molar gas constant
$\overline{R}$	Ratio of muon anomaly difference frequency to free proton NMR frequency
R_{B}	Birge ratio: $R_{\mathrm{B}} = (\chi^2/\nu)^{\frac{1}{2}}$
R_{d}; Rd	Bound-state rms charge radius of the deuteron
R_{K}	von Klitzing constant: $R_{\mathrm{K}} = h/e^2$
$R_{\mathrm{K-90}}$	Conventional value of the von Klitzing constant R_{K}: $R_{\mathrm{K-90}} = 25\,812.807\ \Omega$
R_{p}; Rp	Bound-state rms charge radius of the proton
R_∞	Rydberg constant: $R_\infty = m_{\mathrm{e}}c\alpha^2/2h$
$r(x_i, x_j)$	Correlation coefficient of estimated values x_i and x_j: $r(x_i, x_j) = u(x_i, x_j)/[u(x_i)u(x_j)]$
r_i	Normalized residual of x_i: $r_i = (x_i - \hat{x}_i)/u(x_i)$, $\hat{x}_i$ is the adjusted value of x_i
rms	Root mean square
S_{c}	Self-sensitivity coefficient
SI	Système international d'unités (International System of Units)
Stanford; StanfU	Stanford University, Stanford, California, USA
StPtrsb	St. Petersburg, Russian Federation
SYRTE	Systèmes de référence Temps Espace, Paris, France
T	Thermodynamic temperature
t	Triton (nucleus of tritium T, or ^{3}H)
th	Theory
Type A	Uncertainty evaluation by the statistical analysis of series of observations
Type B	Uncertainty evaluation by means other than the statistical analysis of series of observations
t_{90}	Celsius temperature on the International Temperature Scale of 1990 (ITS-90)
U. Sussex; USus	University of Sussex, Sussex, UK
UK	United Kingdom
USA	United States of America
UWash	University of Washington, Seattle, Washington, USA
u	Unified atomic mass unit (also called the dalton, Da): 1 u $= m_{\mathrm{u}} = m(^{12}\mathrm{C})/12$
$u(x_i)$	Standard uncertainty (*i.e.,* estimated standard deviation) of an estimated value x_i of a quantity X_i (also simply u)
$u(x_i, x_j)$	Covariance of estimated values x_i and x_j
u_{diff}	Standard uncertainty of the difference $x_i - x_j$: $u^2_{\mathrm{diff}} = u^2(x_i) + u^2(x_j) - 2\,u(x_i, x_j)$
$u_{\mathrm{r}}(x_i)$	Relative standard uncertainty of an estimated value x_i of a quantity X_i: $u_{\mathrm{r}}(x_i) = u(x_i)/\|x_i\|,\ x_i \neq 0$ (also simply u_{r})
$u_{\mathrm{r}}(x_i, x_j)$	Relative covariance of estimated values x_i and x_j: $u_{\mathrm{r}}(x_i, x_j) = u(x_i, x_j)/(x_i x_j)$
$V_{\mathrm{m}}(\mathrm{Si})$	Molar volume of naturally occurring silicon
VNIIM	D. I. Mendeleyev All-Russian Research Institute for Metrology, St. Petersburg, Russian Federation
V_{90}	Conventional unit of voltage based on the Josephson effect and $K_{\mathrm{J-90}}$: $V_{90} = (K_{\mathrm{J-90}}/K_{\mathrm{J}})$ V

WGAC	Working Group on the Avogadro Constant of the CIPM Consultative Committee for Mass and Related Quantities (CCM)
W_{90}	Conventional unit of power: $W_{90} = V_{90}^2/\Omega_{90}$
XROI	Combined x-ray and optical interferometer
xu(Cu Kα_1)	Cu x unit: $\lambda(\mathrm{CuK}\alpha_1) = 1\,537.400$ xu(CuKα_1)
xu(Mo Kα_1)	Mo x unit: $\lambda(\mathrm{MoK}\alpha_1) = 707.831$ xu(MoKα_1)
$x(X)$	Amount-of-substance fraction of X
YAG	Yttrium aluminium garnet; $Y_3Al_5O_{12}$
Yale; YaleU	Yale University, New Haven, Connecticut, USA
α	Fine-structure constant: $\alpha = e^2/4\pi\epsilon_0\hbar c \approx 1/137$
$\mathrm{\alpha}$	Alpha particle (nucleus of ^{4}He)
$\Gamma'_{X-90}(\mathrm{lo})$	$\Gamma'_{X-90}(\mathrm{lo}) = (\gamma'_X\, A_{90})\ \mathrm{A}^{-1}$, X = p or h
$\Gamma'_{\mathrm{p}-90}(\mathrm{hi})$	$\Gamma'_{\mathrm{p}-90}(\mathrm{hi}) = (\gamma'_{\mathrm{p}}/A_{90})$ A
γ_{p}	Proton gyromagnetic ratio: $\gamma_{\mathrm{p}} = 2\mu_{\mathrm{p}}/\hbar$
γ'_{p}	Shielded proton gyromagnetic ratio: $\gamma'_{\mathrm{p}} = 2\mu'_{\mathrm{p}}/\hbar$
γ'_{h}	Shielded helion gyromagnetic ratio: $\gamma'_{\mathrm{h}} = 2\lvert\mu'_{\mathrm{h}}\rvert/\hbar$
$\Delta\nu_{\mathrm{Mu}}$	Muonium ground-state hyperfine splitting
δ_{e}	Additive correction to the theoretical expression for the electron magnetic moment anomaly a_{e}
δ_{Mu}	Additive correction to the theoretical expression for the ground-state hyperfine splitting of muonium $\Delta\nu_{\mathrm{Mu}}$
$\delta_{\overline{\mathrm{p}}\,\mathrm{He}}$	Additive correction to the theoretical expression for a particular transition frequency of antiprotonic helium
$\delta_X(n\mathrm{L}_j)$	Additive correction to the theoretical expression for an energy level of either hydrogen H or deuterium D with quantum numbers n, L, and j
$\delta_{\mathrm{\mu}}$	Additive correction to the theoretical expression for the muon magnetic moment anomaly $a_{\mathrm{\mu}}$
ϵ_0	Electric constant: $\epsilon_0 = 1/\mu_0 c^2$
$\lambda(X\,\mathrm{K}\alpha_1)$	Wavelength of Kα_1 x-ray line of element X
λ_{meas}	Measured wavelength of the 2.2 MeV capture γ-ray emitted in the reaction $\mathrm{n} + \mathrm{p} \to \mathrm{d} + \gamma$
$\mathrm{\mu}$	Symbol for either member of the muon-antimuon pair; when necessary, μ^- or μ^+ is used to indicate the negative muon or positive muon
μ_{B}	Bohr magneton: $\mu_{\mathrm{B}} = e\hbar/2m_{\mathrm{e}}$
μ_{N}	Nuclear magneton: $\mu_{\mathrm{N}} = e\hbar/2m_{\mathrm{p}}$
$\mu_X(Y)$	Magnetic moment of particle X in atom or molecule Y.
μ_0	Magnetic constant: $\mu_0 = 4\pi \times 10^{-7}$ N/A^2
μ_X, μ'_X	Magnetic moment, or shielded magnetic moment, of particle X
ν	Degrees of freedom of a particular adjustment
$\nu(f_{\mathrm{p}})$	Difference between muonium hyperfine splitting Zeeman transition frequencies ν_{34} and ν_{12} at a magnetic flux density B corresponding to the free proton NMR frequency f_{p}
σ	Stefan-Boltzmann constant: $\sigma = 2\pi^5 k^4/(15h^3c^2)$
$\mathrm{\tau}$	Symbol for either member of the tau-antitau pair; when necessary, τ^- or τ^+ is used to indicate the negative tau or positive tau
χ^2	The statistic "chi square"
Ω_{90}	Conventional unit of resistance based on the quantum Hall effect and $R_{\mathrm{K}-90}$: $\Omega_{90} = (R_{\mathrm{K}}/R_{\mathrm{K}-90})\ \Omega$
$\doteq$	Symbol used to relate an input datum to its observational equation

1. INTRODUCTION

1. Background

This paper gives the complete 2006 CODATA self-consistent set of recommended values of the fundamental physical constants and describes in detail the 2006 least-squares adjustment, including the selection of the final set of input data based on the results of least-squares analyses. Prepared under the auspices of the CODATA Task Group on Fundamental Constants, this is the fifth such report of the Task Group since its establishment in 1969 [1] and the third in the four-year cycle of reports begun in 1998. The 2006 set of recommended values replaces its immediate predecessor, the 2002 set. The closing date for the availability of the data considered for inclusion in this adjustment was 31 December 2006. As a consequence of the new data that became available in the intervening four years there has been a significant reduction of the uncertainty of many constants. The 2006 set of recommended values first became available on 29 March 2007 at http://physics.nist.gov/constants, a Web site of the NIST Fundamental Constants Data Center (FCDC).

The 1998 and 2002 reports describing the 1998 and 2002 adjustments (Mohr and Taylor, 2000, 2005), referred to as CODATA-98 and CODATA-02 throughout this article, describe in detail much of the currently available data, its analysis, and the techniques used to obtain a set of best values of the constants using the standard method of least squares for correlated input data. This paper focuses mainly on the new information that has become available since 31 December 2002 and references the discussions in CODATA-98 and CODATA-02 for earlier work in the interest of brevity. More specifically, if a potential input datum is not discussed in detail, the reader can assume that it (or a closely related datum) has been reviewed in either CODATA-98 or CODATA-02.

The reader is also referred to these papers for a discussion of the motivation for and the philosophy behind the periodic adjustment of the values of the constants and for descriptions of how units, quantity symbols, numerical values, numerical calculations, and uncertainties are treated, in addition to how the data are characterized, selected, and evaluated. Since the calculations are carried out with more significant figures than are displayed in the text to avoid rounding errors, data with more digits are available on the FCDC Web site for possible independent analysis.

However, because of their importance, we recall in detail the following two points also discussed in these references. First, although it is generally agreed that the

[1] The Committee on Data for Science and Technology was established in 1966 as an interdisciplinary committee of the International Council for Science.

correctness and over-all consistency of the basic theories and experimental methods of physics can be tested by comparing values of particular fundamental constants obtained from widely differing experiments, throughout this adjustment, as a working principle, we assume the validity of the physical theory that necessarily underlies it. This includes special relativity, quantum mechanics, quantum electrodynamics (QED), the Standard Model of particle physics, including combined charge conjugation, parity inversion, and time reversal (CPT) invariance, and the theory of the Josephson and quantum Hall effects, especially the exactness of the relationships between the Josephson and von Klitzing constants $K_{\rm J}$ and $R_{\rm K}$ and the elementary charge e and Planck constant h. In fact, tests of these relations, $K_{\rm J} = 2e/h$ and $R_{\rm K} = h/e^2$, using the input data of the 2006 adjustment are discussed in Sec. 12.2.2.

The second point has to do with the 31 December 2006 closing date for data to be considered for inclusion in the 2006 adjustment. A datum was considered to have met this date, even though not yet reported in an archival journal, as long as a description of the work was available that allowed the Task Group to assign a valid standard uncertainty $u(x_i)$ to the datum. Thus, any input datum labeled with an "07" identifier because it was published in 2007 was, in fact, available by the cutoff date. Also, some references to results that became available after the deadline are included, even though they were not used in the adjustment.

2. Time variation of the constants

This subject, which was briefly touched upon in CODATA-02, continues to be an active field of experimental and theoretical research, because of its importance to our understanding of physics at the most fundamental level. Indeed, a large number of papers relevant to the topic have appeared in the last four years; see the FCDC bibliographic database on the fundamental constants using the keyword "time variation" at http://physics.nist.gov/constantsbib. For example, see Fortier *et al.* (2007); Lea (2007). However, there has been no laboratory observation of time dependence of any constant that might be relevant to the recommended values.

3. Outline of paper

Section 2 touches on special quantities and units, that is, those that have exact values by definition.

Sections 3-11 review all of the available experimental and theoretical data that might be relevant to the 2006 adjustment of the values of the constants. As discussed in Appendix E of CODATA-98, in a least squares analysis of the fundamental constants the numerical data, both experimental and theoretical, also called *observational data* or *input data*, are expressed as functions of a set of independent variables called *adjusted constants*. The functions that relate the input data to the adjusted constants are called *observational equations*, and the least squares procedure provides best estimated values, in the least squares sense, of the adjusted constants. The focus of the review-of-data sections is thus the identification and discussion of the input data and observational equations of interest for the 2006 adjustment. Although not all observational equations that we use are explicitly given in the text, all are summarized in Tables 38, 40, and 42 of Sec. 12.2.

As part of our discussion of a particular datum, we often deduce from it an inferred value of a constant, such as the fine-structure constant α or Planck constant h. It should be understood, however, that these inferred values are for comparison purposes only; the datum from which it is obtained, not the inferred value, is the input datum in the adjustment.

Although just 4 years separate the 31 December closing dates of the 2002 and 2006 adjustments, there are a number of important new results to consider. Experimental advances include the 2003 Atomic Mass Evaluation from the Atomic Mass Data Center (AMDC) that provides new values for the relative atomic masses $A_{\rm r}(X)$ of a number of relevant atoms; a new value of the electron magnetic moment anomaly $a_{\rm e}$ from measurements on a single electron in a cylindrical penning trap that provides a value of the fine-structure constant α; better measurements of the relative atomic masses of ^{2}H, ^{3}H, and ^{4}He; new measurements of transition frequencies in antiprotonic helium ($\bar{\rm p}\,{}^A{\rm He}^+$ atom) that provide a competitive value of the relative atomic mass of the electron $A_{\rm r}({\rm e})$; improved measurements of the nuclear magnetic resonance (NMR) frequencies of the proton and deuteron in the HD molecule and of the proton and triton in the HT molecule; a highly accurate value of the Planck constant obtained from an improved measurement of the product $K_{\rm J}^2 R_{\rm K}$ using a moving-coil watt balance; new results using combined x-ray and optical interferometers for the $\{220\}$ lattice spacing of single crystals of naturally occurring silicon; and an accurate value of the quotient $h/m(^{87}{\rm Rb})$ obtained by measuring the recoil velocity of rubidium-87 atoms upon absorption or emission of photons—a result that provides an accurate value of α that is virtually independent of the electron magnetic moment anomaly.

Theoretical advances include improvements in certain aspects of the theory of the energy levels of hydrogen and deuterium; improvements in the theory of antiprotonic helium transition frequencies that, together with the new transition frequency measurements, have led to the aforementioned competitive value of $A_{\rm r}({\rm e})$; a new theoretical expression for $a_{\rm e}$ that, together with the new experimental value of $a_{\rm e}$, has led to the aforementioned value of α; improvements in the theory of the g-factor of the bound electron in hydrogenic ions with nuclear spin quantum number $i = 0$ relevant to the determination of

$A_r(e)$; and improved theory of the ground state hyperfine splitting of muonium $\Delta\nu_{Mu}$ (the μ^+e^- atom).

Section 12 describes the analysis of the data, with the exception of the Newtonian constant of gravitation which is analyzed in Sec. 10. The consistency of the data and potential impact on the determination of the 2006 recommended values were appraised by comparing measured values of the same quantity, comparing measured values of different quantities through inferred values of a third quantity such as α or h, and finally by using the method of least squares. Based on these investigations, the final set of input data used in the 2006 adjustment was selected.

Section 13 provides, in several tables, the 2006 CODATA recommended values of the basic constants and conversion factors of physics and chemistry, including the covariance matrix of a selected group of constants.

Section 14 concludes the paper with a comparison of the 2006 and 2002 recommended values of the constants, a survey of implications for physics and metrology of the 2006 values and adjustment, and suggestions for future work that can advance our knowledge of the values of the constants.

2. SPECIAL QUANTITIES AND UNITS

Table 1 lists those special quantities whose numerical values are exactly defined. In the International System of Units (SI) (BIPM, 2006), which we use throughout this paper, the definition of the meter fixes the speed of light in vacuum c, the definition of the ampere fixes the magnetic constant (also called the permeability of vacuum) μ_0, and the definition of the mole fixes the molar mass of the carbon 12 atom $M(^{12}\mathrm{C})$ to have the exact values given in the table. Since the electric constant (also called the permittivity of vacuum) is related to μ_0 by $\epsilon_0 = 1/\mu_0 c^2$, it too is known exactly.

The relative atomic mass $A_r(X)$ of an entity X is defined by $A_r(X) = m(X)/m_u$, where $m(X)$ is the mass of X and m_u is the atomic mass constant defined by

$$m_u = \frac{1}{12} m(^{12}\mathrm{C}) = 1\ \mathrm{u} \approx 1.66 \times 10^{-27}\ \mathrm{kg}, \tag{1}$$

where $m(^{12}\mathrm{C})$ is the mass of the carbon 12 atom and u is the unified atomic mass unit (also called the dalton, Da). Clearly, $A_r(X)$ is a dimensionless quantity and $A_r(^{12}\mathrm{C}) = 12$ exactly. The molar mass $M(X)$ of entity X, which is the mass of one mole of X with SI unit kg/mol, is given by

$$M(X) = N_A m(X) = A_r(X) M_u, \tag{2}$$

where $N_A \approx 6.02 \times 10^{23}$/mol is the Avogadro constant and $M_u = 10^{-3}$ kg/mol is the molar mass constant. The numerical value of N_A is the number of entities in one mole, and since the definition of the mole states that one mole contains the same number of entities as there are in 0.012 kg of carbon 12, $M(^{12}\mathrm{C}) = 0.012$ kg/mol exactly.

The Josephson and quantum Hall effects have played and continue to play important roles in adjustments of the values of the constants, because the Josephson and von Klitzing constants K_J and R_K, which underlie these two effects, are related to e and h by

$$K_J = \frac{2e}{h}\,; \qquad R_K = \frac{h}{e^2} = \frac{\mu_0 c}{2\alpha}. \tag{3}$$

Although we assume these relations are exact, and no evidence—either theoretical or experimental—has been put forward that challenges this assumption, the consequences of relaxing it are explored in Sec. 12.2.2. Some references to recent work related to the Josephson and quantum Hall effects may be found in the FCDC bibliographic database (see Sec. 1.2).

The next-to-last two entries in Table 1 are the conventional values of the Josephson and von Klitzing constants adopted by the International Committee for Weights and Measures (CIPM) and introduced on 1 January 1990 to establish worldwide uniformity in the measurement of electrical quantities. In this paper, all electrical quantities are expressed in SI units. However, those measured in terms of the Josephson and quantum Hall effects with the assumption that K_J and R_K have these conventional values are labeled with a subscript 90.

For high-accuracy experiments involving the force of gravity, such as the watt-balance, an accurate measurement of the local acceleration of free fall at the site of the experiment is required. Fortunately, portable and easy-to-use commercial absolute gravimeters are available that can provide a local value of g with a relative standard uncertainty of a few parts in 10^9. That these instruments can achieve such a small uncertainty if properly used is demonstrated by a periodic international comparison of absolute gravimeters (ICAG) carried out at the International Bureau of Weights and Measures (BIPM), Sèvres, France; the seventh and most recent, denoted ICAG-2005, was completed in September 2005 (Vitushkin *et al.*, 2005); the next is scheduled for 2009. In the future, atom interferometry or Bloch oscillations using ultracold atoms could provide a competitive or possibly more accurate method for determining a local value of g (Cladé *et al.*, 2005; McGuirk *et al.*, 2002; Peters *et al.*, 2001).

3. RELATIVE ATOMIC MASSES

Included in the set of adjusted constants are the relative atomic masses $A_r(X)$ of a number of particles, atoms, and ions. Tables 2-6 and the following sections summarize the relevant data.

1. Relative atomic masses of atoms

Most values of the relative atomic masses of neutral atoms used in this adjustment are taken from the 2003 atomic mass evaluation (AME2003) of the Atomic

TABLE 1 Some exact quantities relevant to the 2006 adjustment.

Quantity	Symbol	Value
speed of light in vacuum	c, c_0	$299\,792\,458$ m s^{-1}
magnetic constant	μ_0	$4\pi \times 10^{-7}$ N A^{-2} = $12.566\,370\,614... \times 10^{-7}$ N A^{-2}
electric constant	ϵ_0	$(\mu_0 c^2)^{-1} = 8.854\,187\,817... \times 10^{-12}$ F m^{-1}
relative atomic mass of ^{12}C	$A_{\rm r}(^{12}{\rm C})$	12
molar mass constant	$M_{\rm u}$	10^{-3} kg mol^{-1}
molar mass of ^{12}C $A_{\rm r}(^{12}{\rm C})\,M_{\rm u}$	$M(^{12}{\rm C})$	12×10^{-3} kg mol^{-1}
conventional value of Josephson constant	$K_{\rm J-90}$	$483\,597.9$ GHz V^{-1}
conventional value of von Klitzing constant	$R_{\rm K-90}$	$25\,812.807$ Ω

Mass Data Center, Centre de Spectrométrie Nucléaire et de Spectrométrie de Masse (CSNSM), Orsay, France (AMDC, 2006; Audi *et al.*, 2003; Wapstra *et al.*, 2003). The results of AME2003 supersede those of both the 1993 atomic mass evaluation and the 1995 update. Table 2 lists the values from AME2003 of interest here, while Table 3 gives the covariance for hydrogen and deuterium (AMDC, 2003). Other non-negligible covariances of these values are discussed in the appropriate sections.

Table 4 gives six values of $A_{\rm r}(X)$ relevant to the 2006 adjustment reported since the completion and publication of AME2003 in late 2003 that we use in place of the corresponding values in Table 2.

The ^{3}H and ^{3}He values are those reported by the SMILETRAP group at the Manne Siegbahn Laboratory (MSL), Stockholm, Sweden (Nagy *et al.*, 2006), using a Penning trap and a time of flight technique to detect cyclotron resonances. This new ^{3}He result is in good agreement with a more accurate, but still preliminary, result from the University of Washington group in Seattle, USA (Van Dyck, 2006). The AME2003 values for ^{3}H and ^{3}He were influenced by an earlier result for ^{3}He from the University of Washington group which is in disagreement with their new result.

The values for ^{4}He and ^{16}O are those reported by the University of Washington group (Van Dyck *et al.*, 2006) using their improved mass spectrometer; they are based on a thorough reanalysis of data that yielded preliminary results for these atoms which were used in AME2003. They include an experimentally determined image-charge correction with a relative standard uncertainty $u_{\rm r} = 7.9 \times 10^{-12}$ in the case of ^{4}He and $u_{\rm r} = 4.0 \times 10^{-12}$ in the case of ^{16}O. The value of $A_{\rm r}(^2{\rm H})$ is also from this group and is a near-final result based on the analysis of ten runs carried out over a 4 year period (Van Dyck, 2006). Because the result is not yet final, the total uncertainty is conservatively assigned; $u_{\rm r} = 9.9 \times 10^{-12}$ for the image-charge correction. This value of $A_{\rm r}(^2{\rm H})$ is consistent with the preliminary value reported by Van Dyck *et al.* (2006) based on the analysis of only three runs.

The covariance and correlation coefficient of $A_{\rm r}(^3{\rm H})$ and $A_{\rm r}(^3{\rm He})$ given in Table 5 are due to the common component of uncertainty $u_{\rm r} = 1.4 \times 10^{-10}$ of the relative atomic mass of the H_2^+ reference ion used in the SMILETRAP measurements; the covariances and correlation coefficients of the University of Washington values of $A_{\rm r}(^2{\rm H})$, $A_{\rm r}(^4{\rm He})$, and $A_{\rm r}(^{16}{\rm O})$ given in Table 6 are due to the uncertainties of the image-charge corrections, which are based on the same experimentally determined relation.

The ^{29}Si value is that implied by the ratio $A_{\rm r}(^{29}{\rm Si}^+)/A_{\rm r}(^{28}{\rm Si\,H}^+) = 0.999\,715\,124\,1812(65)$ obtained at the Massachusetts Institute of Technology (MIT), Cambridge, USA, using a recently developed technique of determining mass ratios by directly comparing the cyclotron frequencies of two different ions simultaneously confined in a Penning trap (Rainville *et al.*, 2005). (The relative atomic mass work of the MIT group has now been transferred to Florida State University, Tallahassee, USA.) This approach eliminates many components of uncertainty arising from systematic effects. The value for $A_{\rm r}(^{29}{\rm Si})$ is given in the Supplementary Information to Rainville *et al.* (2005) and has a significantly smaller uncertainty than the corresponding AME2003 value.

2. Relative atomic masses of ions and nuclei

The relative atomic mass $A_{\rm r}(X)$ of a neutral atom X is given in terms of the relative atomic mass of an ion of the atom formed by the removal of n electrons by

$$A_{\rm r}(X) = A_{\rm r}(X^{n+}) + nA_{\rm r}({\rm e}) - \frac{E_{\rm b}(X) - E_{\rm b}(X^{n+})}{m_{\rm u}c^2}\,. \tag{4}$$

Here $E_{\rm b}(X)/m_{\rm u}c^2$ is the relative-atomic-mass equivalent of the total binding energy of the Z electrons of the atom, where Z is the atomic number (proton number), and $E_{\rm b}(X^{n+})/m_{\rm u}c^2$ is the relative-atomic-mass-equivalent of the binding energy of the $Z - n$ electrons of the X^{n+} ion. For a fully stripped atom, that is, for $n = Z$, X^{Z+} is N, where N represents the nucleus of the atom, and $E_{\rm b}(X^{Z+})/m_{\rm u}c^2 = 0$, which yields the first few equations of Table 40 in Sec. 12.2.

The binding energies $E_{\rm b}$ used in this work are the same as those used in the 2002 adjustment; see Table IV of CODATA-02. For tritium, which is not included there, we use the value $1.097\,185\,439 \times 10^7$ m^{-1} (Kotochigova, 2006). The uncertainties of the binding energies are negligible for our application.

TABLE 2 Values of the relative atomic masses of the neutron and various atoms as given in the 2003 atomic mass evaluation together with the defined value for ^{12}C.

Atom	Relative atomic mass $A_r(\mathrm{X})$	Relative standard uncertainty u_r
n	1.008 664 915 74(56)	5.6×10^{-10}
^{1}H	1.007 825 032 07(10)	1.0×10^{-10}
^{2}H	2.014 101 777 85(36)	1.8×10^{-10}
^{3}H	3.016 049 2777(25)	8.2×10^{-10}
^{3}He	3.016 029 3191(26)	8.6×10^{-10}
^{4}He	4.002 603 254 153(63)	1.6×10^{-11}
^{12}C	12	(exact)
^{16}O	15.994 914 619 56(16)	1.0×10^{-11}
^{28}Si	27.976 926 5325(19)	6.9×10^{-11}
^{29}Si	28.976 494 700(22)	7.6×10^{-10}
^{30}Si	29.973 770 171(32)	1.1×10^{-9}
^{36}Ar	35.967 545 105(28)	7.8×10^{-10}
^{38}Ar	37.962 732 39(36)	9.5×10^{-9}
^{40}Ar	39.962 383 1225(29)	7.2×10^{-11}
^{87}Rb	86.909 180 526(12)	1.4×10^{-10}
^{107}Ag	106.905 0968(46)	4.3×10^{-8}
^{109}Ag	108.904 7523(31)	2.9×10^{-8}
^{133}Cs	132.905 451 932(24)	1.8×10^{-10}

TABLE 3 The variances, covariance, and correlation coefficient of the AME2003 values of the relative atomic masses of hydrogen and deuterium. The number in bold above the main diagonal is 10^{18} times the numerical value of the covariance; the numbers in bold on the main diagonal are 10^{18} times the numerical values of the variances; and the number in italics below the main diagonal is the correlation coefficient.

	$A_r(^1\mathrm{H})$	$A_r(^2\mathrm{H})$
$A_r(^1\mathrm{H})$	**0.0107**	**0.0027**
$A_r(^2\mathrm{H})$	*0.0735*	**0.1272**

3. Cyclotron resonance measurement of the electron relative atomic mass $A_r(\mathrm{e})$

A value of $A_r(\mathrm{e})$ is available from a Penning-trap measurement carried out by the University of Washington group (Farnham *et al.*, 1995); it is used as an input datum in the 2006 adjustment, as it was in the 2002 adjustment:

$$A_r(\mathrm{e}) = 0.000\,548\,579\,9111(12) \qquad [2.1 \times 10^{-9}] \, . \quad (5)$$

4. ATOMIC TRANSITION FREQUENCIES

Atomic transition frequencies in hydrogen, deuterium, and anti-protonic helium yield information on the Rydberg constant, the proton and deuteron charge radii, and the relative atomic mass of the electron. The hyperfine splitting in hydrogen and fine-structure splitting in helium do not yield a competitive value of any constant at the current level of accuracy of the relevant experiment and/or theory. All of these topics are discussed in this section.

TABLE 4 Values of the relative atomic masses of various atoms that have become available since the 2003 atomic mass evaluation.

Atom	Relative atomic mass $A_r(X)$	Relative standard uncertainty u_r
^{2}H	2.014 101 778 040(80)	4.0×10^{-11}
^{3}H	3.016 049 2787(25)	8.3×10^{-10}
^{3}He	3.016 029 3217(26)	8.6×10^{-10}
^{4}He	4.002 603 254 131(62)	1.5×10^{-11}
^{16}O	15.994 914 619 57(18)	1.1×10^{-11}
^{29}Si	28.976 494 6625(20)	6.9×10^{-11}

TABLE 5 The variances, covariance, and correlation coefficient of the values of the SMILETRAP relative atomic masses of tritium and helium three. The number in bold above the main diagonal is 10^{18} times the numerical value of the covariance; the numbers in bold on the main diagonal are 10^{18} times the numerical values of the variances; and the number in italics below the main diagonal is the correlation coefficient.

	$A_r(^3\mathrm{H})$	$A_r(^3\mathrm{He})$
$A_r(^3\mathrm{H})$	**6.2500**	**0.1783**
$A_r(^3\mathrm{He})$	*0.0274*	**6.7600**

1. Hydrogen and deuterium transition frequencies, the Rydberg constant R_∞, and the proton and deuteron charge radii R_p, R_d

The Rydberg constant is related to other constants by the definition

$$R_\infty = \alpha^2 \frac{m_e c}{2h} \, . \quad (6)$$

It can be accurately determined by comparing measured resonant frequencies of transitions in hydrogen (H) and deuterium (D) to the theoretical expressions for the energy level differences in which it is a multiplicative factor.

TABLE 6 The variances, covariances, and correlation coefficients of the University of Washington values of the relative atomic masses of deuterium, helium 4, and oxygen 16. The numbers in bold above the main diagonal are 10^{20} times the numerical values of the covariances; the numbers in bold on the main diagonal are 10^{20} times the numerical values of the variances; and the numbers in italics below the main diagonal are the correlation coefficients.

	$A_r(^2\mathrm{H})$	$A_r(^4\mathrm{He})$	$A_r(^{16}\mathrm{O})$
$A_r(^2\mathrm{H})$	**0.6400**	**0.0631**	**0.1276**
$A_r(^4\mathrm{He})$	*0.1271*	**0.3844**	**0.2023**
$A_r(^{16}\mathrm{O})$	*0.0886*	*0.1813*	**3.2400**

1. Theory relevant to the Rydberg constant

The theory of the energy levels of hydrogen and deuterium atoms relevant to the determination of the Rydberg constant R_∞, based on measurements of transition frequencies, is summarized in this section. Complete information necessary to determine the theoretical values of the relevant energy levels is provided, with an emphasis on results that have become available since the previous adjustment described in CODATA-02. For brevity, references to earlier work, which can be found in Eides *et al.* (2001b), for example, are not included here.

An important consideration is that the theoretical values of the energy levels of different states are highly correlated. For example, for S states, the uncalculated terms are primarily of the form of an unknown common constant divided by n^3. This fact is taken into account by calculating covariances between energy levels in addition to the uncertainties of the individual levels as discussed in detail in Sec. 4.1.1.12. In order to take these correlations into account, we distinguish between components of uncertainty that are proportional to $1/n^3$, denoted by u_0, and components of uncertainty that are essentially random functions of n, denoted by u_n.

The energy levels of hydrogen-like atoms are determined mainly by the Dirac eigenvalue, QED effects such as self energy and vacuum polarization, and nuclear size and motion effects, all of which are summarized in the following sections.

1. Dirac eigenvalue The binding energy of an electron in a static Coulomb field (the external electric field of a point nucleus of charge Ze with infinite mass) is determined predominantly by the Dirac eigenvalue

$$E_{\rm D} = f(n,j)\, m_{\rm e}c^2 \ , \tag{7}$$

where

$$f(n,j) = \left[1 + \frac{(Z\alpha)^2}{(n-\delta)^2}\right]^{-1/2} \ , \tag{8}$$

n and j are the principal quantum number and total angular momentum of the state, respectively, and

$$\delta = j + \tfrac{1}{2} - \left[(j+\tfrac{1}{2})^2 - (Z\alpha)^2\right]^{1/2} \ . \tag{9}$$

Although we are interested only in the case where the nuclear charge is e, we retain the atomic number Z in order to indicate the nature of various terms.

Corrections to the Dirac eigenvalue that approximately take into account the finite mass of the nucleus $m_{\rm N}$ are included in the more general expression for atomic energy levels, which replaces Eq. (7) (Barker and Glover, 1955; Sapirstein and Yennie, 1990):

$$\begin{aligned} E_M \;=\; & Mc^2 + [f(n,j)-1]m_{\rm r}c^2 - [f(n,j)-1]^2\frac{m_{\rm r}^2c^2}{2M} \\ & + \frac{1-\delta_{l0}}{\kappa(2l+1)}\frac{(Z\alpha)^4 m_{\rm r}^3c^2}{2n^3m_{\rm N}^2} + \cdots \ , \end{aligned} \tag{10}$$

where l is the nonrelativistic orbital angular momentum quantum number, κ is the angular-momentum-parity quantum number $\kappa = (-1)^{j-l+1/2}(j+\frac{1}{2})$, $M = m_{\rm e}+m_{\rm N}$, and $m_{\rm r} = m_{\rm e}m_{\rm N}/(m_{\rm e}+m_{\rm N})$ is the reduced mass.

2. Relativistic recoil Relativistic corrections to Eq. (10) associated with motion of the nucleus are considered relativistic-recoil corrections. The leading term, to lowest order in $Z\alpha$ and all orders in $m_{\rm e}/m_{\rm N}$, is (Erickson, 1977; Sapirstein and Yennie, 1990)

$$\begin{aligned} E_{\rm S} \;=\; & \frac{m_{\rm r}^3}{m_{\rm e}^2m_{\rm N}}\frac{(Z\alpha)^5}{\pi n^3}m_{\rm e}c^2 \\ & \times\Big\{\tfrac{1}{3}\delta_{l0}\ln(Z\alpha)^{-2} - \tfrac{8}{3}\ln k_0(n,l) - \tfrac{1}{9}\delta_{l0} - \tfrac{7}{3}a_n \\ & - \frac{2}{m_{\rm N}^2-m_{\rm e}^2}\delta_{l0}\left[m_{\rm N}^2\ln\left(\frac{m_{\rm e}}{m_{\rm r}}\right) - m_{\rm e}^2\ln\left(\frac{m_{\rm N}}{m_{\rm r}}\right)\right]\Big\}, \end{aligned} \tag{11}$$

where

$$\begin{aligned} a_n \;=\; & -2\left[\ln\left(\frac{2}{n}\right) + \sum_{i=1}^{n}\frac{1}{i} + 1 - \frac{1}{2n}\right]\delta_{l0} \\ & + \frac{1-\delta_{l0}}{l(l+1)(2l+1)} \ . \end{aligned} \tag{12}$$

To lowest order in the mass ratio, higher-order corrections in $Z\alpha$ have been extensively investigated; the contribution of the next two orders in $Z\alpha$ is

$$\begin{aligned} E_{\rm R} \;=\; & \frac{m_{\rm e}}{m_{\rm N}}\frac{(Z\alpha)^6}{n^3}m_{\rm e}c^2 \\ & \times\left[D_{60} + D_{72}Z\alpha\ln^2(Z\alpha)^{-2} + \cdots\right] \ , \end{aligned} \tag{13}$$

where for $n{\rm S}_{1/2}$ states (Eides and Grotch, 1997c; Pachucki and Grotch, 1995)

$$D_{60} \;=\; 4\ln 2 - \frac{7}{2} \tag{14}$$

and (Melnikov and Yelkhovsky, 1999; Pachucki and Karshenboim, 1999)

$$D_{72} \;=\; -\frac{11}{60\pi} \ , \tag{15}$$

and for states with $l \geq 1$ (Elkhovskiĭ, 1996; Golosov *et al.*, 1995; Jentschura and Pachucki, 1996)

$$D_{60} \;=\; \left[3 - \frac{l(l+1)}{n^2}\right]\frac{2}{(4l^2-1)(2l+3)} \ . \tag{16}$$

In Eq. (16) and subsequent discussion, the first subscript on the coefficient of a term refers to the power of $Z\alpha$ and the second subscript to the power of $\ln(Z\alpha)^{-2}$. The relativistic recoil correction used in the 2006 adjustment is based on Eqs. (11) to (16). The estimated uncertainty

for S states is taken to be 10 % of Eq. (13), and for states with $l \geq 1$, it is taken to be 1 % of that equation.

Numerical values for the complete contribution of Eq. (13) to all orders in $Z\alpha$ have been obtained by (Shabaev *et al.*, 1998). Although the difference between the all-orders calculation and the truncated power series for S states is about three times their quoted uncertainty, the two results are consistent within the uncertainty assigned here. The covariances of the theoretical values are calculated by assuming that the uncertainties are predominately due to uncalculated terms proportional to $(m_e/m_N)/n^3$.

3. Nuclear polarization Interactions between the atomic electron and the nucleus which involve excited states of the nucleus give rise to nuclear polarization corrections. For hydrogen, we use the result (Khriplovich and Sen'kov, 2000)

$$E_P(\mathrm{H}) = -0.070(13)h\frac{\delta_{l0}}{n^3}\ \mathrm{kHz}\ . \tag{17}$$

For deuterium, the sum of the proton polarizability, the neutron polarizability (Khriplovich and Sen'kov, 1998), and the dominant nuclear structure polarizability (Friar and Payne, 1997a), gives

$$E_P(\mathrm{D}) = -21.37(8)h\frac{\delta_{l0}}{n^3}\ \mathrm{kHz}\ . \tag{18}$$

We assume that this effect is negligible in states of higher l.

4. Self energy The one-photon electron self energy is given by

$$E^{(2)}_{\mathrm{SE}} = \frac{\alpha}{\pi}\frac{(Z\alpha)^4}{n^3}F(Z\alpha)\, m_e c^2\ , \tag{19}$$

where

$$\begin{aligned} F(Z\alpha) \;=\;& A_{41}\ln(Z\alpha)^{-2} + A_{40} + A_{50}\,(Z\alpha) \\ &+A_{62}\,(Z\alpha)^2\ln^2(Z\alpha)^{-2} + A_{61}\,(Z\alpha)^2\ln(Z\alpha)^{-2} \\ &+G_{\mathrm{SE}}(Z\alpha)\,(Z\alpha)^2\ . \end{aligned} \tag{20}$$

From Erickson and Yennie (1965) and earlier papers cited therein,

$$\begin{aligned} A_{41} &= \tfrac{4}{3}\,\delta_{l0} \\ A_{40} &= -\tfrac{4}{3}\ln k_0(n,l) + \tfrac{10}{9}\,\delta_{l0} - \frac{1}{2\kappa(2l+1)}(1-\delta_{l0}) \\ A_{50} &= \left(\tfrac{139}{32} - 2\ln 2\right)\pi\,\delta_{l0} \\ A_{62} &= -\delta_{l0} \\ A_{61} &= \left[4\left(1+\frac{1}{2}+\cdots+\frac{1}{n}\right) + \frac{28}{3}\ln 2 - 4\ln n \right.\\ &- \left.\frac{601}{180} - \frac{77}{45n^2}\right]\delta_{l0} + \left(1-\frac{1}{n^2}\right)\left(\frac{2}{15}+\frac{1}{3}\delta_{j\frac{1}{2}}\right)\delta_{l1} \\ &+ \frac{96n^2 - 32l(l+1)}{3n^2(2l-1)(2l)(2l+1)(2l+2)(2l+3)}(1-\delta_{l0})\ . \end{aligned} \tag{21}$$

TABLE 7 Bethe logarithms $\ln k_0(n,l)$ relevant to the determination of R_∞.

n	S	P	D
1	2.984 128 556		
2	2.811 769 893	−0.030 016 709	
3	2.767 663 612		
4	2.749 811 840	−0.041 954 895	−0.006 740 939
6	2.735 664 207		−0.008 147 204
8	2.730 267 261		−0.008 785 043
12			−0.009 342 954

The Bethe logarithms $\ln k_0(n,l)$ in Eq. (21) are given in Table 7 (Drake and Swainson, 1990).

The function $G_{\mathrm{SE}}(Z\alpha)$ in Eq. (20) is the higher-order contribution (in $Z\alpha$) to the self energy, and the values for $G_{\mathrm{SE}}(\alpha)$ that we use here are listed in Table 8. For S and P states with $n \leq 4$ the values in the table are based on direct numerical evaluations by Jentschura and Mohr (2004, 2005); Jentschura *et al.* (1999, 2001). The values of $G_{\mathrm{SE}}(\alpha)$ for the 6S and 8S states are based on the low-Z limit of this function $G_{\mathrm{SE}}(0) = A_{60}$ (Jentschura *et al.*, 2005a) together with extrapolations of the results of complete numerical calculations of $F(Z\alpha)$ [see Eq. (20)] at higher Z (Kotochigova and Mohr, 2006). The values of $G_{\mathrm{SE}}(\alpha)$ for D states are from Jentschura *et al.* (2005b)

The dominant effect of the finite mass of the nucleus on the self energy correction is taken into account by multiplying each term of $F(Z\alpha)$ by the reduced-mass factor $(m_r/m_e)^3$, except that the magnetic moment term $-1/[2\kappa(2l+1)]$ in A_{40} is instead multiplied by the factor $(m_r/m_e)^2$. In addition, the argument $(Z\alpha)^{-2}$ of the logarithms is replaced by $(m_e/m_r)(Z\alpha)^{-2}$ (Sapirstein and Yennie, 1990).

The uncertainty of the self energy contribution to a given level arises entirely from the uncertainty of $G_{\mathrm{SE}}(\alpha)$ listed in Table 8 and is taken to be entirely of type u_n.

5. Vacuum polarization The second-order vacuum-polarization level shift is

$$E^{(2)}_{\mathrm{VP}} = \frac{\alpha}{\pi}\frac{(Z\alpha)^4}{n^3}H(Z\alpha)\, m_e c^2\ , \tag{22}$$

where the function $H(Z\alpha)$ is divided into the part corresponding to the Uehling potential, denoted here by $H^{(1)}(Z\alpha)$, and the higher-order remainder $H^{(\mathrm{R})}(Z\alpha)$, where

$$\begin{aligned} H^{(1)}(Z\alpha) &= V_{40} + V_{50}\,(Z\alpha) + V_{61}\,(Z\alpha)^2\ln(Z\alpha)^{-2} \\ &\quad + G^{(1)}_{\mathrm{VP}}(Z\alpha)\,(Z\alpha)^2 \end{aligned} \tag{23}$$

$$H^{(\mathrm{R})}(Z\alpha) = G^{(\mathrm{R})}_{\mathrm{VP}}(Z\alpha)\,(Z\alpha)^2\ , \tag{24}$$

TABLE 8 Values of the function $G_{\rm SE}(\alpha)$.

n	$\mathrm{S}_{1/2}$	$\mathrm{P}_{1/2}$	$\mathrm{P}_{3/2}$	$\mathrm{D}_{3/2}$	$\mathrm{D}_{5/2}$
1	−30.290 240(20)				
2	−31.185 150(90)	−0.973 50(20)	−0.486 50(20)		
3	−31.047 70(90)				
4	−30.9120(40)	−1.1640(20)	−0.6090(20)		0.031 63(22)
6	−30.711(47)				0.034 17(26)
8	−30.606(47)			0.007 940(90)	0.034 84(22)
12				0.0080(20)	0.0350(30)

with

$$V_{40} = -\frac{4}{15}\,\delta_{l0}$$
$$V_{50} = \frac{5}{48}\pi\,\delta_{l0} \tag{25}$$
$$V_{61} = -\frac{2}{15}\,\delta_{l0}\,.$$

The part $G_{\rm VP}^{(1)}(Z\alpha)$ arises from the Uehling potential with values given in Table 9 (Kotochigova *et al.*, 2002; Mohr, 1982). The higher-order remainder $G_{\rm VP}^{(\rm R)}(Z\alpha)$ has been considered by Wichmann and Kroll, and the leading terms in powers of $Z\alpha$ are (Mohr, 1975, 1983; Wichmann and Kroll, 1956)

$$G_{\rm VP}^{(\rm R)}(Z\alpha) = \left(\frac{19}{45}-\frac{\pi^2}{27}\right)\delta_{l0} + \left(\frac{1}{16}-\frac{31\pi^2}{2880}\right)\pi(Z\alpha)\delta_{l0}+\cdots\,. \tag{26}$$

Higher-order terms omitted from Eq. (26) are negligible.

In a manner similar to that for the self energy, the leading effect of the finite mass of the nucleus is taken into account by multiplying Eq. (22) by the factor $(m_{\rm r}/m_{\rm e})^3$ and including a multiplicative factor of $(m_{\rm e}/m_{\rm r})$ in the argument of the logarithm in Eq. (23).

There is also a second-order vacuum polarization level shift due to the creation of virtual particle pairs other than the $\mathrm{e}^-\mathrm{e}^+$ pair. The predominant contribution for nS states arises from $\mu^+\mu^-$, with the leading term being (Eides and Shelyuto, 1995; Karshenboim, 1995)

$$E_{\mu\rm VP}^{(2)} = \frac{\alpha}{\pi}\frac{(Z\alpha)^4}{n^3}\left(-\frac{4}{15}\right)\left(\frac{m_{\rm e}}{m_\mu}\right)^2\left(\frac{m_{\rm r}}{m_{\rm e}}\right)^3 m_{\rm e}c^2\,. \tag{27}$$

The next order term in the contribution of muon vacuum polarization to nS states is of relative order $Z\alpha m_{\rm e}/m_\mu$ and is therefore negligible. The analogous contribution $E_{\tau\rm VP}^{(2)}$ from $\tau^+\tau^-$ (−18 Hz for the 1S state) is also negligible at the level of uncertainty of current interest.

For the hadronic vacuum polarization contribution, we take the result given by Friar *et al.* (1999) that utilizes all available $\mathrm{e}^+\mathrm{e}^-$ scattering data:

$$E_{\rm had\,VP}^{(2)} = 0.671(15)E_{\mu\rm VP}^{(2)}\,, \tag{28}$$

where the uncertainty is of type u_0.

The muonic and hadronic vacuum polarization contributions are negligible for P and D states.

6. Two-photon corrections Corrections from two virtual photons have been partially calculated as a power series in $Z\alpha$:

$$E^{(4)} = \left(\frac{\alpha}{\pi}\right)^2\frac{(Z\alpha)^4}{n^3}m_{\rm e}c^2F^{(4)}(Z\alpha)\,, \tag{29}$$

where

$$F^{(4)}(Z\alpha) = B_{40}+B_{50}\,(Z\alpha)+B_{63}\,(Z\alpha)^2\ln^3(Z\alpha)^{-2} +B_{62}\,(Z\alpha)^2\ln^2(Z\alpha)^{-2} +B_{61}\,(Z\alpha)^2\ln(Z\alpha)^{-2}+B_{60}\,(Z\alpha)^2 +\cdots\,. \tag{30}$$

The leading term B_{40} is well known:

$$B_{40} = \left[\frac{3\pi^2}{2}\ln 2-\frac{10\pi^2}{27}-\frac{2179}{648}-\frac{9}{4}\zeta(3)\right]\delta_{l0} + \left[\frac{\pi^2\ln 2}{2}-\frac{\pi^2}{12}-\frac{197}{144}-\frac{3\zeta(3)}{4}\right]\frac{1-\delta_{l0}}{\kappa(2l+1)}\,. \tag{31}$$

The second term is (Eides *et al.*, 1997; Eides and Shelyuto, 1995; Pachucki, 1993a, 1994)

$$B_{50} = -21.5561(31)\delta_{l0}\,, \tag{32}$$

and the next coefficient is (Karshenboim, 1993; Manohar and Stewart, 2000; Pachucki, 2001; Yerokhin, 2000)

$$B_{63} = -\frac{8}{27}\delta_{l0}\,. \tag{33}$$

For S states the coefficient B_{62} is given by

$$B_{62} = \frac{16}{9}\left[\frac{71}{60}-\ln 2+\gamma+\psi(n)-\ln n-\frac{1}{n}+\frac{1}{4n^2}\right]\,, \tag{34}$$

where $\gamma = 0.577...$ is Euler's constant and ψ is the psi function (Abramowitz and Stegun, 1965). The difference $B_{62}(1)-B_{62}(n)$ was calculated by Karshenboim (1996)

TABLE 9 Values of the function $G^{(1)}_{\rm VP}(\alpha)$.

n	$\mathrm{S}_{1/2}$	$\mathrm{P}_{1/2}$	$\mathrm{P}_{3/2}$	$\mathrm{D}_{3/2}$	$\mathrm{D}_{5/2}$
1	−0.618 724				
2	−0.808 872	−0.064 006	−0.014 132		
3	−0.814 530				
4	−0.806 579	−0.080 007	−0.017 666		−0.000 000
6	−0.791 450				−0.000 000
8	−0.781 197			−0.000 000	−0.000 000
12				−0.000 000	−0.000 000

and confirmed by Pachucki (2001) who also calculated the n-independent additive constant. For P states the calculated value is (Karshenboim, 1996)

$$B_{62} = \frac{4}{27}\frac{n^2-1}{n^2} \ . \tag{35}$$

This result has been confirmed by Jentschura and Nándori (2002) who also show that for D and higher angular momentum states $B_{62} = 0$.

Recent work has led to new results for B_{61} and higher-order coefficients. In Jentschura *et al.* (2005a) an additional state-independent contribution to the coefficient B_{61} for S states is given, which slightly differs (2 %) from the earlier result of Pachucki (2001) quoted in CODATA 2002. The revised coefficient for S states is

$$\begin{aligned} B_{61} &= \frac{413\,581}{64\,800} + \frac{4N(n\mathrm{S})}{3} + \frac{2027\pi^2}{864} - \frac{616\ \ln 2}{135} \\ &\quad - \frac{2\pi^2 \ln 2}{3} + \frac{40 \ln^2 2}{9} + \zeta(3) + \left(\frac{304}{135} - \frac{32\ \ln 2}{9}\right) \\ &\quad \times \left[\frac{3}{4} + \gamma + \psi(n) - \ln n - \frac{1}{n} + \frac{1}{4n^2}\right] \ , \end{aligned} \tag{36}$$

where ζ is the Riemann zeta function (Abramowitz and Stegun, 1965). The coefficients $N(n\mathrm{S})$ are listed in Table 10. The state-dependent part $B_{61}(n\mathrm{S}) - B_{61}(1\mathrm{S})$ was confirmed by Jentschura *et al.* (2005a) in their Eqs. (4.26) and (6.3). For higher-l states, B_{61} has been calculated by Jentschura *et al.* (2005a); for P states

$$B_{61}(n\mathrm{P}_{1/2}) = \frac{4}{3}N(n\mathrm{P}) + \frac{n^2-1}{n^2}\left(\frac{166}{405} - \frac{8}{27}\ \ln 2\right), \tag{37}$$

$$B_{61}(n\mathrm{P}_{3/2}) = \frac{4}{3}N(n\mathrm{P}) + \frac{n^2-1}{n^2}\left(\frac{31}{405} - \frac{8}{27}\ \ln 2\right), \tag{38}$$

and for D states

$$B_{61}(n\mathrm{D}) = 0 \ . \tag{39}$$

The coefficient B_{61} also vanishes for states with $l > 2$. The necessary values of $N(n\mathrm{P})$ are given in Eq. (17) of Jentschura (2003) and are listed in Table 10.

The next term is B_{60}, and recent work has also been done for this contribution. For S states, the state dependence is considered first, and is given by Czarnecki *et al.* (2005); Jentschura *et al.* (2005a)

$$B_{60}(n\mathrm{S}) - B_{60}(1\mathrm{S}) = b_{\rm L}(n\mathrm{S}) - b_{\rm L}(1\mathrm{S}) + A(n) \ , \tag{40}$$

TABLE 10 Values of N used in the 2006 adjustment

n	$N(n\mathrm{S})$	$N(n\mathrm{P})$
1	17.855 672 03(1)	
2	12.032 141 58(1)	0.003 300 635(1)
3	10.449 809(1)	
4	9.722 413(1)	−0.000 394 332(1)
6	9.031 832(1)	
8	8.697 639(1)	

where

$$\begin{aligned} A(n) &= \left(\frac{38}{45} - \frac{4}{3}\ln 2\right)[N(n\mathrm{S}) - N(1\mathrm{S})] \\ &\quad -\frac{337\,043}{129\,600} - \frac{94\,261}{21\,600\,n} + \frac{902\,609}{129\,600\,n^2} \\ &\quad + \left(\frac{4}{3} - \frac{16}{9\,n} + \frac{4}{9\,n^2}\right)\ln^2 2 \\ &\quad + \left(-\frac{76}{45} + \frac{304}{135\,n} - \frac{76}{135\,n^2}\right)\ln 2 \\ &\quad + \left(-\frac{53}{15} + \frac{35}{2\,n} - \frac{419}{30\,n^2}\right)\zeta(2)\ \ln 2 \\ &\quad + \left(\frac{28\,003}{10\,800} - \frac{11}{2\,n} + \frac{31\,397}{10\,800\,n^2}\right)\zeta(2) \\ &\quad + \left(\frac{53}{60} - \frac{35}{8\,n} + \frac{419}{120\,n^2}\right)\zeta(3) \\ &\quad + \left(\frac{37\,793}{10\,800} + \frac{16}{9}\ln^2 2 - \frac{304}{135}\ln 2 + 8\,\zeta(2)\ \ln 2 \right. \\ &\quad \left. -\frac{13}{3}\zeta(2) - 2\,\zeta(3)\right)[\gamma + \psi(n) - \ln n] \ . \end{aligned} \tag{41}$$

The term $A(n)$ makes a small contribution in the range 0.3 to 0.4 for the states under consideration.

The two-loop Bethe logarithms $b_{\rm L}$ in Eq. (40) are listed in Table 11. The values for $n = 1$ to 6 are from Jentschura (2004); Pachucki and Jentschura (2003), and the value at $n = 8$ is obtained by extrapolation of the calculated values from $n = 4$ to 6 [$b_{\rm L}(5\mathrm{S}) = -60.6(8)$] with a function of the form

$$b_{\rm L}(n\mathrm{S}) = a + \frac{b}{n} + \frac{c}{n(n+1)} \ , \tag{42}$$

which yields

$$b_{\rm L}(n{\rm S}) = -55.8 - \frac{24}{n} \ . \tag{43}$$

It happens that the fit gives $c = 0$. An estimate for B_{60} given by

$$B_{60}(n{\rm S}) = b_{\rm L}(n{\rm S}) + \frac{10}{9}N(n{\rm S}) + \cdots \tag{44}$$

was derived by Pachucki (2001). The dots represent uncalculated contributions at the relative level of 15 % (Pachucki and Jentschura, 2003). Equation (44) gives $B_{60}(1{\rm S}) = -61.6(9.2)$. However, more recently Yerokhin *et al.* (2003, 2005a,b, 2007) have calculated the 1S-state two-loop self energy correction for $Z \geq 10$. This is expected to give the main contribution to the higher-order two-loop correction. Their results extrapolated to $Z = 1$ yield a value for the contribution of all terms of order B_{60} or higher of $-127 \times (1 \pm 0.3)$, which corresponds to a value of roughly $B_{60} = -129(39)$, assuming a linear extrapolation from $Z = 1$ to $Z = 0$. This differs by about a factor of two from the result given by Eq. (44). In view of this difference between the two calculations, for the 2006 adjustment, we use the average of the two values with an uncertainty that is half the difference, which gives

$$B_{60}(1{\rm S}) = -95.3(0.3)(33.7) \ . \tag{45}$$

In Eq. (45), the first number in parentheses is the state-dependent uncertainty $u_n(B_{60})$ associated with the two-loop Bethe logarithm, and the second number in parentheses is the state-independent uncertainty $u_0(B_{60})$ that is common to all S-state values of B_{60}. Values of B_{60} for all relevant S-states are given in Table 11. For higher-l states, B_{60} has not been calculated, so we take it to be zero, with uncertainties $u_n[B_{60}(n{\rm P})] = 5.0$ and $u_n[B_{60}(n{\rm D})] = 1.0$. We assume that these uncertainties account for higher-order P and D state uncertainties as well. For S states, higher-order terms have been estimated by Jentschura *et al.* (2005a) with an effective potential model. They find that the next term has a coefficient of B_{72} and is state independent. We thus assume that the uncertainty $u_0[B_{60}(n{\rm S})]$ is sufficient to account for the uncertainty due to omitting such a term and higher-order state-independent terms. In addition, they find an estimate for the state dependence of the next term, given by

$$\begin{aligned} \Delta B_{71}(n{\rm S}) &= B_{71}(n{\rm S}) - B_{71}(1{\rm S}) = \pi\left(\frac{427}{36} - \frac{16}{3}\ln 2\right) \\ &\times \left[\frac{3}{4} - \frac{1}{n} + \frac{1}{4n^2} + \gamma + \psi(n) - \ln n\right] \end{aligned} \tag{46}$$

with a relative uncertainty of 50 %. We include this additional term, which is listed in Table 11, along with the estimated uncertainty $u_n(B_{71}) = B_{71}/2$.

The disagreement of the analytic and numerical calculations results in an uncertainty of the two-photon contribution that is larger than the estimated uncertainty used in the 2002 adjustment. As a result, the uncertainties of the recommended values of the Rydberg constant and proton and deuteron radii are slightly larger in the 2006 adjustment, although the 2002 and 2006 recommended values are consistent with each other. On the other hand, the uncertainty of the 2P state fine structure is reduced as a result of the new analytic calculations.

TABLE 11 Values of $b_{\rm L}$, B_{60}, and ΔB_{71} used in the 2006 adjustment

n	$b_{\rm L}(n{\rm S})$	$B_{60}(n{\rm S})$	$\Delta B_{71}(n{\rm S})$
1	−81.4(0.3)	−95.3(0.3)(33.7)	
2	−66.6(0.3)	−80.2(0.3)(33.7)	16(8)
3	−63.5(0.6)	−77.0(0.6)(33.7)	22(11)
4	−61.8(0.8)	−75.3(0.8)(33.7)	25(12)
6	−59.8(0.8)	−73.3(0.8)(33.7)	28(14)
8	−58.8(2.0)	−72.3(2.0)(33.7)	29(15)

As in the case of the order α self-energy and vacuum-polarization contributions, the dominant effect of the finite mass of the nucleus is taken into account by multiplying each term of the two-photon contribution by the reduced-mass factor $(m_{\rm r}/m_{\rm e})^3$, except that the magnetic moment term, the second line of Eq. (31), is instead multiplied by the factor $(m_{\rm r}/m_{\rm e})^2$. In addition, the argument $(Z\alpha)^{-2}$ of the logarithms is replaced by $(m_{\rm e}/m_{\rm r})(Z\alpha)^{-2}$.

7. Three-photon corrections The leading contribution from three virtual photons is expected to have the form

$$E^{(6)} = \left(\frac{\alpha}{\pi}\right)^3 \frac{(Z\alpha)^4}{n^3} m_{\rm e}c^2 \left[C_{40} + C_{50}(Z\alpha) + \cdots\right] \ , \tag{47}$$

in analogy with Eq. (29) for two photons. The leading term C_{40} is (Baikov and Broadhurst, 1995; Eides and Grotch, 1995a; Laporta and Remiddi, 1996; Melnikov and

van Ritbergen, 2000)

$$
\begin{aligned}
C_{40} \;=\; \bigg[& -\frac{568\,\mathrm{a}_4}{9} + \frac{85\,\zeta(5)}{24} \\
& -\frac{121\,\pi^2\,\zeta(3)}{72} - \frac{84\,071\,\zeta(3)}{2304} - \frac{71\,\ln^4 2}{27} \\
& -\frac{239\,\pi^2\,\ln^2 2}{135} + \frac{4787\,\pi^2\,\ln 2}{108} + \frac{1591\,\pi^4}{3240} \\
& -\frac{252\,251\,\pi^2}{9720} + \frac{679\,441}{93\,312}\bigg]\delta_{l0} \\
& + \bigg[-\frac{100\,\mathrm{a}_4}{3} + \frac{215\,\zeta(5)}{24} \\
& -\frac{83\,\pi^2\,\zeta(3)}{72} - \frac{139\,\zeta(3)}{18} - \frac{25\,\ln^4 2}{18} \\
& +\frac{25\,\pi^2\,\ln^2 2}{18} + \frac{298\,\pi^2\,\ln 2}{9} + \frac{239\,\pi^4}{2160} \\
& -\frac{17\,101\,\pi^2}{810} - \frac{28\,259}{5184}\bigg]\frac{1-\delta_{l0}}{\kappa(2l+1)} \,,
\end{aligned}
\tag{48}
$$

where $a_4 = \sum_{n=1}^{\infty} 1/(2^n\,n^4) = 0.517\,479\,061\ldots$. Higher-order terms have not been calculated, although partial results have been obtained (Eides and Shelyuto, 2007). An uncertainty is assigned by taking $u_0(C_{50}) = 30\delta_{l0}$ and $u_n(C_{63}) = 1$, where C_{63} is defined by the usual convention. The dominant effect of the finite mass of the nucleus is taken into account by multiplying the term proportional to δ_{l0} by the reduced-mass factor $(m_\mathrm{r}/m_\mathrm{e})^3$ and the term proportional to $1/[\kappa(2l+1)]$, the magnetic moment term, by the factor $(m_\mathrm{r}/m_\mathrm{e})^2$.

The contribution from four photons is expected to be of order

$$
\left(\frac{\alpha}{\pi}\right)^4 \frac{(Z\alpha)^4}{n^3} m_\mathrm{e}c^2 \,, \tag{49}
$$

which is about 10 Hz for the 1S state and is negligible at the level of uncertainty of current interest.

8. Finite nuclear size At low Z, the leading contribution due to the finite size of the nucleus is

$$
E_\mathrm{NS}^{(0)} = \mathcal{E}_\mathrm{NS}\delta_{l0} \,, \tag{50}
$$

with

$$
\mathcal{E}_\mathrm{NS} = \frac{2}{3}\left(\frac{m_\mathrm{r}}{m_\mathrm{e}}\right)^3 \frac{(Z\alpha)^2}{n^3}\, m_\mathrm{e}c^2 \left(\frac{Z\alpha R_\mathrm{N}}{\lambda\!\!\!^{-}_\mathrm{C}}\right)^2 \,, \tag{51}
$$

where R_N is the bound-state root-mean-square (rms) charge radius of the nucleus and $\lambda\!\!\!^{-}_\mathrm{C}$ is the Compton wavelength of the electron divided by 2π. The leading higher-order contributions have been examined by Friar (1979b); Friar and Payne (1997b); Karshenboim (1997) [see also Borisoglebsky and Trofimenko (1979); Mohr (1983)]. The expressions that we employ to evaluate the nuclear size correction are the same as those discussed in more detail in CODATA-98.

For S states the leading and next-order corrections are given by

$$
\begin{aligned}
E_\mathrm{NS} \;=\; & \mathcal{E}_\mathrm{NS}\bigg\{1 - C_\eta \frac{m_\mathrm{r}}{m_\mathrm{e}}\frac{R_\mathrm{N}}{\lambda\!\!\!^{-}_\mathrm{C}} Z\alpha - \bigg[\ln\left(\frac{m_\mathrm{r}}{m_\mathrm{e}}\frac{R_\mathrm{N}}{\lambda\!\!\!^{-}_\mathrm{C}}\frac{Z\alpha}{n}\right) \\
& + \psi(n) + \gamma - \frac{(5n+9)(n-1)}{4n^2} - C_\theta\bigg](Z\alpha)^2\bigg\} \,,
\end{aligned}
\tag{52}
$$

where C_η and C_θ are constants that depend on the details of the assumed charge distribution in the nucleus. The values used here are $C_\eta = 1.7(1)$ and $C_\theta = 0.47(4)$ for hydrogen or $C_\eta = 2.0(1)$ and $C_\theta = 0.38(4)$ for deuterium.

For the $\mathrm{P}_{1/2}$ states in hydrogen the leading term is

$$
E_\mathrm{NS} = \mathcal{E}_\mathrm{NS}\frac{(Z\alpha)^2(n^2-1)}{4n^2} \,. \tag{53}
$$

For $\mathrm{P}_{3/2}$ states and D states the nuclear-size contribution is negligible.

9. Nuclear-size correction to self energy and vacuum polarization For the self energy, the additional contribution due to the finite size of the nucleus is (Eides and Grotch, 1997b; Milstein *et al.*, 2002, 2003a; Pachucki, 1993b)

$$
E_\mathrm{NSE} = \left(4\ln 2 - \frac{23}{4}\right)\alpha(Z\alpha)\mathcal{E}_\mathrm{NS}\delta_{l0} \,, \tag{54}
$$

and for the vacuum polarization it is (Eides and Grotch, 1997b; Friar, 1979a, 1981; Hylton, 1985)

$$
E_\mathrm{NVP} = \frac{3}{4}\alpha(Z\alpha)\mathcal{E}_\mathrm{NS}\delta_{l0} \,. \tag{55}
$$

For the self-energy term, higher-order size corrections for S states (Milstein *et al.*, 2002) and size corrections for P states have been calculated (Jentschura, 2003; Milstein *et al.*, 2003b), but these corrections are negligible for the current work, and are not included. The D-state corrections are assumed to be negligible.

10. Radiative-recoil corrections The dominant effect of nuclear motion on the self energy and vacuum polarization has been taken into account by including appropriate reduced-mass factors. The additional contributions beyond this prescription are termed radiative-recoil effects with leading terms given by

$$
\begin{aligned}
E_\mathrm{RR} \;=\; & \frac{m_\mathrm{r}^3}{m_\mathrm{e}^2 m_\mathrm{N}}\frac{\alpha(Z\alpha)^5}{\pi^2\,n^3} m_\mathrm{e}c^2\delta_{l0} \\
& \times\bigg[6\,\zeta(3) - 2\,\pi^2\ln 2 + \frac{35\,\pi^2}{36} - \frac{448}{27} \\
& + \frac{2}{3}\pi(Z\alpha)\,\ln^2(Z\alpha)^{-2} + \cdots\bigg] \,.
\end{aligned}
\tag{56}
$$

The constant term in Eq. (56) is the sum of the analytic result for the electron-line contribution (Czarnecki and Melnikov, 2001; Eides *et al.*, 2001a) and the vacuum-polarization contribution (Eides and Grotch, 1995b; Pachucki, 1995). This term agrees with the numerical value (Pachucki, 1995) used in CODATA-98. The log-squared term has been calculated by Pachucki and Karshenboim (1999) and by Melnikov and Yelkhovsky (1999).

For the uncertainty, we take a term of order $(Z\alpha)\ln(Z\alpha)^{-2}$ relative to the square brackets in Eq. (56) with numerical coefficients 10 for u_0 and 1 for u_n. These coefficients are roughly what one would expect for the higher-order uncalculated terms. For higher-l states in the present evaluation, we assume that the uncertainties of the two- and three-photon corrections are much larger than the uncertainty of the radiative-recoil correction. Thus, we assign no uncertainty for the radiative-recoil correction for P and D states.

11. Nucleus self energy An additional contribution due to the self energy of the nucleus has been given by Pachucki (1995):

$$E_{\rm SEN} = \frac{4Z^2\alpha(Z\alpha)^4}{3\pi n^3}\frac{m_{\rm r}^3}{m_{\rm N}^2}c^2 \times\left[\ln\left(\frac{m_{\rm N}}{m_{\rm r}(Z\alpha)^2}\right)\delta_{l0} - \ln k_0(n,l)\right] . \quad (57)$$

This correction has also been examined by Eides *et al.* (2001b), who consider how it is modified by the effect of structure of the proton. The structure effect would lead to an additional model-dependent constant in the square brackets in Eq. (57).

To evaluate the nucleus self-energy correction, we use Eq. (57) and assign an uncertainty u_0 that corresponds to an additive constant of 0.5 in the square brackets for S states. For P and D states, the correction is small and its uncertainty, compared to other uncertainties, is negligible.

12. Total energy and uncertainty The total energy $E^X_{n{\rm L}j}$ of a particular level (where L = S, P, ... and X = H, D) is the sum of the various contributions listed above plus an additive correction $\delta^X_{n{\rm L}j}$ that accounts for the uncertainty in the theoretical expression for $E^X_{n{\rm L}j}$. Our theoretical estimate of the value of $\delta^X_{n{\rm L}j}$ for a particular level is zero with a standard uncertainty of $u(\delta^X_{n{\rm L}j})$ equal to the square root of the sum of the squares of the individual uncertainties of the contributions; as they are defined above, the contributions to the energy of a given level are independent. (Components of uncertainty associated with the fundamental constants are not included here, because they are determined by the least squares adjustment itself.) Thus, we have for the square of the uncertainty, or variance, of a particular level

$$u^2(\delta^X_{n{\rm L}j}) = \sum_i \frac{u^2_{0i}(XLj) + u^2_{ni}(XLj)}{n^6} , \quad (58)$$

where the individual values $u_{0i}(XLj)/n^3$ and $u_{ni}(XLj)/n^3$ are the components of uncertainty from each of the contributions, labeled by i, discussed above. (The factors of $1/n^3$ are isolated so that $u_{0i}(XLj)$ is explicitly independent of n.)

The covariance of any two δ's follows from Eq. (F7) of Appendix F of CODATA-98. For a given isotope X, we have

$$u(\delta^X_{n_1{\rm L}j}, \delta^X_{n_2{\rm L}j}) = \sum_i \frac{u^2_{0i}(XLj)}{(n_1n_2)^3} , \quad (59)$$

which follows from the fact that $u(u_{0i}, u_{ni}) = 0$ and $u(u_{n_1i}, u_{n_2i}) = 0$ for $n_1 \neq n_2$. We also set

$$u(\delta^X_{n_1{\rm L}_1j_1}, \delta^X_{n_2{\rm L}_2j_2}) = 0 , \quad (60)$$

if $\rm L_1 \neq L_2$ or $j_1 \neq j_2$.

For covariances between δ's for hydrogen and deuterium, we have for states of the same n

$$u(\delta^{\rm H}_{n{\rm L}j}, \delta^{\rm D}_{n{\rm L}j}) = \sum_{i=i_{\rm c}} \frac{u_{0i}({\rm HL}j)u_{0i}({\rm DL}j) + u_{ni}({\rm HL}j)u_{ni}({\rm DL}j)}{n^6} , \quad (61)$$

and for $n_1 \neq n_2$

$$u(\delta^{\rm H}_{n_1{\rm L}j}, \delta^{\rm D}_{n_2{\rm L}j}) = \sum_{i=i_{\rm c}} \frac{u_{0i}({\rm HL}j)u_{0i}({\rm DL}j)}{(n_1n_2)^3} , \quad (62)$$

where the summation is over the uncertainties common to hydrogen and deuterium. In most cases, the uncertainties can in fact be viewed as common except for a known multiplicative factor that contains all of the mass dependence. We assume

$$u(\delta^{\rm H}_{n_1{\rm L}_1j_1}, \delta^{\rm D}_{n_2{\rm L}_2j_2}) = 0 , \quad (63)$$

if $\rm L_1 \neq L_2$ or $j_1 \neq j_2$.

The values of $u(\delta^X_{n{\rm L}j})$ of interest for the 2006 adjustment are given in Table 28 of Sec. 12, and the non negligible covariances of the δ's are given in the form of correlation coefficients in Table 29 of that section. These coefficients are as large as 0.9999.

Since the transitions between levels are measured in frequency units (Hz), in order to apply the above equations for the energy level contributions we divide the theoretical expression for the energy difference ΔE of the transition by the Planck constant h to convert it to a frequency. Further, since we take the Rydberg constant $R_\infty = \alpha^2 m_{\rm e}c/2h$ (expressed in m^{-1}) rather than the electron mass $m_{\rm e}$ to be an adjusted constant, we replace the group of constants $\alpha^2 m_{\rm e}c^2/2h$ in $\Delta E/h$ by cR_∞.

13. Transition frequencies between levels with $\boldsymbol{n = 2}$ As an indication of the consistency of the theory summarized above and the experimental data, we list below values of the transition frequencies between levels with $n = 2$ in hydrogen. These results are based on values of the constants obtained in a variation of the 2006 least squares adjustment in which the measurements of the directly related transitions (items $A38$, $A39.1$, and $A39.2$ in Table 28) are not included, and the weakly coupled constants $A_{\rm r}({\rm e})$, $A_{\rm r}({\rm p})$, $A_{\rm r}({\rm d})$, and α, are assigned their 2006 adjusted values. The results are

$$\begin{aligned}
\nu_{\rm H}(2{\rm P}_{1/2} - 2{\rm S}_{1/2}) &= 1\,057\,843.9(2.5)\ {\rm kHz} \quad [2.3 \times 10^{-6}] \\
\nu_{\rm H}(2{\rm S}_{1/2} - 2{\rm P}_{3/2}) &= 9\,911\,197.6(2.5)\ {\rm kHz} \quad [2.5 \times 10^{-7}] \\
\nu_{\rm H}(2{\rm P}_{1/2} - 2{\rm P}_{3/2}) & \\
&= 10\,969\,041.475(99)\ {\rm kHz} \quad [9.0 \times 10^{-9}]\ , \qquad (64)
\end{aligned}$$

which agree well with the relevant experimental results of Table 28. Although the first two values in Eq. (64) have changed only slightly from the results of the 2002 adjustment, the third value, the fine-structure splitting, has an uncertainty that is almost an order-of-magnitude smaller than the 2002 value, due mainly to improvements in the theory of the two-photon correction.

A value of the fine structure constant α can be obtained from the data on the hydrogen and deuterium transitions. This is done by running a variation of the 2006 least-squares adjustment that includes all the transition frequency data in Table 28 and the 2006 adjusted values of $A_{\rm r}({\rm e})$, $A_{\rm r}({\rm p})$, and $A_{\rm r}({\rm d})$. The resulting value is

$$\alpha^{-1} = 137.036\,002(48) \qquad [3.5 \times 10^{-7}]\ , \qquad (65)$$

which is consistent with the 2006 recommended value, although substantially less accurate. This result is included in Table 34.

2. Experiments on hydrogen and deuterium

Table 12 summarizes the transition frequency data relevant to the determination of R_∞. With the exception of the first entry, which is the most recent result for the $1{\rm S}_{1/2} - 2{\rm S}_{1/2}$ transition frequency in hydrogen from the group at the Max-Planck-Institute für Quantenoptik (MPQ), Garching, Germany, all of these data are the same as those used in the 2002 adjustment. Since these data are reviewed in CODATA-98 or CODATA-02, they are not discussed here. For a brief discussion of data not included in Table 12, see Sec. II.B.3 of CODATA-02.

The new MPQ result,

$$\nu_{\rm H}(1{\rm S}_{1/2} - 2{\rm S}_{1/2}) = 2\,466\,061\,413\,187.074(34)\ {\rm kHz} \quad [1.4 \times 10^{-14}], \qquad (66)$$

was obtained in the course of an experiment to search for a temporal variation of the fine-structure constant α (Fischer *et al.*, 2004; Hänsch *et al.*, 2005; Poirier *et al.*, 2004; Udem, 2006). It is consistent with, but has a somewhat smaller uncertainty than, the previous result from the MPQ group, $\nu_{\rm H}(1{\rm S}_{1/2} - 2{\rm S}_{1/2}) = 2.466\,061\,413\,187.103(46)$ kHz $[1.9 \times 10^{-14}]$ (Niering *et al.*, 2000), which was the value used in the 2002 adjustment. The improvements that led to the reduction in uncertainty include a more stable external reference cavity for locking the 486 nm cw dye laser, thereby reducing its linewidth; an upgraded vacuum system that lowered the background gas pressure in the interaction region, thereby reducing the background gas pressure shift and its associated uncertainty; and a significantly reduced within-day Type A (*i.e.,* statistical) uncertainty due to the narrower laser linewidth and better signal-to-noise ratio.

The MPQ result in Eq. (66) and Table 12 for $\nu_{\rm H}(1{\rm S}_{1/2} - 2{\rm S}_{1/2})$ was provided by Udem (2006) of the MPQ group. It follows from the measured value $\nu_{\rm H}(1{\rm S}_{1/2} - 2{\rm S}_{1/2}) = 2.466\,061\,102\,474.851(34)$ kHz $[1.4 \times 10^{-14}]$ obtained for the $(1{\rm S},\ F = 1,\ m_F = \pm 1) \longrightarrow (2{\rm S},\ F' = 1,\ m'_F = \pm 1)$ transition frequency (Fischer *et al.*, 2004; Hänsch *et al.*, 2005; Poirier *et al.*, 2004) by using the well known 1S and 2S hyperfine splittings (Kolachevsky *et al.*, 2004; Ramsey, 1990) to convert it to the frequency corresponding to the hyperfine centroid.

3. Nuclear radii

The theoretical expressions for the finite nuclear size correction to the energy levels of hydrogen H and deuterium D (see Sec. 4.1.1.8) are functions of the bound-state nuclear rms charge radius for the proton, $R_{\rm p}$, and for the deuteron, $R_{\rm d}$. These values are treated as variables in the adjustment, so the transition frequency data, together with theory, determine values for the radii. The radii are also determined by elastic electron-proton scattering data in the case of $R_{\rm p}$ and from elastic electron-deuteron scattering data in the case of $R_{\rm d}$. These independently determined values are used as additional information on the radii. There have been no new results during the last 4 years and thus we take as input data for these two radii the values used in the 2002 adjustment:

$$R_{\rm p} = 0.895(18)\ {\rm fm} \qquad (67)$$

$$R_{\rm d} = 2.130(10)\ {\rm fm}. \qquad (68)$$

The result for $R_{\rm p}$ is due to Sick (2003) [see also Sick (2007b)]. The result for $R_{\rm d}$ is that given in Sec. III.B.7 of CODATA-98 based on the analysis of Sick and Trautmann (1998).

An experiment currently underway to measure the Lamb shift in muonic hydrogen may eventually provide a significantly improved value of $R_{\rm p}$ and hence an improved value of R_∞ (Nebel *et al.*, 2007).

TABLE 12 Summary of measured transition frequencies ν considered in the present work for the determination of the Rydberg constant R_∞ (H is hydrogen and D is deuterium).

Authors	Laboratory	Frequency interval(s)	Reported value ν/kHz	Rel. stand. uncert. u_r
(Fischer *et al.*, 2004)	MPQ	$\nu_H(1S_{1/2}-2S_{1/2})$	2 466 061 413 187.074(34)	1.4×10^{-14}
(Weitz *et al.*, 1995)	MPQ	$\nu_H(2S_{1/2}-4S_{1/2})-\frac{1}{4}\nu_H(1S_{1/2}-2S_{1/2})$	4 797 338(10)	2.1×10^{-6}
		$\nu_H(2S_{1/2}-4D_{5/2})-\frac{1}{4}\nu_H(1S_{1/2}-2S_{1/2})$	6 490 144(24)	3.7×10^{-6}
		$\nu_D(2S_{1/2}-4S_{1/2})-\frac{1}{4}\nu_D(1S_{1/2}-2S_{1/2})$	4 801 693(20)	4.2×10^{-6}
		$\nu_D(2S_{1/2}-4D_{5/2})-\frac{1}{4}\nu_D(1S_{1/2}-2S_{1/2})$	6 494 841(41)	6.3×10^{-6}
(Huber *et al.*, 1998)	MPQ	$\nu_D(1S_{1/2}-2S_{1/2})-\nu_H(1S_{1/2}-2S_{1/2})$	670 994 334.64(15)	2.2×10^{-10}
(de Beauvoir *et al.*, 1997)	LKB/SYRTE	$\nu_H(2S_{1/2}-8S_{1/2})$	770 649 350 012.0(8.6)	1.1×10^{-11}
		$\nu_H(2S_{1/2}-8D_{3/2})$	770 649 504 450.0(8.3)	1.1×10^{-11}
		$\nu_H(2S_{1/2}-8D_{5/2})$	770 649 561 584.2(6.4)	8.3×10^{-12}
		$\nu_D(2S_{1/2}-8S_{1/2})$	770 859 041 245.7(6.9)	8.9×10^{-12}
		$\nu_D(2S_{1/2}-8D_{3/2})$	770 859 195 701.8(6.3)	8.2×10^{-12}
		$\nu_D(2S_{1/2}-8D_{5/2})$	770 859 252 849.5(5.9)	7.7×10^{-12}
(Schwob *et al.*, 1999, 2001)	LKB/SYRTE	$\nu_H(2S_{1/2}-12D_{3/2})$	799 191 710 472.7(9.4)	1.2×10^{-11}
		$\nu_H(2S_{1/2}-12D_{5/2})$	799 191 727 403.7(7.0)	8.7×10^{-12}
		$\nu_D(2S_{1/2}-12D_{3/2})$	799 409 168 038.0(8.6)	1.1×10^{-11}
		$\nu_D(2S_{1/2}-12D_{5/2})$	799 409 184 966.8(6.8)	8.5×10^{-12}
(Bourzeix *et al.*, 1996)	LKB	$\nu_H(2S_{1/2}-6S_{1/2})-\frac{1}{4}\nu_H(1S_{1/2}-3S_{1/2})$	4 197 604(21)	4.9×10^{-6}
		$\nu_H(2S_{1/2}-6D_{5/2})-\frac{1}{4}\nu_H(1S_{1/2}-3S_{1/2})$	4 699 099(10)	2.2×10^{-6}
(Berkeland *et al.*, 1995)	Yale	$\nu_H(2S_{1/2}-4P_{1/2})-\frac{1}{4}\nu_H(1S_{1/2}-2S_{1/2})$	4 664 269(15)	3.2×10^{-6}
		$\nu_H(2S_{1/2}-4P_{3/2})-\frac{1}{4}\nu_H(1S_{1/2}-2S_{1/2})$	6 035 373(10)	1.7×10^{-6}
(Hagley and Pipkin, 1994)	Harvard	$\nu_H(2S_{1/2}-2P_{3/2})$	9 911 200(12)	1.2×10^{-6}
(Lundeen and Pipkin, 1986)	Harvard	$\nu_H(2P_{1/2}-2S_{1/2})$	1 057 845.0(9.0)	8.5×10^{-6}
(Newton *et al.*, 1979)	U. Sussex	$\nu_H(2P_{1/2}-2S_{1/2})$	1 057 862(20)	1.9×10^{-5}

2. Antiprotonic helium transition frequencies and $A_r(e)$

The antiprotonic helium atom is a three-body system consisting of a ^{4}He or ^{3}He nucleus, an antiproton, and an electron, denoted by $\bar{p}\,He^+$. Even though the Bohr radius for the antiproton in the field of the nucleus is about 1836 times smaller than the electron Bohr radius, in the highly-excited states studied experimentally, the average orbital radius of the antiproton is comparable to the electron Bohr radius, giving rise to relatively long-lived states. Also, for the high-l states studied, because of the vanishingly small overlap of the antiproton wavefunction with the helium nucleus, strong interactions between the antiproton and the nucleus are negligible.

One of the goals of the experiments is to measure the antiproton-electron mass ratio. However, since we assume that CPT is a valid symmetry, for the purpose of the least squares adjustment we take the masses of the antiproton and proton to be equal and use the data to determine the proton-electron mass ratio. Since the proton mass is known more accurately than the electron mass from other experiments, the mass ratio yields information primarily on the electron mass. Other experiments have demonstrated the equality of the charge-to-mass ratio of p and $\bar{p}$ to within 9 parts in 10^{11}; see Gabrielse (2006).

1. Theory relevant to antiprotonic helium

Calculations of transition frequencies of antiprotonic helium have been done by Kino *et al.* (2003) and by Korobov (2003, 2005). The uncertainties of calculations by Korobov (2005) are of the order of 1 MHz to 2 MHz, while the uncertainties and scatter relative to the experimental values of the results of Kino *et al.* (2003) are substantially larger, so we use the results Korobov (2005) in the 2006 adjustment. [See also the remarks in Hayano (2007) concerning the theory.]

The dominant contribution to the energy levels is just the non-relativistic solution of the Schrödinger equation for the three-body system together with relativistic and radiative corrections treated as perturbations. The non-relativistic levels are resonances, because the states can decay by the Auger effect in which the electron is ejected. Korobov (2005) calculates the nonrelativistic energy by using one of two formalisms, depending on whether the Auger rate is small or large. In the case where the rate is small, the Feshbach formalism is used with an optical potential. The optical potential is omitted in the calculation of higher-order relativistic and radiative corrections. For broad resonances with a higher Auger rate, the non-relativistic energies are calculated with the Complex Coordinate rotation method. In checking the convergence of

the nonrelativistic levels, attention was paid to the convergence of the expectation value of the the delta function operators used in the evaluation of the relativistic and radiative corrections.

Korobov (2005) evaluated the relativistic and radiative corrections as perturbations to the nonrelativistic levels, including relativistic corrections of order $\alpha^2 R_\infty$, anomalous magnetic moment corrections of order $\alpha^3 R_\infty$ and higher, one-loop self-energy and vacuum-polarization corrections of order $\alpha^3 R_\infty$, higher-order one-loop and leading two-loop corrections of order $\alpha^4 R_\infty$. Higher-order relativistic corrections of order $\alpha^4 R_\infty$ and radiative corrections of order $\alpha^5 R_\infty$ were estimated with effective operators. The uncertainty estimates account for uncalculated terms of order $\alpha^5 \ln\alpha\, R_\infty$.

Transition frequencies obtained by Korobov (2005, 2006) using the CODATA-02 values of the relevant constants are listed in Table 13 under the column header "Calculated Value." We denote these values of the frequencies by $\nu^{(0)}_{\bar{\mathrm{p}}\,\mathrm{He}}(n,l:n',l')$, where He is either $^3\mathrm{He}^+$ or $^4\mathrm{He}^+$. Also calculated are the leading-order changes in the theoretical values of the transition frequencies as a function of the relative changes in the mass ratios $A_\mathrm{r}(\bar{\mathrm{p}})/A_\mathrm{r}(\mathrm{e})$ and $A_\mathrm{r}(N)/A_\mathrm{r}(\bar{\mathrm{p}})$; here N is either $^3\mathrm{He}^{2+}$ or $^4\mathrm{He}^{2+}$. If we denote the transition frequencies as functions of these mass ratios by $\nu_{\bar{\mathrm{p}}\,\mathrm{He}}(n,l:n',l')$, then the changes can be written as

$$a_{\bar{\mathrm{p}}\,\mathrm{He}}(n,l:n',l') = \left(\frac{A_\mathrm{r}(\bar{\mathrm{p}})}{A_\mathrm{r}(\mathrm{e})}\right)^{(0)} \frac{\partial \nu_{\bar{\mathrm{p}}\,\mathrm{He}}(n,l:n',l')}{\partial\left(\frac{A_\mathrm{r}(\bar{\mathrm{p}})}{A_\mathrm{r}(\mathrm{e})}\right)} \tag{69}$$

$$b_{\bar{\mathrm{p}}\,\mathrm{He}}(n,l:n',l') = \left(\frac{A_\mathrm{r}(He)}{A_\mathrm{r}(\bar{\mathrm{p}})}\right)^{(0)} \frac{\partial \nu_{\bar{\mathrm{p}}\,\mathrm{He}}(n,l:n',l')}{\partial\left(\frac{A_\mathrm{r}(N)}{A_\mathrm{r}(\bar{\mathrm{p}})}\right)}\,. \tag{70}$$

Values of these derivatives, in units of $2cR_\infty$, are listed in Table 13 in the columns with the headers "a" and "b," respectively. The zero-order frequencies and the derivatives are used in the expression

$$\begin{aligned}\nu_{\bar{\mathrm{p}}\,\mathrm{He}}(n,l:n',l') &= \nu^{(0)}_{\bar{\mathrm{p}}\,\mathrm{He}}(n,l:n',l')\\ &+a_{\bar{\mathrm{p}}\,\mathrm{He}}(n,l:n',l')\left[\left(\frac{A_\mathrm{r}(\mathrm{e})}{A_\mathrm{r}(\bar{\mathrm{p}})}\right)^{(0)}\left(\frac{A_\mathrm{r}(\bar{\mathrm{p}})}{A_\mathrm{r}(\mathrm{e})}\right)-1\right]\\ &+b_{\bar{\mathrm{p}}\,\mathrm{He}}(n,l:n',l')\left[\left(\frac{A_\mathrm{r}(\bar{\mathrm{p}})}{A_\mathrm{r}(N)}\right)^{(0)}\left(\frac{A_\mathrm{r}(N)}{A_\mathrm{r}(\bar{\mathrm{p}})}\right)-1\right]+\dots,\end{aligned} \tag{71}$$

which provides a first-order approximation to the transition frequencies as a function of changes to the mass ratios. This expression is used to incorporate the experimental data and the calculations for the antiprotonic system as a function of the mass ratios into the least-squares adjustment. It should be noted that even though the mass ratios are the independent variables in Eq. (71) and the atomic relative masses $A_\mathrm{r}(\mathrm{e})$, $A_\mathrm{r}(\mathrm{p})$, and $A_\mathrm{r}(N)$ are the adjusted constants in the 2006 least-squares adjustment, the primary effect of including this data in the adjustment is on the electron relative atomic mass, because independent data in the adjustment constrains the proton and helium nuclei relative atomic masses with smaller uncertainties.

The uncertainties in the theoretical expressions for the transition frequencies are included in the adjustment as additive constants $\delta_{\bar{\mathrm{p}}\,\mathrm{He}}(n,l:n',l')$. Values for the theoretical uncertainties and covariances used in the adjustment are given in Sec. 12, Tables 32 and 33, respectively (Korobov, 2006).

2. Experiments on antiprotonic helium

Experimental work on antiprotonic helium began in the early 1990s and it continues to be an active field of research; a comprehensive review through 2000 is given by Yamazaki *et al.* (2002) and a very concise review through 2006 by Hayano (2007). The first measurements of $\bar{\mathrm{p}}\,\mathrm{He}^+$ transition frequencies at CERN with $u_\mathrm{r} < 10^{-6}$ were reported in 2001 (Hori *et al.*, 2001), improved results were reported in 2003 (Hori *et al.*, 2003), and transition frequencies with uncertainties sufficiently small that they can, together with the theory of the transitions, provide a competitive value of $A_\mathrm{r}(\mathrm{e})$, were reported in 2006 (Hori *et al.*, 2006).

The 12 transition frequencies—seven for ^{4}He and five for ^{3}He given by Hori *et al.* (2006)—which we take as input data in the 2006 adjustment are listed in column 2 of Table 13 with the corresponding transitions indicated in column 1. To reduce rounding errors, an additional digit for both the frequencies and their uncertainties as provided by Hori (2006) have been included. All twelve frequencies are correlated; their correlation coefficients, based on detailed uncertainty budgets for each, also provided by Hori (2006), are given in Table 33 in Sec 12.

In the current version of the experiment, 5.3 MeV antiprotons from the CERN Antiproton Decelerator (AD) are decelerated using a radio-frequency quadrupole decelerator (RFQD) to energies in the range 10 keV to 120 keV controlled by a dc potential bias on the RFQD's electrodes. The decelerated antiprotons, about 30 % of the antiprotons entering the RFQD, are then diverted to a low pressure cryogenic helium gas target at 10 K by an achromatic momentum analyzer, the purpose of which is to eliminate the large background that the remaining 70 % of undecelerated antiprotons would have produced.

About 3 % of the $\bar{\mathrm{p}}$ stopped in the target form $\bar{\mathrm{p}}\,\mathrm{He}^+$, in which a $\bar{\mathrm{p}}$ with large principle quantum number ($n \approx 38$) and angular momentum quantum number ($l \approx n$) circulates in a localized, nearly circular orbit around the He^{2+} nucleus while the electron occupies the distributed 1S state. These $\bar{\mathrm{p}}$ energy levels are metastable with lifetimes of several microseconds and de-excite radiatively. There are also short lived $\bar{\mathrm{p}}$ states with similar values of n and l but with lifetimes on the order of 10 ns and which

TABLE 13 Summary of data related to the determination of $A_r(e)$ from measurements on antiprotonic helium

Transition $(n,l) \to (n',l')$	Experimental Value (MHz)	Calculated Value (MHz)	a $(2cR_\infty)$	b $(2cR_\infty)$
$\bar{p}^4He^+$: $(32,31) \to (31,30)$	1 132 609 209(15)	1 132 609 223.50(82)	0.2179	0.0437
$\bar{p}^4He^+$: $(35,33) \to (34,32)$	804 633 059.0(8.2)	804 633 058.0(1.0)	0.1792	0.0360
$\bar{p}^4He^+$: $(36,34) \to (35,33)$	717 474 004(10)	717 474 001.1(1.2)	0.1691	0.0340
$\bar{p}^4He^+$: $(37,34) \to (36,33)$	636 878 139.4(7.7)	636 878 151.7(1.1)	0.1581	0.0317
$\bar{p}^4He^+$: $(39,35) \to (38,34)$	501 948 751.6(4.4)	501 948 755.4(1.2)	0.1376	0.0276
$\bar{p}^4He^+$: $(40,35) \to (39,34)$	445 608 557.6(6.3)	445 608 569.3(1.3)	0.1261	0.0253
$\bar{p}^4He^+$: $(37,35) \to (38,34)$	412 885 132.2(3.9)	412 885 132.8(1.8)	−0.1640	−0.0329
$\bar{p}^3He^+$: $(32,31) \to (31,30)$	1 043 128 608(13)	1 043 128 579.70(91)	0.2098	0.0524
$\bar{p}^3He^+$: $(34,32) \to (33,31)$	822 809 190(12)	822 809 170.9(1.1)	0.1841	0.0460
$\bar{p}^3He^+$: $(36,33) \to (35,32)$	646 180 434(12)	646 180 408.2(1.2)	0.1618	0.0405
$\bar{p}^3He^+$: $(38,34) \to (37,33)$	505 222 295.7(8.2)	505 222 280.9(1.1)	0.1398	0.0350
$\bar{p}^3He^+$: $(36,34) \to (37,33)$	414 147 507.8(4.0)	414 147 509.3(1.8)	−0.1664	−0.0416

de-excite by Auger transitions to form $\bar{p}\,He^{2+}$ hydrogen-like ions. These undergo Stark collisions, which cause the rapid annihilation of the $\bar{p}$ in the helium nucleus. The annihilation rate vs. time elapsed since $\bar{p}\,He^+$ formation, or delayed annihilation time spectrum (DATS), is measured using Cherenkov counters.

With the exception of the $(36,34) \to (35,33)$ transition frequency, all of the frequencies given in Table 13 were obtained by stimulating transitions from the $\bar{p}\,He^+$ metastable states with values of n and l indicated in column one on the left-hand side of the arrow to the short lived, Auger-decaying states with values of n and l indicated on the right-hand side of the arrow.

The megawatt-scale light intensities needed to induce the $\bar{p}\,He^+$ transitions, which cover the wavelength range 265 nm to 726 nm, can only be provided by a pulsed laser. Frequency and linewidth fluctuations and frequency calibration problems associated with such lasers were overcome by starting with a cw "seed" laser beam of frequency ν_{cw}, known with $u_r < 4 \times 10^{-10}$ through its stabilization by an optical frequency comb, and then amplifying the intensity of the laser beam by a factor of 10^6 in a cw pulse amplifier consisting of three dye cells pumped by a pulsed Nd:YAG laser. The 1 W seed laser beam with wavelength in the range 574 nm to 673 nm was obtained from a pumped cw dye laser, and the 1 W seed laser beam with wavelength in the range 723 nm to 941 nm was obtained from a pumped cw Ti:sapphire laser. The shorter wavelengths (265 nm to 471 nm) for inducing transitions were obtained by frequency doubling the amplifier output at 575 nm and 729 nm to 941 nm or by frequency tripling its 794 nm output. The frequency of the seed laser beam ν_{cw}, and thus the frequency ν_{pl} of the pulse amplified beam, was scanned over a range of ± 4 GHz around the $\bar{p}\,He^+$ transition frequency by changing the repetition frequency f_{rep} of the frequency comb.

The resonance curve for a transition was obtained by plotting the area under the resulting DATS peak vs. ν_{pl}. Because of the approximate 400 MHz Doppler broadening of the resonance due to the 10 K thermal motion of the $\bar{p}\,He^+$ atoms, a rather sophisticated theoretical line shape that takes into account many factors must be used to obtained the desired transition frequency.

Two other effects of major importance are the so-called chirp effect and linear shifts in the transition frequencies due to collisions between the $\bar{p}\,He^+$ and background helium atoms. The frequency ν_{pl} can deviate from ν_{cw} due to sudden changes in the index of refraction of the dye in the cells of the amplifier. This chirp, which can be expressed as $\Delta\nu_c(t) = \nu_{pl}(t) - \nu_{cw}$, can shift the measured $\bar{p}\,He^+$ frequencies from their actual values. Hori *et al.* (2006) eliminated this effect by measuring $\Delta\nu_c(t)$ in real time and applying a frequency shift to the seed laser, thereby canceling the dye-cell chirp. This effect is the predominant contributor to the correlations among the 12 transitions (Hori, 2006). The collisional shift was eliminated by measuring the frequencies of ten transitions in helium gas targets with helium atom densities ρ in the range $2 \times 10^{18}/cm^3$ to $3 \times 10^{21}/cm^3$ to determine $d\nu/d\rho$. The *in vacuo* ($\rho = 0$) values were obtained by applying a suitable correction in the range −14 MHz to 1 MHz to the initially measured frequencies obtained at $\rho \approx 2 \times 10^{18}/cm^3$.

In contrast to the other 11 transition frequencies in Table 13, which were obtained by inducing a transition from a long-lived, metastable state to a short-lived, Auger-decaying state, the $(36,34) \to (35,33)$ transition frequency was obtained by inducing a transition from the (36, 34) metastable state to the (35, 33) metastable state using three different lasers. This was done by first depopulating at time t_1 the (35, 33) metastable state by inducing the $(35,33) \to (34,32)$ metastable-state to short-lived-state transition, then at time t_2 inducing the $(36,34) \to (35,33)$ transition using the cw pulse-amplified laser, and then at time t_3 again inducing the $(35,33) \to (34,32)$ transition. The resonance curve for the $(36,34) \to (35,33)$ transition was obtained from the DATS peak resulting from this last induced transition.

The 4 MHz to 15 MHz standard uncertainties of the transition frequencies in Table 13 arise from the reso-

nance line shape fit (3 MHz to 13 MHz, statistical or Type A), not completely eliminating the chirp effect (2 MHz to 4 MHz, nonstatistical or Type B), collisional shifts (0.1 MHz to 2 MHz, Type B), and frequency doubling or tripling (1 MHz to 2 MHz, Type B).

3. Values of $A_r(e)$ inferred from antiprotonic helium

From the theory of the 12 antiprotonic transition frequencies discussed in Sec 4.2.1, the 2006 recommended values of the relative atomic masses of the proton, alpha particle (nucleus of the ^{4}He atom), and the helion (nucleus of the ^{3}He atom), $A_r(p)$, $A_r(\text{alpha})$, and $A_r(h)$, respectively, together with the 12 experimental values for these frequencies given in Table 13, we find the following three values for $A_r(e)$ from the seven $\bar{p}\,^4\text{He}^+$ frequencies alone, from the five $\bar{p}\,^3\text{He}^+$ frequencies alone, and from the 12 frequencies together:

$$A_r(e) = 0.000\,548\,579\,9103(12) \quad [2.1\times10^{-9}] \qquad (72)$$

$$A_r(e) = 0.000\,548\,579\,9053(15) \quad [2.7\times10^{-9}] \qquad (73)$$

$$A_r(e) = 0.000\,548\,579\,908\,81(91) \quad [1.7\times10^{-9}]\,. \qquad (74)$$

The separate inferred values from the $\bar{p}\,^4\text{He}^+$ and $\bar{p}\,^3\text{He}^+$ frequencies differ somewhat, but the value from all 12 frequencies not only agrees with the three other available results for $A_r(e)$ (see Table 36, Sec 12.1), but has a competitive level of uncertainty as well.

3. Hyperfine structure and fine structure

1. Hyperfine structure

Because the ground-state hyperfine transition frequencies $\Delta\nu_{\rm H}$, $\Delta\nu_{\rm Mu}$, and $\Delta\nu_{\rm Ps}$ of the comparatively simple atoms hydrogen, muonium, and positronium, respectively, are proportional to $\alpha^2 R_\infty c$, in principle a value of α can be obtained by equating an experimental value of one of these transition frequencies to its presumed readily calculable theoretical expression. However, currently only measurements of $\Delta\nu_{\rm Mu}$ and the theory of the muonium hyperfine structure have sufficiently small uncertainties to provide a useful result for the 2006 adjustment, and even in this case the result is not a competitive value of α, but rather the most accurate value of the electron-muon mass ratio m_e/m_μ. Indeed, we discuss the relevant experiments and theory in Sec.6.2.

Although the ground-state hyperfine transition frequency of hydrogen has long been of interest as a potential source of an accurate value of α because it is experimentally known with $u_r \approx 10^{-12}$ (Ramsey, 1990), the relative uncertainty of the theory is still of the order of 10^{-6}. Thus, $\Delta\nu_{\rm H}$ cannot yet provide a competitive value of the fine-structure constant. At present, the main sources of uncertainty in the theory arise from the internal structure of the proton, namely (i) the electric charge and magnetization densities of the proton, which are taken into account by calculating the proton's so-called Zemach radius; and (ii) the polarizability of the proton (that is, protonic excited states). For details of the progress made over the last four years in reducing the uncertainties from both sources, see (Carlson, 2007; Pachucki, 2007; Sick, 2007a) and the references cited therein. Because the muon is a structureless point-like particle, the theory of $\Delta\nu_{\rm Mu}$ is free of such uncertainties.

It is also not yet possible to obtain a useful value of α from $\Delta\nu_{\rm Ps}$ since the most accurate experimental result has $u_r = 3.6\times10^{-6}$ (Ritter *et al.*, 1984). The uncertainty of the theory of $\Delta\nu_{\rm Ps}$ is not significantly smaller and may in fact be larger (Adkins *et al.*, 2002; Penin, 2004).

2. Fine structure

As in the case of hyperfine splittings, fine-structure transition frequencies are proportional to $\alpha^2 R_\infty c$ and could be used to deduce a value of α. Some data related to the fine structure of hydrogen and deuterium are discussed in Sec. 4.1.2 in connection with the Rydberg constant. They are included in the adjustment because of their influence on the adjusted value of R_∞. However, the value of α that can be derived from these data is not competitive; see Eq. (65). See also Sec. III.B.3 of CODATA-02 for a discussion of why earlier fine structure-related results in H and D are not considered.

Because the transition frequencies corresponding to the differences in energy of the three 2^3P levels of ^{4}He can be both measured and calculated with reasonable accuracy, the fine structure of ^{4}He has long been viewed as a potential source of a reliable value of α. The three frequencies of interest are $\nu_{01} \approx 29.6$ GHz, $\nu_{12} \approx 2.29$ GHz, and $\nu_{02} \approx 31.9$ GHz, which correspond to the intervals $2^3\text{P}_1 - 2^3\text{P}_0, 2^3\text{P}_2 - 2^3\text{P}_1$, and $2^3\text{P}_2 - 2^3\text{P}_0$, respectively. The value with the smallest uncertainty for any of these frequencies was obtained at Harvard (Zelevinsky *et al.*, 2005):

$$\nu_{01} = 29\,616\,951.66(70)\ \text{kHz} \qquad [2.4\times10^{-8}]\,. \qquad (75)$$

It is consistent with the value of ν_{01} reported by George *et al.* (2001) with $u_r = 3.0\times10^{-8}$, and that reported by Giusfredi *et al.* (2005) with $u_r = 3.4\times10^{-8}$. If the theoretical expression for ν_{01} were exactly known, the weighted mean of the three results would yield a value of α with $u_r \approx 8\times10^{-9}$.

However, as discussed in CODATA-02, the theory of the 2^3P_J transition frequencies is far from satisfactory. First, different calculations disagree, and because of the considerable complexity of the calculations and the history of their evolution, there is general agreement that results that have not been confirmed by independent evaluation should be taken as tentative. Second, there are significant disagreements between theory and experiment. Recently, Pachucki (2006) has advanced the theory by calculating the complete contribution to the 2^3P_J fine-structure levels of order $m\alpha^7$ (or α^5 Ryd), with the final

theoretical result for ν_{01} being

$$\nu_{01} = 29\,616\,943.01(17)\ \text{kHz} \qquad [5.7\times10^{-9}]\ . \tag{76}$$

This value disagrees with the experimental value given in Eq (75) as well as with the theoretical value $\nu_{01} = 29\,616\,946.42(18)$ kHz $[6.1\times10^{-9}]$ given by Drake (2002), which also disagrees with the experimental value. These disagreements suggest that there is a problem with theory and/or experiment which must be resolved before a meaningful value of α can be obtained from the helium fine structure (Pachucki, 2006). Therefore, as in the 2002 adjustment, we do not include ^{4}He fine-structure data in the 2006 adjustment.

5. MAGNETIC MOMENT ANOMALIES AND g-FACTORS

In this section, the theory and experiment for the magnetic moment anomalies of the free electron and muon and the bound-state g-factor of the electron in hydrogenic carbon ($^{12}C^{5+}$) and in hydrogenic oxygen ($^{16}O^{7+}$) are reviewed.

The magnetic moment of any of the three charged leptons $\ell = \mathrm{e}, \mu, \tau$ is written as

$$\boldsymbol{\mu}_\ell = g_\ell \frac{e}{2m_\ell}\boldsymbol{s}\ , \tag{77}$$

where g_ℓ is the g-factor of the particle, m_ℓ is its mass, and $\boldsymbol{s}$ is its spin. In Eq. (77), e is the elementary charge and is positive. For the negatively charged leptons ℓ^-, g_ℓ is negative, and for the corresponding antiparticles ℓ^+, g_ℓ is positive. CPT invariance implies that the masses and absolute values of the g-factors are the same for each particle-antiparticle pair. These leptons have eigenvalues of spin projection $s_z = \pm\hbar/2$, and it is conventional to write, based on Eq. (77),

$$\mu_\ell = \frac{g_\ell}{2}\frac{e\hbar}{2m_\ell}\ , \tag{78}$$

where in the case of the electron, $\mu_\mathrm{B} = e\hbar/2m_\mathrm{e}$ is the Bohr magneton.

The free lepton magnetic moment anomaly a_ℓ is defined as

$$|g_\ell| \;=\; 2(1 + a_\ell)\ , \tag{79}$$

where $g_\mathrm{D} = -2$ is the value predicted by the free-electron Dirac equation. The theoretical expression for a_ℓ may be written as

$$a_\ell(\text{th}) = a_\ell(\text{QED}) + a_\ell(\text{weak}) + a_\ell(\text{had})\ , \tag{80}$$

where the terms denoted by QED, weak, and had account for the purely quantum electrodynamic, predominantly electroweak, and predominantly hadronic (that is, strong interaction) contributions to a_ℓ, respectively.

The QED contribution may be written as (Kinoshita *et al.*, 1990)

$$\begin{aligned} a_\ell(\text{QED}) \;=\; & A_1 + A_2(m_\ell/m_{\ell'}) + A_2(m_\ell/m_{\ell''}) \\ & + A_3(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})\ , \end{aligned} \tag{81}$$

where for the electron, $(\ell, \ell', \ell'') = (\mathrm{e}, \mu, \tau)$, and for the muon, $(\ell, \ell', \ell'') = (\mu, \mathrm{e}, \tau)$. The anomaly for the τ, which is poorly known experimentally (Yao *et al.*, 2006), is not considered here. For recent work on the theory of a_τ, see Eidelman and Passera (2007). In Eq. (81), the term A_1 is mass independent, and the mass dependence of A_2 and A_3 arises from vacuum polarization loops with lepton ℓ', ℓ'', or both. Each of the four terms on the right-hand side of Eq. (81) can be expressed as a power series in the fine-structure constant α:

$$\begin{aligned} A_i \;=\; & A_i^{(2)}\left(\frac{\alpha}{\pi}\right) + A_i^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + A_i^{(6)}\left(\frac{\alpha}{\pi}\right)^3 \\ & + A_i^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + A_i^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \cdots\ , \end{aligned} \tag{82}$$

where $A_2^{(2)} = A_3^{(2)} = A_3^{(4)} = 0$. Coefficients proportional to $(\alpha/\pi)^n$ are of order e^{2n} and are referred to as $2n$th-order coefficients.

The second-order coefficient is known exactly, and the fourth- and sixth-order coefficients are known analytically in terms of readily evaluated functions:

$$A_1^{(2)} \;=\; \tfrac{1}{2} \tag{83}$$

$$A_1^{(4)} \;=\; -0.328\,478\,965\,579\ldots \tag{84}$$

$$A_1^{(6)} \;=\; 1.181\,241\,456\ldots\ . \tag{85}$$

A total of 891 Feynman diagrams give rise to the mass-independent eighth-order coefficient $A_1^{(8)}$, and only a few of these are known analytically. However, in an effort that has its origins in the 1960s, Kinoshita and collaborators have calculated all of $A_1^{(8)}$ numerically, with the result of this ongoing project that was used in the 2006 adjustment being (Gabrielse *et al.*, 2006, 2007; Kinoshita and Nio, 2006a)

$$A_1^{(8)} = -1.7283(35)\ . \tag{86}$$

Work was done in the evaluation and checking of this coefficient in an effort to obtain a reliable quantitative result. A subset of 373 diagrams containing closed electron loops was verified by more than one independent formulation. The remaining 518 diagrams with no closed electron loops were formulated in only one way. As a check on this set, extensive cross checking was performed on the renormalization terms both among themselves and with lower-order diagrams that are known exactly (Kinoshita and Nio, 2006a) [see also Gabrielse *et al.* (2006, 2007)]. For the final numerical integrations, an adaptive-iterative Monte Carlo routine was used. A time-consuming part of the work was checking for round-off error in the integration.

TABLE 14 Summary of data related to magnetic moments of the electron and muon and inferred values of the fine structure constant. (The source data and not the inferred values given here are used in the adjustment.)

Quantity	Value	Relative standard uncertainty u_r	Identification	Sect. and Eq.
a_e	$1.159\,652\,1883(42) \times 10^{-3}$	3.7×10^{-9}	UWash-87	5.1.2.1 (102)
$\alpha^{-1}(a_e)$	$137.035\,998\,83(50)$	3.7×10^{-9}		5.1.3 (104)
a_e	$1.159\,652\,180\,85(76) \times 10^{-3}$	6.6×10^{-10}	HarvU-06	5.1.2.2 (103)
$\alpha^{-1}(a_e)$	$137.035\,999\,711(96)$	7.0×10^{-10}		5.1.3 (105)
$\overline{R}$	$0.003\,707\,2064(20)$	5.4×10^{-7}	BNL-06	5.2.2 (128)
a_μ	$1.165\,920\,93(63) \times 10^{-3}$	5.4×10^{-7}		5.2.2 (129)
$\alpha^{-1}(\overline{R})$	$137.035\,67(26)$	1.9×10^{-6}		5.2.2.1 (132)

The 0.0035 standard uncertainty of $A_1^{(8)}$ contributes a standard uncertainty to $a_e(\text{th})$ of $0.88 \times 10^{-10}\, a_e$, which is smaller than the uncertainty due to uncalculated higher-order contributions. Independent work is in progress on analytic calculations of eighth-order integrals. See, for example, Laporta (2001); Laporta *et al.* (2004); Mastrolia and Remiddi (2001).

Little is known about the tenth-order coefficient $A_1^{(10)}$ and higher-order coefficients, although Kinoshita *et al.* (2006) are starting the numerical evaluation of the 12 672 Feynman diagrams for this coefficient. To evaluate the contribution to the uncertainty of $a_e(\text{th})$ due to lack of knowledge of $A_1^{(10)}$, we follow CODATA-98 to obtain $A_1^{(10)} = 0.0(3.7)$. The 3.7 standard uncertainty of $A_1^{(10)}$ contributes a standard uncertainty component to $a_e(\text{th})$ of $2.2 \times 10^{-10}\, a_e$; the uncertainty contributions to $a_e(\text{th})$ from all other higher-order coefficients, which should be significantly smaller, are assumed to be negligible.

The 2006 least-squares adjustment was carried out using the theoretical results given above, including the value of $A_1^{(8)}$ given in Eq. (86). Well after the deadline for new data and the recommended values from the adjustment were made public (Mohr *et al.*, 2007), it was discovered by Aoyama *et al.* (2007) that 2 of the 47 integrals representing 518 QED diagrams that had not previously been confirmed independently required a corrected treatment of infrared divergences. The revised value they give is

$$A_1^{(8)} = -1.9144(35)\,, \tag{87}$$

although the new calculation is still tentative (Aoyama *et al.*, 2007). This result would lead to the value

$$\alpha^{-1} = 137.035\,999\,070(98) \qquad [7.1 \times 10^{-10}] \tag{88}$$

for the inverse fine-structure constant derived from the electron anomaly using the Harvard measurement result for a_e (Gabrielse *et al.*, 2006, 2007). This number is shifted down from the previous result by 641×10^{-9} and its uncertainty is increased from (96) to (98) (see Sec. 5.1.3), but it is still consistent with the values obtained from recoil experiments (see Table 26). If this result for $A_1^{(8)}$ had been used in the 2006 adjustment, the recommended value of the inverse fine-structure constant would differ by a similar, although slightly smaller, change. The effect on the muon anomaly theory is completely negligible.

The mass independent term A_1 contributes equally to the free electron and muon anomalies and the bound-electron g-factors. The mass-dependent terms are different for the electron and muon and are considered separately in the following. For the bound-electron g-factor, there are bound-state corrections in addition to the free-electron value of the g-factor, as discussed below.

1. Electron magnetic moment anomaly a_e and the fine-structure constant α

The combination of theory and experiment for the electron magnetic moment anomaly yields the value for the fine-structure constant α with the smallest estimated uncertainty (see Table 14 for the values corresponding to the 2006 adjustment).

1. Theory of a_e

The mass-dependent coefficients of interest and corresponding contributions to the theoretical value of the anomaly $a_e(\text{th})$, based on the 2006 recommended values of the mass ratios, are

$$A_2^{(4)}(m_e/m_\mu) = 5.197\,386\,78(26) \times 10^{-7} \rightarrow 24.182 \times 10^{-10} a_e \quad (89)$$

$$A_2^{(4)}(m_e/m_\tau) = 1.837\,63(60) \times 10^{-9} \rightarrow 0.085 \times 10^{-10} a_e \quad (90)$$

$$A_2^{(6)}(m_e/m_\mu) = -7.373\,941\,72(27) \times 10^{-6} \rightarrow -0.797 \times 10^{-10} a_e \quad (91)$$

$$A_2^{(6)}(m_e/m_\tau) = -6.5819(19) \times 10^{-8} \rightarrow -0.007 \times 10^{-10} a_e \,, \quad (92)$$

where the standard uncertainties of the coefficients are due to the uncertainties of the mass ratios, which are negligible. The contributions from $A_3^{(6)}(m_e/m_\mu, m_e/m_\tau)$ and all higher-order mass-dependent terms are negligible as well.

The value for $A_2^{(6)}(m_e/m_\mu)$ in Eq. (91) has been updated from the value in CODATA-02 and is in agreement with the result of Passera (2007) based on a calculation to all orders in the mass ratio. The change is given by the term

$$\frac{17\,x^6\,\zeta(3)}{36} - \frac{4381\,x^6\,\ln^2 x}{30240} + \frac{24761\,x^6\,\ln x}{158760} - \frac{13\,\pi^2\,x^6}{1344} - \frac{1840256147\,x^6}{3556224000} \,, \quad (93)$$

where $x = m_e/m_\mu$, which was not included in CODATA-02. The earlier result was based on Eq. (4) of Laporta and Remiddi (1993), which only included terms to order x^4. The additional term was kindly provided by Laporta and Remiddi (2006).

For the electroweak contribution we have

$$a_e(\text{weak}) = 0.029\,73(52) \times 10^{-12} = 0.2564(45) \times 10^{-10} a_e \,, \quad (94)$$

as calculated in CODATA-98 but with the current values of G_F and $\sin^2\theta_W$ (see Sec. 11.2).

The hadronic contribution is

$$a_e(\text{had}) = 1.682(20) \times 10^{-12} = 1.450(17) \times 10^{-9} a_e \,. \quad (95)$$

It is the sum of the following three contributions: $a_e^{(4)}(\text{had}) = 1.875(18) \times 10^{-12}$ obtained by Davier and Höcker (1998); $a_e^{(6a)}(\text{had}) = -0.225(5) \times 10^{-12}$ given by Krause (1997); and $a_e^{(\gamma\gamma)}(\text{had}) = 0.0318(58) \times 10^{-12}$ calculated by multiplying the corresponding result for the muon given in Sec. 5.2.1 by the factor $(m_e/m_\mu)^2$, since $a_e^{(\gamma\gamma)}(\text{had})$ is assumed to vary approximately as the square of the mass.

Since the dependence on α of any contribution other than $a_e(\text{QED})$ is negligible, the anomaly as a function of α is given by combining terms that have like powers of α/π to yield

$$a_e(\text{th}) = a_e(\text{QED}) + a_e(\text{weak}) + a_e(\text{had}) \,, \quad (96)$$

where

$$a_e(\text{QED}) = C_e^{(2)}\left(\frac{\alpha}{\pi}\right) + C_e^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_e^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_e^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_e^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \cdots \,, \quad (97)$$

with

$$\begin{aligned} C_e^{(2)} &= 0.5 \\ C_e^{(4)} &= -0.328\,478\,444\,00 \\ C_e^{(6)} &= 1.181\,234\,017 \\ C_e^{(8)} &= -1.7283(35) \\ C_e^{(10)} &= 0.0(3.7) \,, \end{aligned} \quad (98)$$

and where $a_e(\text{weak})$ and $a_e(\text{had})$ are given in Eqs. (94) and (95).

The standard uncertainty of $a_e(\text{th})$ from the uncertainties of the terms listed above, other than that due to α, is

$$u[a_e(\text{th})] = 0.27 \times 10^{-12} = 2.4 \times 10^{-10}\,a_e, \quad (99)$$

and is dominated by the uncertainty of the coefficient $C_e^{(10)}$.

For the purpose of the least-squares calculations carried out in Sec. 12.2, we define an additive correction δ_e to $a_e(\text{th})$ to account for the lack of exact knowledge of $a_e(\text{th})$, and hence the complete theoretical expression for the electron anomaly is

$$a_e(\alpha, \delta_e) = a_e(\text{th}) + \delta_e \,. \quad (100)$$

Our theoretical estimate of δ_e is zero and its standard uncertainty is $u[a_e(\text{th})]$:

$$\delta_e = 0.00(27) \times 10^{-12} \,. \quad (101)$$

2. Measurements of a_e

1. Measurement of a_e: University of Washington. The classic series of measurements of the electron and positron anomalies carried out at the University of Washington by Van Dyck *et al.* (1987) yield the value

$$a_e = 1.159\,652\,1883(42) \times 10^{-3} \quad [3.7 \times 10^{-9}] \,, \quad (102)$$

as discussed in CODATA-98. This result assumes that CPT invariance holds for the electron-positron system.

2. Measurement of a_e: Harvard University. A new determination of the electron anomaly using a cylindrical Penning trap has been carried out by Odom *et al.* (2006) at Harvard University, yielding the value

$$a_e = 1.159\,652\,180\,85(76) \times 10^{-3} \quad [6.6 \times 10^{-10}] \,, \tag{103}$$

which has an uncertainty that is nearly six times smaller than that of the University of Washington result.

As in the University of Washington experiment, the anomaly is obtained in essence from the relation $a_e = f_a/f_c$ by determining, in the same magnetic flux density B (about 5 T), the anomaly difference frequency $f_a = f_s - f_c$ and cyclotron frequency $f_c = eB/2\pi m_e$, where $f_s = g_e \mu_B B/h$ is the electron spin-flip (often called precession) frequency. The marked improvement achieved by the Harvard group, the culmination of a 20 year effort, is due in large part to the use of a cylindrical Penning trap with a resonant cavity that interacts with the trapped electron in a readily calculable way, and through its high Q resonances, significantly increases the lifetime of the electron in its lowest few energy states by inhibiting the decay of these states through spontaneous emission. Further, cooling the trap and its vacuum enclosure to 100 mK by means of a dilution refrigerator eliminates blackbody radiation that could excite the electron from these states.

The frequencies f_a and f_c are determined by applying quantum-jump spectroscopy (QJS) to transitions between the lowest spin ($m_s = \pm 1/2$) and cyclotron ($n = 0, 1, 2$) quantum states of the electron in the trap. (In QJS, the quantum jumps per attempt to drive them are measured as a function of drive frequency.) The transitions are induced by applying a signal of frequency $\approx f_a$ to trap electrodes or by transmitting microwaves of frequency $\approx f_c$ into the trap cavity. A change in the cyclotron or spin state of the electron is reflected in a shift in $\overline{\nu}_z$, the self excited axial oscillation of the electron. (The trap axis and B are in the z direction.) This oscillation induces a signal in a resonant circuit that is amplified and fed back to the trap to drive the oscillation. Saturated nickel rings surrounding the trap produce a small magnetic bottle that provides quantum nondemolition couplings of the spin and cyclotron energies to $\overline{\nu}_z$. Failure to resolve the cyclotron energy levels would result in an increase of uncertainty due to the leading relativistic correction $\delta/f_c \equiv hf_c/mc^2 \approx 10^{-9}$.

Another unique feature of the Harvard experiment is that the effect of the trap cavity modes on f_c, and hence on the measured value of a_e, are directly observed for the first time. The modes are quantitatively identified as the familiar transverse electric (TE) and transverse magnetic (TM) modes by observing the response of a cloud of electrons to an axial parametric drive, and, based on the work of Brown and Gabrielse (1986), the range of possible shifts of f_c for a cylindrical cavity with a $Q > 500$ as used in the Harvard experiment can be readily calculated. Two measurements of a_e were made: one, which resulted in the value of a_e given in Eq. (103), was at a value of B for which $f_c = 149$ GHz, far from modes that couple to the cyclotron motion; the other was at 146.8 GHz, close to mode TE_{127}. Within the calibration and identification uncertainties for the mode frequencies, very good agreement was found between the measured and predicted difference in the two values. Indeed, their weighted mean gives a value of a_e that is larger than the value in Eq. (103) by only the fractional amount 0.5×10^{-10}, with u_r slightly reduced to 6.5×10^{-10}.

The largest component of uncertainty, 5.2×10^{-10}, in the 6.6×10^{-10} u_r of the Harvard result for a_e arises from fitting the resonance line shapes for f_a and f_c obtained from the quantum jump spectroscopy data. It is based on the consistency of three different methods of extracting these frequencies from the line shapes. The method that yielded the best fits and which was used to obtain the reported value of a_e weights each drive frequency, spin flip or cyclotron, by the number of quantum jumps it produces, and then uses the weighted average of the resulting spin flip and cyclotron frequencies in the final calculation of a_e. Although the cavity shifts are well characterized, they account for the second largest fractional uncertainty component, 3.4×10^{-10}. The statistical (Type A) component, which is the next largest, is only 1.5×10^{-10}.

3. Values of α inferred from a_e

Equating the theoretical expression with the two experimental values of a_e given in Eqs. (102) and (103) yields

$$\alpha^{-1}(a_e) = 137.035\,998\,83(50) \quad [3.7 \times 10^{-9}] \tag{104}$$

from the University of Washington result and

$$\alpha^{-1}(a_e) = 137.035\,999\,711(96) \quad [7.0 \times 10^{-10}] \tag{105}$$

from the Harvard University result. The contribution of the uncertainty in a_e(th) to the relative uncertainty of either of these results is 2.4×10^{-10}. The value in Eq. (105) has the smallest uncertainty of any value of alpha currently available. Both values are included in Table 14.

2. Muon magnetic moment anomaly a_μ

Comparison of theory and experiment for the muon magnetic moment anomaly gives a test of the theory of the hadronic contributions, with the possibility of revealing physics beyond the Standard Model.

1. Theory of a_μ

The current theory of a_μ has been throughly reviewed in a number of recent publications by different authors,

including a book devoted solely to the subject; see, for example, Davier *et al.* (2006); Jegerlehner (2007); Melnikov and Vainshtein (2006); Miller *et al.* (2007); Passera (2005).

The relevant mass-dependent terms and the corresponding contributions to $a_\mu(\mathrm{th})$, based on the 2006 recommended values of the mass ratios, are

$$A_2^{(4)}(m_\mu/m_\mathrm{e}) = 1.094\,258\,3088(82) \to 506\,386.4561(38)\times 10^{-8}a_\mu\ , \tag{106}$$

$$A_2^{(4)}(m_\mu/m_\tau) = 0.000\,078\,064(25) \to 36.126(12)\times 10^{-8}a_\mu\ , \tag{107}$$

$$A_2^{(6)}(m_\mu/m_\mathrm{e}) = 22.868\,379\,97(19) \to 24\,581.766\,16(20)\times 10^{-8}a_\mu\ , \tag{108}$$

$$A_2^{(6)}(m_\mu/m_\tau) = 0.000\,360\,51(21) \to 0.387\,52(22)\times 10^{-8}a_\mu\ , \tag{109}$$

$$A_2^{(8)}(m_\mu/m_\mathrm{e}) = 132.6823(72) \to 331.288(18)\times 10^{-8}a_\mu\ , \tag{110}$$

$$A_2^{(10)}(m_\mu/m_\mathrm{e}) = 663(20) \tag{111}$$
$$\to 3.85(12)\times 10^{-8}a_\mu\ , \tag{112}$$

$$A_3^{(6)}(m_\mu/m_\mathrm{e}, m_\mu/m_\tau) = 0.000\,527\,66(17) \to 0.567\,20(18)\times 10^{-8}a_\mu\ , \tag{113}$$

$$A_3^{(8)}(m_\mu/m_\mathrm{e}, m_\mu/m_\tau) = 0.037\,594(83) \to 0.093\,87(21)\times 10^{-8}a_\mu\ . \tag{114}$$

These contributions and their uncertainties, as well as the values (including their uncertainties) of $a_\mu(\mathrm{weak})$ and $a_\mu(\mathrm{had})$ given below, should be compared with the $54\times 10^{-8}a_\mu$ standard uncertainty of the experimental value of a_μ from Brookhaven National Laboratory (BNL) (see next section).

Some of the above terms reflect the results of recent calculations. The value of $A_2^{(6)}(m_\mu/m_\tau)$ in Eq. (109) includes an additional contribution as discussed in connection with Eq. (91). The terms $A_2^{(8)}(m_\mu/m_\mathrm{e})$ and $A_3^{(8)}(m_\mu/m_\mathrm{e}, m_\mu/m_\tau)$ have been updated by Kinoshita and Nio (2004), with the resulting value for $A_2^{(8)}(m_\mu/m_\mathrm{e})$ in Eq. (110) differing from the previous value of 127.50(41) due to the elimination of various problems with the earlier calculations, and the resulting value for $A_3^{(8)}(m_\mu/m_\mathrm{e}, m_\mu/m_\tau)$ in Eq. (114) differing from the previous value of 0.079(3), because diagrams that were thought to be negligible do in fact contribute to the result. Further, the value for $A_2^{(10)}(m_\mu/m_\mathrm{e})$ in Eq. (111) from Kinoshita and Nio (2006b) replaces the previous value, 930(170). These authors believe that their result, obtained from the numerical evaluation of all of the integrals from 17 key subsets of Feynman diagrams, accounts for the leading contributions to $A_2^{(10)}(m_\mu/m_\mathrm{e})$, and the work of Kataev (2006), based on the so-called renormalization group-inspired scheme-invariant approach, strongly supports this view.

The electroweak contribution to $a_\mu(\mathrm{th})$ is taken to be

$$a_\mu(\mathrm{weak}) = 154(2)\times 10^{-11}\ , \tag{115}$$

as given by Czarnecki *et al.* (2003, 2006). This value was used in the 2002 adjustment and is discussed in CODATA-02.

The hadronic contribution to $a_\mu(\mathrm{th})$ may be written as

$$a_\mu(\mathrm{had}) = a_\mu^{(4)}(\mathrm{had}) + a_\mu^{(6a)}(\mathrm{had}) + a_\mu^{(\gamma\gamma)}(\mathrm{had}) + \cdots\ , \tag{116}$$

where $a_\mu^{(4)}(\mathrm{had})$ and $a_\mu^{(6a)}(\mathrm{had})$ arise from hadronic vacuum polarization and are of order $(\alpha/\pi)^2$ and $(\alpha/\pi)^3$, respectively; and $a_\mu^{(\gamma\gamma)}(\mathrm{had})$, which arises from hadronic light-by-light vacuum polarization, is also of order $(\alpha/\pi)^3$.

Values of $a_\mu^{(4)}(\mathrm{had})$ are obtained from calculations that evaluate dispersion integrals over measured cross sections for the scattering of $\mathrm{e}^+\mathrm{e}^-$ into hadrons. In addition, in some such calculations, data on decays of the τ into hadrons is used to replace the $\mathrm{e}^+\mathrm{e}^-$ data in certain parts of the calculation. In the 2002 adjustment, results from both types of calculation were averaged to obtain a value that would be representative of both approaches.

There have been improvements in the calculations that use only $\mathrm{e}^+\mathrm{e}^-$ data with the addition of new data from the detectors CMD-2 at Novosibirsk, KLOE at Frascati, BaBar at the Stanford Linear Accelerator Center, and corrected data from the detector SND at Novosibirsk (Davier, 2007; Hagiwara *et al.*, 2007; Jegerlehner, 2007). However, there is a persistent disagreement between the results that include the τ decay data and those that use only $\mathrm{e}^+\mathrm{e}^-$ data. In view of the improvements in the results based solely on $\mathrm{e}^+\mathrm{e}^-$ data and the unresolved questions concerning the assumptions required to incorporate the τ data into the analysis (Davier, 2007; Davier *et al.*, 2006; Melnikov and Vainshtein, 2006), we use in the 2006 adjustment results based solely on $\mathrm{e}^+\mathrm{e}^-$ data. The value employed is

$$a_\mu^{(4)}(\mathrm{had}) = 690(21)\times 10^{-10}\ , \tag{117}$$

which is the unweighted mean of the values $a_\mu^{(4)}(\mathrm{had}) = 689.4(4.6)\times 10^{-10}$ (Hagiwara *et al.*, 2007) and $a_\mu^{(4)}(\mathrm{had}) = 690.9(4.4)\times 10^{-10}$ (Davier, 2007). The uncertainty assigned the value of $a_\mu^{(4)}(\mathrm{had})$, as expressed in Eq (117), is essentially the difference between the values that include τ data and those that do not. In particular,

the result that includes τ data that we use to estimate the uncertainty is $711.0(5.8) \times 10^{-11}$ from Davier *et al.* (2003); the value of $a_{\mu}^{(4)}(\text{had})$ used in the 2002 adjustment was based in part on this result. Although there is the smaller value $701.8(5.8) \times 10^{-11}$ from Trocóniz and Ynduráin (2005), we use only the larger value in order to obtain an uncertainty that covers the possibility of physics beyond the Standard Model not included in the calculation of $a_{\mu}(\text{th})$. Other, mostly older results for $a_{\mu}^{(4)}(\text{had})$, but which in general agree with the two values we have averaged, are summarized in Table III of Jegerlehner (2007).

For the second term in Eq. (116), we employ the value

$$a_{\mu}^{(6a)}(\text{had}) = -97.90(95) \times 10^{-11} \tag{118}$$

calculated by Hagiwara *et al.* (2004), which was also used in the 2002 adjustment.

The light-by-light contribution in Eq. (116) has been calculated by Melnikov and Vainshtein (2004, 2006), who obtain the value

$$a_{\mu}^{(\gamma\gamma)}(\text{had}) = 136(25) \times 10^{-11} \; . \tag{119}$$

It is somewhat larger than earlier results, because it includes short distance constraints imposed by quantum chromodynamics (QCD) that were not included in the previous calculations. It is consistent with the 95 % confidence limit upper bound of 159×10^{-11} for $a_{\mu}^{(\gamma\gamma)}(\text{had})$ obtained by Erler and Sánchez (2006), the value $110(40) \times 10^{-11}$ proposed by Bijnens and Prades (2007), and the value $125(35) \times 10^{-11}$ suggested by Davier and Marciano (2004).

The total hadronic contribution is

$$\begin{aligned} a_{\mu}(\text{had}) &= 694(21) \times 10^{-10} \\ &= 595(18) \times 10^{-7} a_{\mu} \; . \end{aligned} \tag{120}$$

Combining terms in $a_{\mu}(\text{QED})$ that have like powers of α/π, we summarize the theory of a_{μ} as follows:

$$a_{\mu}(\text{th}) = a_{\mu}(\text{QED}) + a_{\mu}(\text{weak}) + a_{\mu}(\text{had}) \; , \tag{121}$$

where

$$\begin{aligned} a_{\mu}(\text{QED}) &= C_{\mu}^{(2)}\left(\frac{\alpha}{\pi}\right) + C_{\mu}^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_{\mu}^{(6)}\left(\frac{\alpha}{\pi}\right)^3 \\ &\quad + C_{\mu}^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_{\mu}^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \cdots \; , \end{aligned} \tag{122}$$

with

$$\begin{aligned} C_{\mu}^{(2)} &= 0.5 \\ C_{\mu}^{(4)} &= 0.765\,857\,408(27) \\ C_{\mu}^{(6)} &= 24.050\,509\,59(42) \\ C_{\mu}^{(8)} &= 130.9916(80) \\ C_{\mu}^{(10)} &= 663(20) \; , \end{aligned} \tag{123}$$

and where $a_{\mu}(\text{weak})$ and $a_{\mu}(\text{had})$ are as given in Eqs. (115) and (120). The standard uncertainty of $a_{\mu}(\text{th})$ from the uncertainties of the terms listed above, other than that due to α, is

$$u[a_{\mu}(\text{th})] = 2.1 \times 10^{-9} = 1.8 \times 10^{-6}\, a_{\mu}, \tag{124}$$

and is primarily due to the uncertainty of $a_{\mu}(\text{had})$.

For the purpose of the least-squares calculations carried out in Sec. 12.2, we define an additive correction δ_{μ} to $a_{\mu}(\text{th})$ to account for the lack of exact knowledge of $a_{\mu}(\text{th})$, and hence the complete theoretical expression for the muon anomaly is

$$a_{\mu}(\alpha, \delta_{\mu}) = a_{\mu}(\text{th}) + \delta_{\mu} \; . \tag{125}$$

Our theoretical estimate of δ_{μ} is zero and its standard uncertainty is $u[a_{\mu}(\text{th})]$:

$$\delta_{\mu} = 0.0(2.1) \times 10^{-9} \; . \tag{126}$$

Although $a_{\mu}(\text{th})$ and $a_{\text{e}}(\text{th})$ have some common components of uncertainty, the covariance of δ_{μ} and δ_{e} is negligible.

2. Measurement of a_{μ}: Brookhaven.

Experiment E821 at Brookhaven National Laboratory (BNL), Upton, New York, was initiated by the Muon $g-2$ Collaboration in the early-1980s with the goal of measuring a_{μ} with a significantly smaller uncertainty than $u_{\text{r}} = 7.2 \times 10^{-6}$. This is the uncertainty achieved in the third $g-2$ experiment carried out at the European Organization for Nuclear Research (CERN), Geneva, Switzerland, in the mid-1970s using both positive and negative muons and which was the culmination of nearly 20 years of effort (Bailey *et al.*, 1979).

The basic principle of the experimental determination of a_{μ} is similar to that used to determine a_{e} and involves measuring the anomaly difference frequency $f_{\text{a}} = f_{\text{s}} - f_{\text{c}}$, where $f_{\text{s}} = |g_{\mu}|(e\hbar/2m_{\mu})B/h$ is the muon spin-flip (often called precession) frequency in the applied magnetic flux density B and where $f_{\text{c}} = eB/2\pi m_{\mu}$ is the corresponding muon cyclotron frequency. However, instead of eliminating B by measuring f_{c} as is done for the electron, B is determined from proton nuclear magnetic resonance (NMR) measurements. As a consequence, the value of μ_{μ}/μ_{p} is required to deduce the value of a_{μ} from the data. The relevant equation is

$$a_{\mu} = \frac{\overline{R}}{|\mu_{\mu}/\mu_{\text{p}}| - \overline{R}} \; , \tag{127}$$

where $\overline{R} = f_{\text{a}}/\overline{f}_{\text{p}}$, and $\overline{f}_{\text{p}}$ is the free proton NMR frequency corresponding to the average flux density seen by the muons in their orbits in the muon storage ring used in the experiment. (Of course, in the corresponding experiment for the electron, a Penning trap is employed rather than a storage ring.)

The BNL a_μ experiment was discussed in both CODATA-98 and CODATA-02. In the 1998 adjustment, the CERN final result for $\overline{R}$ with $u_r = 7.2\times10^{-6}$, and the first BNL result for $\overline{R}$, obtained from the 1997 engineering run using positive muons and with $u_r = 13 \times 10^{-6}$, were taken as input data. By the time of the 2002 adjustment, the BNL experiment had progressed to the point where the CERN result was no longer competitive, and the input datum used was the BNL mean value of $\overline{R}$ with $u_r = 6.7 \times 10^{-7}$ obtained from the 1998, 1999, and 2000 runs using μ^+. The final run of the BNL E821 experiment was carried out in 2001 with μ^- and achieved an uncertainty for $\overline{R}$ of $u_r = 7.0 \times 10^{-7}$, but the result only became available in early 2004, well after the closing date of the 2002 adjustment.

Based on the data obtained in all five runs and assuming CPT invariance, an assumption justified by the consistency of the values of $\overline{R}$ obtained from either μ^+ or μ^-, the final report on the E821 experiment gives as the final value of $\overline{R}$ (Bennett *et al.*, 2006) [see also (Miller *et al.*, 2007)]

$$\overline{R} \;=\; 0.003\,707\,2064(20) \quad [5.4 \times 10^{-7}] \,, \qquad (128)$$

which we take as an input datum in the 2006 adjustment. A new BNL experiment to obtain a value of $\overline{R}$ with a smaller uncertainty is under discussion (Hertzog, 2007).

The experimental value of a_μ implied by this value of $\overline{R}$ is, from Eq. (127) and the 2006 recommended value of μ_μ/μ_p, the uncertainty of which is inconsequential in this application,

$$a_\mu(\mathrm{exp}) = 1.165\,920\,93(63) \times 10^{-3} \quad [5.4 \times 10^{-7}] \,. \quad (129)$$

Further, with the aid of Eq. (217) in Sec. 6.2, Eq. (127) can be written as

$$\overline{R} = -\frac{a_\mu(\alpha, \delta_\mu)}{1 + a_e(\alpha, \delta_e)} \frac{m_e}{m_\mu} \frac{\mu_{e^-}}{\mu_p} \,, \qquad (130)$$

where we have used the relations $g_e = -2(1 + a_e)$ and $g_\mu = -2(1 + a_\mu)$ and replaced a_e and a_μ with their complete theoretical expressions $a_e(\alpha, \delta_e)$ and $a_\mu(\alpha, \delta_\mu)$, which are discussed in Sec. 5.1.1 and Sec. 5.2.1, respectively. Equation (130) is, in fact, the observational equation for the input datum $\overline{R}$.

1. Theoretical value of a_μ and inferred value of α Evaluation of the theoretical expression for a_μ in Eq. (121) with the 2006 recommended value of α, the uncertainty of which is negligible in this context, yields

$$a_\mu(\mathrm{th}) = 1.165\,9181(21) \times 10^{-3} \quad [1.8 \times 10^{-6}] \,, \quad (131)$$

which may be compared to the value in Eq. (129) deduced from the BNL result for $\overline{R}$ given in Eq. (128). The experimental value exceeds the theoretical value by $1.3\,u_{\mathrm{diff}}$, where u_{diff} is the standard uncertainty of the difference. It should be recognized, however, that this reasonable agreement is a consequence of the comparatively large uncertainty we have assigned to $a_\mu^{(4)}(\mathrm{had})$ [see Eq. (124)]. If the result for $a_\mu^{(4)}(\mathrm{had})$ that includes tau data were ignored and the uncertainty of $a_\mu^{(4)}(\mathrm{had})$ were based on the estimated uncertainties of the calculated values using only e^+e^- data, then the experimental value would exceed the theoretical value by $3.5\,u_{\mathrm{diff}}$. This inconsistency is well known to the high-energy physics community and is of considerable interest because it may be an indication of “New Physics” beyond the Standard Model, such as supersymmetry (Stöckinger, 2007).

One might ask, why include the theoretical value for a_μ in the 2006 adjustment given its current problems? By retaining the theoretical expression with an increased uncertainty, we ensure that the 2006 recommended value of a_μ reflects, even though with a comparatively small weight, the existence of the theoretical value.

The consistency between theory and experiment may also be examined by considering the value of α obtained by equating the theoretical expression for a_μ with the BNL experimental value, as was done for a_e in Sec. 5.1.3. The result is

$$\alpha^{-1} = 137.035\,67(26) \quad [1.9 \times 10^{-6}] \,, \qquad (132)$$

which is the value included in Table 14.

3. Bound electron g-factor in $^{12}C^{5+}$ and in $^{16}O^{7+}$ and $A_r(e)$

Precise measurements and theoretical calculations of the g-factor of the electron in hydrogenic ^{12}C and in hydrogenic ^{16}O lead to values of $A_r(e)$ that contribute to the determination of the 2006 recommended value of this important constant.

For a ground-state hydrogenic ion $^AX^{(Z-1)+}$ with mass number A, atomic number (proton number) Z, nuclear spin quantum number $i = 0$, and g-factor $g_{e^-}(^AX^{(Z-1)+})$ in an applied magnetic flux density B, the ratio of the electron's spin-flip (often called precession) frequency $f_s = |g_{e^-}(^AX^{(Z-1)+})|(e\hbar/2m_e)B/h$ to the cyclotron frequency of the ion $f_c = (Z-1)eB/2\pi m(^AX^{(Z-1)+})$ in the same magnetic flux density is

$$\frac{f_s(^AX^{(Z-1)+})}{f_c(^AX^{(Z-1)+})} \;=\; -\frac{g_{e^-}(^AX^{(Z-1)+})}{2(Z-1)} \frac{A_r(^AX^{(Z-1)+})}{A_r(e)} \,, \qquad (133)$$

where as usual, $A_r(X)$ is the relative atomic mass of particle X. If the frequency ratio f_s/f_c is determined experimentally with high accuracy, and $A_r(^AX^{(Z-1)+})$ of the ion is also accurately known, then this expression can be used to determine an accurate value of $A_r(e)$, assuming the bound-state electron g-factor can be calculated from QED theory with sufficient accuracy; or the g-factor can be determined if $A_r(e)$ is accurately known from another

experiment. In fact, a broad program involving workers from a number of European laboratories has been underway since the mid-1990s to measure the frequency ratio and calculate the g-factor for different ions, most notably (to date) $^{12}\mathrm{C}^{5+}$ and $^{16}\mathrm{O}^{7+}$. The measurements themselves are being performed at the Gesellschaft für Schwerionenforschung, Darmstadt, Germany (GSI) by GSI and University of Mainz researchers, and we discuss the experimental determinations of $f_\mathrm{s}/f_\mathrm{c}$ for $^{12}\mathrm{C}^{5+}$ and $^{16}\mathrm{O}^{7+}$ at GSI in Secs. 5.3.2.1 and 5.3.2.2. The theoretical expressions for the bound-electron g-factors of these two ions are reviewed in the next section.

1. Theory of the bound electron g-factor

In this section, we consider an electron in the 1S state of hydrogen like carbon 12 or oxygen 16 within the framework of bound-state QED. The measured quantity is the transition frequency between the two Zeeman levels of the atom in an externally applied magnetic field.

The energy of a free electron with spin projection s_z in a magnetic flux density B in the z direction is

$$E = -\boldsymbol{\mu}\cdot\boldsymbol{B} = -g_{\mathrm{e}^-}\frac{e}{2m_\mathrm{e}}s_z B \ , \tag{134}$$

and hence the spin-flip energy difference is

$$\Delta E = -g_{\mathrm{e}^-}\mu_\mathrm{B}B \ . \tag{135}$$

(In keeping with the definition of the g-factor in Sec. 5, the quantity g_{e^-} is negative.) The analogous expression for ions with no nuclear spin is

$$\Delta E_\mathrm{b}(X) = -g_{\mathrm{e}^-}(X)\mu_\mathrm{B}B \ , \tag{136}$$

which defines the bound-state electron g-factor, and where X is either $^{12}\mathrm{C}^{5+}$ or $^{16}\mathrm{O}^{7+}$.

The theoretical expression for $g_{\mathrm{e}^-}(X)$ is written as

$$g_{\mathrm{e}^-}(\mathrm{X}) = g_\mathrm{D} + \Delta g_\mathrm{rad} + \Delta g_\mathrm{rec} + \Delta g_\mathrm{ns} + \cdots \ , \tag{137}$$

where the individual terms are the Dirac value, the radiative corrections, the recoil corrections, and the nuclear size corrections, respectively. These theoretical contributions are discussed in the following paragraphs; numerical results based on the 2006 recommended values are summarized in Tables 15 and 16. In the 2006 adjustment α in the expression for g_D is treated as a variable, but the constants in the rest of the calculation of the g-factors are taken as fixed quantities.

(Breit, 1928) obtained the exact value

$$\begin{aligned} g_\mathrm{D} &= -\frac{2}{3}\left[1 + 2\sqrt{1-(Z\alpha)^2}\right] \\ &= -2\left[1 - \frac{1}{3}(Z\alpha)^2 - \frac{1}{12}(Z\alpha)^4 - \frac{1}{24}(Z\alpha)^6 + \cdots\right] \end{aligned} \tag{138}$$

from the Dirac equation for an electron in the field of a fixed point charge of magnitude Ze, where the only uncertainty is that due to the uncertainty in α.

The radiative corrections may be written as

$$\Delta g_\mathrm{rad} = -2\left[C_\mathrm{e}^{(2)}(Z\alpha)\left(\frac{\alpha}{\pi}\right) + C_\mathrm{e}^{(4)}(Z\alpha)\left(\frac{\alpha}{\pi}\right)^2 + \cdots\right] \ , \tag{139}$$

where the coefficients $C_\mathrm{e}^{(2n)}(Z\alpha)$, corresponding to n virtual photons, are slowly varying functions of $Z\alpha$. These coefficients are defined in direct analogy with the corresponding coefficients for the free electron $C_\mathrm{e}^{(2n)}$ given in Eq. (98) so that

$$\lim_{Z\alpha\to 0} C_\mathrm{e}^{(2n)}(Z\alpha) = C_\mathrm{e}^{(2n)} \ . \tag{140}$$

The first two terms of the coefficient $C_\mathrm{e}^{(2)}(Z\alpha)$ have been known for some time (Close and Osborn, 1971; Faustov, 1970; Grotch, 1970). Recently, Pachucki *et al.* (2005a, 2004, 2005b) have calculated additional terms with the result

$$\begin{aligned} C_\mathrm{e,SE}^{(2)}(Z\alpha) &= \frac{1}{2}\Bigg\{1 + \frac{(Z\alpha)^2}{6} + (Z\alpha)^4\left[\frac{32}{9}\ln(Z\alpha)^{-2}\right. \\ &\quad \left. + \frac{247}{216} - \frac{8}{9}\ln k_0 - \frac{8}{3}\ln k_3\right] \\ &\quad + (Z\alpha)^5 R_\mathrm{SE}(Z\alpha)\Bigg\} \ , \end{aligned} \tag{141}$$

where

$$\ln k_0 = 2.984\,128\,556 \tag{142}$$
$$\ln k_3 = 3.272\,806\,545 \tag{143}$$
$$R_\mathrm{SE}(6\alpha) = 22.160(10) \tag{144}$$
$$R_\mathrm{SE}(8\alpha) = 21.859(4) \ . \tag{145}$$

The quantity $\ln k_0$ is the Bethe logarithm for the 1S state (see Table 7) and $\ln k_3$ is a generalization of the Bethe logarithm relevant to the g-factor calculation. The remainder function $R_\mathrm{SE}(Z\alpha)$ was obtained by Pachucki *et al.* (2004, 2005b) by extrapolation of the results of numerical calculations of the self energy for $Z > 8$ by Yerokhin *et al.* (2002) using Eq. (141) to remove the lower-order terms. For $Z = 6$ and $Z = 8$ this yields

$$\begin{aligned} C_\mathrm{e,SE}^{(2)}(6\alpha) &= 0.500\,183\,606\,65(80) \\ C_\mathrm{e,SE}^{(2)}(8\alpha) &= 0.500\,349\,2887(14) \ . \end{aligned} \tag{146}$$

The lowest-order vacuum-polarization correction consists of a wave-function correction and a potential correction. The wave-function correction has been calculated numerically by Beier *et al.* (2000), with the result (in our notation)

$$\begin{aligned} C_\mathrm{e,VPwf}^{(2)}(6\alpha) &= -0.000\,001\,840\,3431(43) \ . \\ C_\mathrm{e,VPwf}^{(2)}(8\alpha) &= -0.000\,005\,712\,028(26) \ . \end{aligned} \tag{147}$$

Each of these values is the sum of the Uehling potential contribution and the higher-order Wichmann-Kroll contribution, which were calculated separately with the uncertainties added linearly, as done by Beier *et al.* (2000). The values in Eq. (147) are consistent with the result of an evaluation of the correction in powers of $Z\alpha$ (Karshenboim, 2000; Karshenboim *et al.*, 2001a,b). For the potential correction, Beier *et al.* (2000) found that the Uehling potential contribution is zero and calculated the Wichmann-Kroll contribution numerically over a wide range of Z (Beier, 2000). An extrapolation of the numerical values from higher-Z, taken together with the analytic result of Karshenboim and Milstein (2002),

$$C_{\mathrm{e,VPp}}^{(2)}(Z\alpha) = \frac{7\pi}{432}(Z\alpha)^5 + \cdots , \tag{148}$$

for the lowest-order Wichmann-Kroll contribution, yields

$$\begin{aligned} C_{\mathrm{e,VPp}}^{(2)}(6\alpha) &= 0.000\,000\,007\,9595(69) \\ C_{\mathrm{e,VPp}}^{(2)}(8\alpha) &= 0.000\,000\,033\,235(29) . \end{aligned} \tag{149}$$

More recently, Lee *et al.* (2005) have obtained the result

$$\begin{aligned} C_{\mathrm{e,VPp}}^{(2)}(6\alpha) &= 0.000\,000\,008\,201(11) \\ C_{\mathrm{e,VPp}}^{(2)}(8\alpha) &= 0.000\,000\,034\,23(11) . \end{aligned} \tag{150}$$

The values in Eq. (149) and Eq. (150) disagree somewhat, so in the present analysis, we use a value that is an unweighted average of the two, with half the difference for the uncertainty. These average values are

$$\begin{aligned} C_{\mathrm{e,VPp}}^{(2)}(6\alpha) &= 0.000\,000\,008\,08(12) \\ C_{\mathrm{e,VPp}}^{(2)}(8\alpha) &= 0.000\,000\,033\,73(50) . \end{aligned} \tag{151}$$

The total one-photon vacuum polarization coefficients are given by the sum of Eqs. (147) and (151):

$$\begin{aligned} C_{\mathrm{e,VP}}^{(2)}(6\alpha) &= C_{\mathrm{e,VPwf}}^{(2)}(6\alpha) + C_{\mathrm{e,VPp}}^{(2)}(6\alpha) \\ &= -0.000\,001\,832\,26(12) \\ C_{\mathrm{e,VP}}^{(2)}(8\alpha) &= C_{\mathrm{e,VPwf}}^{(2)}(8\alpha) + C_{\mathrm{e,VPp}}^{(2)}(8\alpha) \\ &= -0.000\,005\,678\,30(50) . \end{aligned} \tag{152}$$

The total for the one-photon coefficient $C_{\mathrm{e}}^{(2)}(Z\alpha)$, given by the sum of Eqs. (146) and (152), is

$$\begin{aligned} C_{\mathrm{e}}^{(2)}(6\alpha) &= C_{\mathrm{e,SE}}^{(2)}(6\alpha) + C_{\mathrm{e,VP}}^{(2)}(6\alpha) \\ &= 0.500\,181\,774\,38(81) \\ C_{\mathrm{e}}^{(2)}(8\alpha) &= C_{\mathrm{e,SE}}^{(2)}(8\alpha) + C_{\mathrm{e,VP}}^{(2)}(8\alpha) \\ &= 0.500\,343\,6104(14) , \end{aligned} \tag{153}$$

and the total one-photon contribution $\Delta g^{(2)}$ to the g-factor is thus

$$\begin{aligned} \Delta g^{(2)} &= -2\,C_{\mathrm{e}}^{(2)}(Z\alpha)\left(\frac{\alpha}{\pi}\right) \\ &= -0.002\,323\,663\,914(4) \quad \text{for } Z=6 \\ &= -0.002\,324\,415\,746(7) \quad \text{for } Z=8 . \end{aligned} \tag{154}$$

The separate one-photon self energy and vacuum polarization contributions to the g-factor are given in Tables 15 and 16.

Calculations by Eides and Grotch (1997a) using the Bargmann-Michel-Telegdi equation and by Czarnecki *et al.* (2001) using an effective potential approach yield

$$C_{\mathrm{e}}^{(2n)}(Z\alpha) = C_{\mathrm{e}}^{(2n)}\left(1 + \frac{(Z\alpha)^2}{6} + \cdots\right) \tag{155}$$

as the leading binding correction to the free electron coefficients $C_{\mathrm{e}}^{(2n)}$ for any order n. For $C_{\mathrm{e}}^{(2)}(Z\alpha)$, this correction was known for some time. For higher-order terms, it provides the leading binding effect.

The two-loop contribution of relative order $(Z\alpha)^4$ has recently been calculated by Jentschura *et al.* (2006); Pachucki *et al.* (2005a) for any S state. Their result for the ground-state correction is

$$\begin{aligned} C_{\mathrm{e}}^{(4)}(Z\alpha) &= C_{\mathrm{e}}^{(4)}\left(1 + \frac{(Z\alpha)^2}{6}\right) \\ &+ (Z\alpha)^4\left[\frac{14}{9}\ln{(Z\alpha)^{-2}} + \frac{991343}{155520} - \frac{2}{9}\ln k_0 - \frac{4}{3}\ln k_3\right. \\ &\left. + \frac{679\,\pi^2}{12960} - \frac{1441\,\pi^2}{720}\ln 2 + \frac{1441}{480}\zeta(3)\right] + \mathcal{O}(Z\alpha)^5 \\ &= -0.328\,5778(23) \quad \text{for } Z=6 \\ &= -0.328\,6578(97) \quad \text{for } Z=8, \end{aligned} \tag{156}$$

where $\ln k_0$ and $\ln k_3$ are given in Eqs. (142) and (143). The uncertainty due to uncalculated terms is estimated by assuming that the unknown higher-order terms, of order $(Z\alpha)^5$ or higher for two loops, are comparable to the higher-order one-loop terms scaled by the free-electron coefficients in each case, with an extra factor of 2 included (Pachucki *et al.*, 2005a):

$$u\left[C_{\mathrm{e}}^{(4)}(Z\alpha)\right] = 2\left|(Z\alpha)^5\, C_{\mathrm{e}}^{(4)}\, R_{\mathrm{SE}}(Z\alpha)\right| . \tag{157}$$

The three- and four-photon terms are calculated with the leading binding correction included:

$$\begin{aligned} C_{\mathrm{e}}^{(6)}(Z\alpha) &= C_{\mathrm{e}}^{(6)}\left(1 + \frac{(Z\alpha)^2}{6} + \cdots\right) \\ &= 1.181\,611\ldots \quad \text{for } Z=6 \\ &= 1.181\,905\ldots \quad \text{for } Z=8 , \end{aligned} \tag{158}$$

where $C_{\mathrm{e}}^{(6)} = 1.181\,234\ldots$, and

$$\begin{aligned} C_{\mathrm{e}}^{(8)}(Z\alpha) &= C_{\mathrm{e}}^{(8)}\left(1 + \frac{(Z\alpha)^2}{6} + \cdots\right) \\ &= -1.7289(35)\ldots \quad \text{for } Z=6 \\ &= -1.7293(35)\ldots \quad \text{for } Z=8 , \end{aligned} \tag{159}$$

where $C_{\mathrm{e}}^{(8)} = -1.7283(35)$ (Kinoshita and Nio, 2006a). This value would shift somewhat if the more recent tentative value $C_{\mathrm{e}}^{(8)} = -1.9144(35)$ (Aoyama *et al.*, 2007)

were used (see Sec. 5). An uncertainty estimate

$$C_{\rm e}^{(10)}(Z\alpha) \;\approx\; C_{\rm e}^{(10)} = 0.0(3.7) \tag{160}$$

is included for the five-loop correction.

The recoil correction to the bound-state g-factor associated with the finite mass of the nucleus is denoted by $\Delta g_{\rm rec}$, which we write here as the sum $\Delta g_{\rm rec}^{(0)} + \Delta g_{\rm rec}^{(2)}$ corresponding to terms that are zero- and first-order in α/π, respectively. For $\Delta g_{\rm rec}^{(0)}$, we have

$$\begin{aligned}\Delta g_{\rm rec}^{(0)} &= \Bigg\{ -(Z\alpha)^2 + \frac{(Z\alpha)^4}{3[1+\sqrt{1-(Z\alpha)^2}]^2} \\ &\quad -(Z\alpha)^5\,P(Z\alpha)\Bigg\}\frac{m_{\rm e}}{m_{\rm N}} + \mathcal{O}\left(\frac{m_{\rm e}}{m_{\rm N}}\right)^2 \\ &= -0.000\,000\,087\,71(1)\ldots \text{ for } Z=6 \\ &= -0.000\,000\,117\,11(1)\ldots \text{ for } Z=8\ ,\end{aligned} \tag{161}$$

where $m_{\rm N}$ is the mass of the nucleus. The mass ratios, obtained from the 2006 adjustment, are $m_{\rm e}/m(^{12}{\rm C}^{6+}) = 0.000\,045\,727\,5\ldots$ and $m_{\rm e}/m(^{16}{\rm O}^{8+}) = 0.000\,034\,306\,5\ldots$. The recoil terms are the same as in CODATA-02 and references to the original calculations are given there. An additional term of the order of the mass ratio squared is included as

$$S_Z(Z\alpha)^2\left(\frac{m_{\rm e}}{m_{\rm N}}\right)^2 , \tag{162}$$

where S_Z is taken to be the average of the disagreeing values $1+Z$, obtained by Eides (2002); Eides and Grotch (1997a), and $Z/3$ obtained by Martynenko and Faustov (2001, 2002) for this term. The uncertainty in S_Z is taken to be half the difference of the two values.

For $\Delta g_{\rm rec}^{(2)}$, we have

$$\begin{aligned}\Delta g_{\rm rec}^{(2)} &= \frac{\alpha}{\pi}\frac{(Z\alpha)^2}{3}\frac{m_{\rm e}}{m_{\rm N}} + \cdots \\ &= 0.000\,000\,000\,06\ldots \text{ for } Z=6 \\ &= 0.000\,000\,000\,09\ldots \text{ for } Z=8\ .\end{aligned} \tag{163}$$

There is a small correction to the bound-state g-factor due to the finite size of the nucleus, of order

$$\Delta g_{\rm ns} = -\frac{8}{3}(Z\alpha)^4\left(\frac{R_{\rm N}}{\lambda\!\!\!^{-}_{\rm C}}\right)^2 + \cdots\ , \tag{164}$$

where $R_{\rm N}$ is the bound-state nuclear rms charge radius and $\lambda\!\!\!^{-}_{\rm C}$ is the Compton wavelength of the electron divided by 2π. This term is calculated as in CODATA-02 (Glazov and Shabaev, 2002) with updated values for the nuclear radii $R_{\rm N} = 2.4703(22)$ fm and $R_{\rm N} = 2.7013(55)$ from the compilation of Angeli (2004) for ^{12}C and ^{16}O, respectively. This yields the correction

$$\begin{aligned}\Delta g_{\rm ns} &= -0.000\,000\,000\,408(1) \quad \text{for } ^{12}\text{C} \\ \Delta g_{\rm ns} &= -0.000\,000\,001\,56(1) \quad \text{for } ^{16}\text{O}\ .\end{aligned} \tag{165}$$

The theoretical value for the g-factor of the electron in hydrogenic carbon 12 or oxygen 16 is the sum of the individual contributions discussed above and summarized in Tables 15 and 16:

$$\begin{aligned}g_{\rm e^-}(^{12}{\rm C}^{5+}) &= -2.001\,041\,590\,203(28) \\ g_{\rm e^-}(^{16}{\rm O}^{7+}) &= -2.000\,047\,020\,38(11)\ .\end{aligned} \tag{166}$$

For the purpose of the least-squares calculations carried out in Sec. 12.2, we define $g_{\rm C}({\rm th})$ to be the sum of $g_{\rm D}$ as given in Eq. (138), the term $-2(\alpha/\pi)C_{\rm e}^{(2)}$, and the numerical values of the remaining terms in Eq. (137) as given in Table 15, where the standard uncertainty of these latter terms is

$$u[g_{\rm C}({\rm th})] \;=\; 0.3\times10^{-10} = 1.4\times10^{-11}|g_{\rm C}({\rm th})|\ . \tag{167}$$

The uncertainty in $g_{\rm C}$(th) due to the uncertainty in α enters the adjustment primarily through the functional dependence of $g_{\rm D}$ and the term $-2(\alpha/\pi)C_{\rm e}^{(2)}$ on α. Therefore this particular component of uncertainty is not explicitly included in $u[g_{\rm C}({\rm th})]$. To take the uncertainty $u[g_{\rm C}({\rm th})]$ into account we employ as the theoretical expression for the g-factor

$$g_{\rm C}(\alpha,\delta_{\rm C}) \;=\; g_{\rm C}({\rm th}) + \delta_{\rm C}\ , \tag{168}$$

where the input value of the additive correction $\delta_{\rm C}$ is taken to be zero and its standard uncertainty is $u[g_{\rm C}({\rm th})]$:

$$\delta_{\rm C} = 0.00(27)\times10^{-10}\ . \tag{169}$$

Analogous considerations apply for the g-factor in oxygen:

$$u[g_{\rm O}({\rm th})] \;=\; 1.1\times10^{-10} = 5.3\times10^{-11}|g_{\rm O}({\rm th})| \tag{170}$$

$$g_{\rm O}(\alpha,\delta_{\rm O}) \;=\; g_{\rm O}({\rm th}) + \delta_{\rm O} \tag{171}$$

$$\delta_{\rm O} \;=\; 0.0(1.1)\times10^{-10}\ . \tag{172}$$

Since the uncertainties of the theoretical values of the carbon and oxygen g-factors arise primarily from the same sources, the quantities $\delta_{\rm C}$ and $\delta_{\rm O}$ are highly correlated. Their covariance is

$$u(\delta_{\rm C},\delta_{\rm O}) = 27\times10^{-22}\ , \tag{173}$$

which corresponds to a correlation coefficient of $r(\delta_{\rm C},\delta_{\rm O}) = 0.92$.

The theoretical value of the ratio of the two g-factors, which is relevant to the comparison to experiment in Sec. 5.3.2.3, is

$$\frac{g_{\rm e^-}(^{12}{\rm C}^{5+})}{g_{\rm e^-}(^{16}{\rm O}^{7+})} \;=\; 1.000\,497\,273\,218(41)\ , \tag{174}$$

where the covariance, including the contribution from the uncertainty in α for this case, is taken into account.

TABLE 15 Theoretical contributions and total for the g-factor of the electron in hydrogenic carbon 12 based on the 2006 recommended values of the constants.

Contribution	Value	Source
Dirac $g_{\rm D}$	$-1.998\,721\,354\,402(2)$	Eq. (138)
$\Delta g_{\rm SE}^{(2)}$	$-0.002\,323\,672\,426(4)$	Eq. (146)
$\Delta g_{\rm VP}^{(2)}$	$0.000\,000\,008\,512(1)$	Eq. (152)
$\Delta g^{(4)}$	$0.000\,003\,545\,677(25)$	Eq. (156)
$\Delta g^{(6)}$	$-0.000\,000\,029\,618$	Eq. (158)
$\Delta g^{(8)}$	$0.000\,000\,000\,101$	Eq. (159)
$\Delta g^{(10)}$	$0.000\,000\,000\,000(1)$	Eq. (160)
$\Delta g_{\rm rec}$	$-0.000\,000\,087\,639(10)$	Eqs. (161)-(163)
$\Delta g_{\rm ns}$	$-0.000\,000\,000\,408(1)$	Eq. (165)
$g_{\rm e^-}(^{12}{\rm C}^{5+})$	$-2.001\,041\,590\,203(28)$	Eq. (166)

TABLE 16 Theoretical contributions and total for the g-factor of the electron in hydrogenic oxygen 16 based on the 2006 recommended values of the constants.

Contribution	Value	Source
Dirac $g_{\rm D}$	$-1.997\,726\,003\,08$	Eq. (138)
$\Delta g_{\rm SE}^{(2)}$	$-0.002\,324\,442\,12(1)$	Eq. (146)
$\Delta g_{\rm VP}^{(2)}$	$0.000\,000\,026\,38$	Eq. (152)
$\Delta g^{(4)}$	$0.000\,003\,546\,54(11)$	Eq. (156)
$\Delta g^{(6)}$	$-0.000\,000\,029\,63$	Eq. (158)
$\Delta g^{(8)}$	$0.000\,000\,000\,10$	Eq. (159)
$\Delta g^{(10)}$	$0.000\,000\,000\,00$	Eq. (160)
$\Delta g_{\rm rec}$	$-0.000\,000\,117\,02(1)$	Eqs. (161)-(163)
$\Delta g_{\rm ns}$	$-0.000\,000\,001\,56(1)$	Eq. (165)
$g_{\rm e^-}(^{16}{\rm O}^{7+})$	$-2.000\,047\,020\,38(11)$	Eq. (166)

2. Measurements of $g_{\rm e}(^{12}{\rm C}^{5+})$ and $g_{\rm e}(^{16}{\rm O}^{7+})$.

The experimental data on the electron bound-state g-factor in hydrogenic carbon and oxygen and the inferred values of $A_{\rm r}({\rm e})$ are summarized in Table 17.

1. Experiment on $g_{\rm e}(^{12}{\rm C}^{5+})$. The accurate determination of the frequency ratio $f_{\rm s}(^{12}{\rm C}^{5+})/f_{\rm c}(^{12}{\rm C}^{5+})$ at GSI based on the double Penning-trap technique was discussed in CODATA-02. [See also the recent concise review by Werth *et al.* (2006).] Since the result used as an input datum in the 2002 adjustment is unchanged, we take it as an input datum in the 2006 adjustment as well (Beier *et al.*, 2002; Häffner *et al.*, 2003; Werth, 2003):

$$\frac{f_{\rm s}\left(^{12}{\rm C}^{5+}\right)}{f_{\rm c}\left(^{12}{\rm C}^{5+}\right)} = 4376.210\,4989(23)\,. \tag{175}$$

From Eq. (133) and Eq. (4) we have

$$\frac{f_{\rm s}\left(^{12}{\rm C}^{5+}\right)}{f_{\rm c}\left(^{12}{\rm C}^{5+}\right)} = -\frac{g_{\rm e^-}\left(^{12}{\rm C}^{5+}\right)}{10A_{\rm r}({\rm e})} \times\left[12-5A_{\rm r}({\rm e})+\frac{E_{\rm b}\left(^{12}{\rm C}\right)-E_{\rm b}\left(^{12}{\rm C}^{5+}\right)}{m_{\rm u}c^2}\right]\,, \tag{176}$$

which is the basis for the observational equation for the $^{12}{\rm C}^{5+}$ frequency-ratio input datum.

Evaluation of this expression using the result for $f_{\rm s}(^{12}{\rm C}^{5+})/f_{\rm c}(^{12}{\rm C}^{5+})$ in Eq. (175), the theoretical result for $g_{\rm e^-}(^{12}{\rm C}^{5+})$ in Table 15, and the relevant binding energies in Table IV of CODATA-02, yields

$$A_{\rm r}({\rm e}) = 0.000\,548\,579\,909\,32(29)\quad[5.2\times10^{-10}]\,. \tag{177}$$

This value is consistent with that from antiprotonic helium given in Eq. (74) and that from the University of Washington given in Eq. (5), but has about a factor of three to four smaller uncertainty.

2. Experiment on $g_{\rm e}(^{16}{\rm O}^{7+})$. The double Penning-trap determination of the frequency ratio $f_{\rm s}(^{16}{\rm O}^{7+})/f_{\rm c}(^{16}{\rm O}^{7+})$ at GSI was also discussed in CODATA-02, but the value used as an input datum was not quite final (Verdú *et al.*, 2003, 2002; Werth, 2003). A slightly different value for the ratio was given in the final report of the measurement (Tomaselli *et al.*, 2002), which is the value we take as the input datum in the 2006 adjustment but modified slightly as follows based on information provided by Verdú (2006): (i) an unrounded instead of a rounded value for the correction due to extrapolating the axial temperature T_z to 0 K was added to the uncorrected ratio ($-0.000\,004\,7$ in place of $-0.000\,005$); and (ii) a more detailed uncertainty budget was employed to evaluate the uncertainty of the ratio. The resulting value is

$$\frac{f_{\rm s}\left(^{16}{\rm O}^{7+}\right)}{f_{\rm c}\left(^{16}{\rm O}^{7+}\right)} = 4164.376\,1837(32)\,. \tag{178}$$

In analogy with what was done above with the ratio $f_{\rm s}(^{12}{\rm C}^{5+})/f_{\rm c}(^{12}{\rm C}^{5+})$, from Eq. (133) and Eq. (4) we have

$$\frac{f_{\rm s}\left(^{16}{\rm O}^{7+}\right)}{f_{\rm c}\left(^{16}{\rm O}^{7+}\right)} = -\frac{g_{\rm e^-}\left(^{16}{\rm O}^{7+}\right)}{14A_{\rm r}({\rm e})}\,A_{\rm r}\left(^{16}{\rm O}^{7+}\right) \tag{179}$$

with

$$A_{\rm r}\left(^{16}{\rm O}\right) = A_{\rm r}\left(^{16}{\rm O}^{7+}\right)+7A_{\rm r}({\rm e}) -\frac{E_{\rm b}\left(^{16}{\rm O}\right)-E_{\rm b}\left(^{16}{\rm O}^{7+}\right)}{m_{\rm u}c^2}\,, \tag{180}$$

which are the basis for the observational equations for the oxygen frequency ratio and $A_{\rm r}(^{16}{\rm O})$, respectively. The first expression, evaluated using the result

TABLE 17 Summary of experimental data on the electron bound-state g-factor in hydrogenic carbon and oxygen and inferred values of the relative atomic mass of the electron.

Input datum	Value	Relative standard uncertainty u_r	Identification	Sec. and Eq.
$f_s(^{12}C^{5+})/f_c(^{12}C^{5+})$	4376.210 4989(23)	5.2×10^{-10}	GSI-02	5.3.2.1 (175)
$A_r(e)$	0.000 548 579 909 32(29)	5.2×10^{-10}		5.3.2.1 (177)
$f_s(^{16}O^{7+})/f_c(^{16}O^{7+})$	4164.376 1837(32)	7.6×10^{-10}	GSI-02	5.3.2.2 (178)
$A_r(e)$	0.000 548 579 909 58(42)	7.6×10^{-10}		5.3.2.2 (181)

for $f_s(^{16}O^{7+})/f_c(^{16}O^{7+})$ in Eq. (178) and the theoretical result for $g_{e^-}(^{16}O^{7+})$ in Table 16, in combination with the second expression, evaluated using the value of $A_r(^{16}O)$ in Table 4 and the relevant binding energies in Table IV of CODATA-02, yields

$$A_r(e) = 0.000\,548\,579\,909\,58(42) \quad [7.6\times10^{-10}] \ . \tag{181}$$

It is consistent with both the University of Washington value in Eq. (5) and the value in Eq. (177) obtained from $f_s(^{12}C^{5+})/f_c(^{12}C^{5+})$.

3. Relations between $g_e(^{12}C^{5+})$ and $g_e(^{16}O^{7+})$. It should be noted that the GSI frequency ratios for $^{12}C^{5+}$ and $^{16}O^{7+}$ are correlated. Based on the detailed uncertainty budgets of the two results (Verdú, 2006; Werth, 2003), we find the correlation coefficient to be

$$r\left[\frac{f_s\left(^{12}C^{5+}\right)}{f_c\left(^{12}C^{5+}\right)}, \frac{f_s\left(^{16}O^{7+}\right)}{f_c\left(^{16}O^{7+}\right)}\right] = 0.082 \ . \tag{182}$$

Finally, as a consistency test, it is of interest to compare the experimental and theoretical values of the ratio of $g_{e^-}(^{12}C^{5+})$ to $g_{e^-}(^{16}O^{7+})$ (Karshenboim and Ivanov, 2002). The main reason is that the experimental value of the ratio is only weakly dependent on the value of $A_r(e)$. The theoretical value of the ratio is given in Eq. (174) and takes into account the covariance of the two theoretical values. The experimental value of the ratio can be obtained by combining Eqs. (175), (176), (178) to (180) and (182), and using the 2006 recommended value for $A_r(e)$. Because of the weak dependence of the experimental ratio on $A_r(e)$, the value used is not at all critical. The result is

$$\frac{g_{e^-}(^{12}C^{5+})}{g_{e^-}(^{16}O^{7+})} = 1.000\,497\,273\,68(89)\ [8.9\times10^{-10}] \ , \tag{183}$$

in agreement with the theoretical value.

6. MAGNETIC MOMENT RATIOS AND THE MUON-ELECTRON MASS RATIO

Magnetic moment ratios and the muon-electron mass ratio are determined by experiments on bound states of the relevant particles. The free electron and muon magnetic moments are discussed in Sec. 5 and the theory of the g-factor of an electron bound in an atom with no nuclear spin is considered in Sec. 5.3.1.

For nucleons or nuclei with spin $\boldsymbol{I}$, the magnetic moment can be written as

$$\boldsymbol{\mu} = g\frac{e}{2m_p}\boldsymbol{I} \ , \tag{184}$$

or

$$\mu = g\mu_N i \ . \tag{185}$$

In Eq. (185), $\mu_N = e\hbar/2m_p$ is the nuclear magneton, defined in analogy with the Bohr magneton, and i is the spin quantum number of the nucleus defined by $\boldsymbol{I}^2 = i(i+1)\hbar^2$ and $I_z = -i\hbar, ..., (i-1)\hbar, i\hbar$, where I_z is the spin projection. However, in some publications, moments of nucleons are expressed in terms of the Bohr magneton with a corresponding change in the definition of the g-factor.

For atoms with a nonzero nuclear spin, bound state g-factors are defined by considering the contribution to the Hamiltonian from the interaction of the atom with an applied magnetic flux density B. For example, for hydrogen, in the framework of the Pauli approximation, we have

$$\begin{aligned}\mathcal{H} &= \beta(\mathrm{H})\boldsymbol{\mu}_{e^-}\cdot\boldsymbol{\mu}_p - \boldsymbol{\mu}_{e^-}(\mathrm{H})\cdot\boldsymbol{B} - \boldsymbol{\mu}_p(\mathrm{H})\cdot\boldsymbol{B} \\ &= \frac{2\pi}{\hbar}\Delta\nu_H \boldsymbol{s}\cdot\boldsymbol{I} - g_{e^-}(\mathrm{H})\,\frac{\mu_B}{\hbar}\,\boldsymbol{s}\cdot\boldsymbol{B} - g_p(\mathrm{H})\,\frac{\mu_N}{\hbar}\,\boldsymbol{I}\cdot\boldsymbol{B} \ ,\end{aligned} \tag{186}$$

where $\beta(\mathrm{H})$ characterizes the strength of the hyperfine interaction, $\Delta\nu_H$ is the ground-state hyperfine frequency, $\boldsymbol{s}$ is the spin of the electron, and $\boldsymbol{I}$ is the spin of the nucleus, that is, the proton. Equation (186), or its analog for other combinations of particles, serves to define the corresponding bound-state g-factors, which are $g_{e^-}(\mathrm{H})$ and $g_p(\mathrm{H})$ in this case.

1. Magnetic moment ratios

A number of magnetic moment ratios are of interest for the 2006 adjustment. The results of measurements and the inferred values of various quantities are summarized in Sec. 6.1.2, and the measurement results themselves are also summarized in Table 19.

The inferred moment ratios depend on the relevant theoretical binding corrections that relate the g-factor

measured in the bound state to the corresponding free-particle g-factor. To use the results of these experiments in the 2006 adjustment, we employ theoretical expressions that give predictions for the moments and g-factors of the bound particles in terms of free-particle moments and g-factors as well as adjusted constants; this is discussed in the following section. However, in a number of cases, the differences between the bound-state and free-state values are sufficiently small that the adjusted constants can be taken as exactly known.

1. Theoretical ratios of atomic bound-particle to free-particle g-factors

Theoretical g-factor-related quantities used in the 2006 adjustment are the ratio of the g-factor of the electron in the ground state of hydrogen to that of the free electron $g_{e^-}(\mathrm{H})/g_{e^-}$; the ratio of the g-factor of the proton in hydrogen to that of the free proton $g_p(\mathrm{H})/g_p$; the analogous ratios for the electron and deuteron in deuterium, $g_{e^-}(\mathrm{D})/g_{e^-}$ and $g_d(\mathrm{D})/g_d$, respectively; and the analogous ratios for the electron and positive muon in muonium, $g_{e^-}(\mathrm{Mu})/g_{e^-}$ and $g_{\mu^+}(\mathrm{Mu})/g_{\mu^+}$, respectively.

These ratios and the references for the relevant calculations are discussed in CODATA-98 and CODATA-02; only a summary of the results is included here.

For the electron in hydrogen, we have

$$\begin{aligned}\frac{g_{e^-}(\mathrm{H})}{g_{e^-}} &= 1-\tfrac{1}{3}(Z\alpha)^2-\tfrac{1}{12}(Z\alpha)^4+\tfrac{1}{4}(Z\alpha)^2\left(\frac{\alpha}{\pi}\right)\\ &\quad +\tfrac{1}{2}(Z\alpha)^2\frac{m_e}{m_p}+\tfrac{1}{2}\left(A_1^{(4)}-\tfrac{1}{4}\right)(Z\alpha)^2\left(\frac{\alpha}{\pi}\right)^2\\ &\quad -\tfrac{5}{12}(Z\alpha)^2\left(\frac{\alpha}{\pi}\right)\frac{m_e}{m_p}+\cdots\ , \qquad (187)\end{aligned}$$

where $A_1^{(4)}$ is given in Eq. (84). For the proton in hydrogen, we have

$$\begin{aligned}\frac{g_p(\mathrm{H})}{g_p} &= 1-\tfrac{1}{3}\alpha(Z\alpha)-\tfrac{97}{108}\alpha(Z\alpha)^3\\ &\quad +\tfrac{1}{6}\alpha(Z\alpha)\frac{m_e}{m_p}\frac{3+4a_p}{1+a_p}+\cdots\ , \qquad (188)\end{aligned}$$

where the proton magnetic moment anomaly a_p is defined by

$$a_p = \frac{\mu_p}{(e\hbar/2m_p)}-1\approx 1.793\ . \qquad (189)$$

For deuterium, similar expressions apply for the electron

$$\begin{aligned}\frac{g_{e^-}(\mathrm{D})}{g_{e^-}} &= 1-\tfrac{1}{3}(Z\alpha)^2-\tfrac{1}{12}(Z\alpha)^4+\tfrac{1}{4}(Z\alpha)^2\left(\frac{\alpha}{\pi}\right)\\ &\quad +\tfrac{1}{2}(Z\alpha)^2\frac{m_e}{m_d}+\tfrac{1}{2}\left(A_1^{(4)}-\tfrac{1}{4}\right)(Z\alpha)^2\left(\frac{\alpha}{\pi}\right)^2\\ &\quad -\tfrac{5}{12}(Z\alpha)^2\left(\frac{\alpha}{\pi}\right)\frac{m_e}{m_d}+\cdots\ , \qquad (190)\end{aligned}$$

TABLE 18 Theoretical values for various bound-particle to free-particle g-factor ratios relevant to the 2006 adjustment based on the 2006 recommended values of the constants.

Ratio	Value
$g_{e^-}(\mathrm{H})/g_{e^-}$	$1-17.7054\times10^{-6}$
$g_p(\mathrm{H})/g_p$	$1-17.7354\times10^{-6}$
$g_{e^-}(\mathrm{D})/g_{e^-}$	$1-17.7126\times10^{-6}$
$g_d(\mathrm{D})/g_d$	$1-17.7461\times10^{-6}$
$g_{e^-}(\mathrm{Mu})/g_{e^-}$	$1-17.5926\times10^{-6}$
$g_{\mu^+}(\mathrm{Mu})/g_{\mu^+}$	$1-17.6254\times10^{-6}$

and deuteron

$$\begin{aligned}\frac{g_d(\mathrm{D})}{g_d} &= 1-\tfrac{1}{3}\alpha(Z\alpha)-\tfrac{97}{108}\alpha(Z\alpha)^3\\ &\quad +\tfrac{1}{6}\alpha(Z\alpha)\frac{m_e}{m_d}\frac{3+4a_d}{1+a_d}+\cdots\ , \qquad (191)\end{aligned}$$

where the deuteron magnetic moment anomaly a_d is defined by

$$a_d = \frac{\mu_d}{(e\hbar/m_d)}-1\approx -0.143\ . \qquad (192)$$

In the case of muonium Mu, some additional higher-order terms are included because of the larger mass ratio. For the electron in muonium, we have

$$\begin{aligned}\frac{g_{e^-}(\mathrm{Mu})}{g_{e^-}} &= 1-\tfrac{1}{3}(Z\alpha)^2-\tfrac{1}{12}(Z\alpha)^4+\tfrac{1}{4}(Z\alpha)^2\left(\frac{\alpha}{\pi}\right)\\ &\quad +\tfrac{1}{2}(Z\alpha)^2\frac{m_e}{m_\mu}+\tfrac{1}{2}\left(A_1^{(4)}-\tfrac{1}{4}\right)(Z\alpha)^2\left(\frac{\alpha}{\pi}\right)^2\\ &\quad -\tfrac{5}{12}(Z\alpha)^2\left(\frac{\alpha}{\pi}\right)\frac{m_e}{m_\mu}-\tfrac{1}{2}(1+Z)(Z\alpha)^2\left(\frac{m_e}{m_\mu}\right)^2\\ &\quad +\cdots\ , \qquad (193)\end{aligned}$$

and for the muon in muonium, the ratio is

$$\begin{aligned}\frac{g_{\mu^+}(\mathrm{Mu})}{g_{\mu^+}} &= 1-\tfrac{1}{3}\alpha(Z\alpha)-\tfrac{97}{108}\alpha(Z\alpha)^3\\ &\quad +\tfrac{1}{2}\alpha(Z\alpha)\frac{m_e}{m_\mu}+\tfrac{1}{12}\alpha(Z\alpha)\left(\frac{\alpha}{\pi}\right)\frac{m_e}{m_\mu}\\ &\quad -\tfrac{1}{2}(1+Z)\alpha(Z\alpha)\left(\frac{m_e}{m_\mu}\right)^2+\cdots\ . \qquad (194)\end{aligned}$$

The numerical values of the corrections in Eqs. (187) to (194), based on the 2006 adjusted values of the relevant constants, are listed in Table 18. Uncertainties are negligible at the level of uncertainty of the relevant experiments.

2. Ratio measurements

1. Electron to proton magnetic moment ratio μ_e/μ_p. The ratio μ_e/μ_p is obtained from measurements of the ratio of the magnetic moment of the electron to the magnetic moment of the proton in the 1S state of hydrogen $\mu_{e^-}(\mathrm{H})/\mu_p(\mathrm{H})$. We use the value obtained by Winkler *et al.* (1972) at MIT:

$$\frac{\mu_{e^-}(\mathrm{H})}{\mu_p(\mathrm{H})} = -658.210\,7058(66) \quad [1.0\times10^{-8}] \,, \quad (195)$$

where a minor typographical error in the original publication has been corrected (Kleppner, 1997). The free-particle ratio μ_e/μ_p follows from the bound-particle ratio and the relation

$$\begin{aligned}\frac{\mu_{e^-}}{\mu_p} &= \frac{g_p(\mathrm{H})}{g_p}\left(\frac{g_{e^-}(\mathrm{H})}{g_{e^-}}\right)^{-1}\frac{\mu_{e^-}(\mathrm{H})}{\mu_p(\mathrm{H})} \\ &= -658.210\,6860(66) \quad [1.0\times10^{-8}] \,, \quad (196)\end{aligned}$$

where the bound-state g-factor ratios are given in Table 18.

2. Deuteron to electron magnetic moment ratio μ_d/μ_e. From measurements of the ratio $\mu_d(\mathrm{D})/\mu_{e^-}(\mathrm{D})$ in the 1S state of deuterium, Phillips *et al.* (1984) at MIT obtained

$$\frac{\mu_d(\mathrm{D})}{\mu_{e^-}(\mathrm{D})} = -4.664\,345\,392(50)\times10^{-4} \quad [1.1\times10^{-8}] \,. \quad (197)$$

Although this result has not been published, as in the 1998 and 2002 adjustments, we include it as an input datum, because the method is described in detail by Winkler *et al.* (1972) in connection with their measurement of $\mu_{e^-}(\mathrm{H})/\mu_p(\mathrm{H})$. The free-particle ratio is given by

$$\begin{aligned}\frac{\mu_d}{\mu_{e^-}} &= \frac{g_{e^-}(\mathrm{D})}{g_{e^-}}\left(\frac{g_d(\mathrm{D})}{g_d}\right)^{-1}\frac{\mu_d(\mathrm{D})}{\mu_{e^-}(\mathrm{D})} \\ &= -4.664\,345\,548(50)\times10^{-4} \quad [1.1\times10^{-8}] \,, \quad (198)\end{aligned}$$

with the bound-state g-factor ratios given in Table 18.

3. Proton to deuteron and triton to proton magnetic moment ratios μ_p/μ_d and μ_t/μ_p The ratios μ_p/μ_d and μ_t/μ_p can be determined by nuclear magnetic resonance (NMR) measurements on the HD molecule (bound state of hydrogen and deuterium) and the HT molecule (bound state of hydrogen and tritium, ^{3}H), respectively. The relevant expressions are (see CODATA-98)

$$\begin{aligned}\frac{\mu_p(\mathrm{HD})}{\mu_d(\mathrm{HD})} &= [1+\sigma_d(\mathrm{HD})-\sigma_p(\mathrm{HD})]\,\frac{\mu_p}{\mu_d} \quad (199)\\ \frac{\mu_t(\mathrm{HT})}{\mu_p(\mathrm{HT})} &= [1-\sigma_t(\mathrm{HT})+\sigma_p(\mathrm{HT})]\,\frac{\mu_t}{\mu_p} \,, \quad (200)\end{aligned}$$

where $\mu_p(\mathrm{HD})$ and $\mu_d(\mathrm{HD})$ are the proton and deuteron magnetic moments in HD, respectively, and $\sigma_p(\mathrm{HD})$ and $\sigma_d(\mathrm{HD})$ are the corresponding nuclear magnetic shielding corrections. Similarly, $\mu_t(\mathrm{HT})$ and $\mu_p(\mathrm{HT})$ are the triton (nucleus of tritium) and proton magnetic moments in HT, respectively, and $\sigma_t(\mathrm{HT})$ and $\sigma_p(\mathrm{HT})$ are the corresponding nuclear magnetic shielding corrections. [Note that $\mu(\text{bound}) = (1-\sigma)\mu(\text{free})$ and the nuclear magnetic shielding corrections are small.]

The determination of μ_d/μ_p from NMR measurements on HD by Wimett (1953) and by a Russian group working in St. Petersburg (Gorshkov *et al.*, 1989; Neronov *et al.*, 1975) was discussed in CODATA-98. However, for reasons given there, mainly the lack of sufficient information to assign a reliable uncertainty to the reported values of $\mu_d(\mathrm{HD})/\mu_p(\mathrm{HD})$ and also to the nuclear magnetic shielding correction difference $\sigma_d(\mathrm{HD}) - \sigma_p(\mathrm{HD})$, the results were not used in the 1998 or 2002 adjustments. Further, since neither of these adjustments addressed quantities related to the triton, the determination of μ_t/μ_p from measurements on HT by the Russian group (Neronov and Barzakh, 1977) was not considered in either of these adjustments. It may be recalled that a systematic error related to the use of separate inductance coils for the proton and deuteron NMR resonances in the measurements of Neronov *et al.* (1975) was eliminated in the HT measurements of Neronov and Barzakh (1977) as well as in the HD measurements of Gorshkov *et al.* (1989).

Recently, a member of the earlier St. Petersburg group together with one or more other Russian colleagues in St. Petersburg published the following results based in part on new measurements and re-examination of relevant theory (Karshenboim *et al.*, 2005; Neronov and Karshenboim, 2003):

$$\begin{aligned}\frac{\mu_p(\mathrm{HD})}{\mu_d(\mathrm{HD})} &= 3.257\,199\,531(29) \qquad [8.9\times10^{-9}] \quad (201)\\ \frac{\mu_t(\mathrm{HT})}{\mu_p(\mathrm{HT})} &= 1.066\,639\,887(10) \qquad [9.4\times10^{-9}] \quad (202)\end{aligned}$$

$$\begin{aligned}\sigma_{dp} &\equiv \sigma_d(\mathrm{HD})-\sigma_p(\mathrm{HD}) = 15(2)\times10^{-9} \quad (203)\\ \sigma_{tp} &\equiv \sigma_t(\mathrm{HT})-\sigma_p(\mathrm{HT}) = 20(3)\times10^{-9} \,, \quad (204)\end{aligned}$$

which together with Eqs. (199) and (200) yield

$$\begin{aligned}\frac{\mu_p}{\mu_d} &= 3.257\,199\,482(30) \qquad [9.1\times10^{-9}] \quad (205)\\ \frac{\mu_t}{\mu_p} &= 1.066\,639\,908(10) \qquad [9.8\times10^{-9}] \,. \quad (206)\end{aligned}$$

The purpose of the new work (Karshenboim *et al.*, 2005; Neronov and Karshenboim, 2003) was (i) to check whether rotating the NMR sample and using a high-pressure gas as the sample (60 to 130 atmospheres), which was the case in most of the older Russian experiments, influenced the results and to report a value of $\mu_p(\mathrm{HD})/\mu_d(\mathrm{HD})$ with a reliable uncertainty; and (ii) to

TABLE 19 Summary of data for magnetic moment ratios of various bound particles.

Quantity	Value	Relative standard uncertainty u_r	Identification	Sect. and Eq.
$\mu_{e^-}(H)/\mu_p(H)$	$-658.210\,7058(66)$	1.0×10^{-8}	MIT-72	6.1.2.1 (195)
$\mu_d(D)/\mu_{e^-}(D)$	$-4.664\,345\,392(50)\times10^{-4}$	1.1×10^{-8}	MIT-84	6.1.2.2 (197)
$\mu_p(HD)/\mu_d(HD)$	$3.257\,199\,531(29)$	8.9×10^{-9}	StPtrsb-03	6.1.2.3 (201)
σ_{dp}	$15(2)\times10^{-9}$		StPtrsb-03	6.1.2.3 (203)
$\mu_t(HT)/\mu_p(HT)$	$1.066\,639\,887(10)$	9.4×10^{-9}	StPtrsb-03	6.1.2.3 (202)
σ_{tp}	$20(3)\times10^{-9}$		StPtrsb-03	6.1.2.3 (204)
$\mu_{e^-}(H)/\mu'_p$	$-658.215\,9430(72)$	1.1×10^{-8}	MIT-77	6.1.2.4 (209)
μ'_h/μ'_p	$-0.761\,786\,1313(33)$	4.3×10^{-9}	NPL-93	6.1.2.5 (211)
μ_n/μ'_p	$-0.684\,996\,94(16)$	2.4×10^{-7}	ILL-79	6.1.2.6 (212)

re-examine the theoretical values of the nuclear magnetic shielding correction differences σ_{dp} and σ_{tp} and their uncertainties. It was also anticipated that based on this new work, a value of $\mu_t(HT)/\mu_p(HT)$ with a reliable uncertainty could be obtained from the highly precise measurements of Neronov and Barzakh (1977). However, Gorshkov *et al.* (1989), as part of their experiment to determine μ_d/μ_p, compared the result from a 100 atmosphere HD rotating sample with a 100 atmosphere HD non-rotating sample and found no statistically significant difference.

To test the effect of sample rotation and sample pressure, Neronov and Karshenboim (2003) performed measurements using a commercial NMR spectrometer operating at a magnetic flux density of about 7 T and a non-rotating 10 atmosphere HD gas sample. Because of the relatively low pressure, the NMR signals were comparatively weak and a measurement time of 1 h was required. To simplify the measurements, the frequency of the proton NMR signal from HD was determined relative to the frequency of the more easily measured proton NMR signal from acetone, $(CH_3)_2CO$. Similarly, the frequency of the deuteron NMR signal from HD was determined relative to the frequency of the more easily measured deuteron NMR signal from deuterated acetone, $(CD_3)_2CO$. A number of tests involving the measurement of the hyperfine interaction constant in the case of the proton triplet NMR spectrum, and the isotopic shift in the case of the deuteron, where the deuteron HD doublet NMR spectrum was compared with the singlet spectrum of D_2, were carried out to investigate the reliability of the new data. The results of the tests were in good agreement with the older results obtained with sample rotation and high gas pressure.

The more recent result for $\mu_p(HD)/\mu_d(HD)$ reported by Karshenboim *et al.* (2005), which was obtained with the same NMR spectrometer employed by Neronov and Karshenboim (2003) but with a 20 atmosphere non-rotating gas sample, agrees with the 10 atmosphere non-rotating sample result of the latter researchers and is interpreted by Karshenboim *et al.* (2005) as confirming the 2003 result. Although the values of $\mu_p(HD)/\mu_d(HD)$ reported by the Russian researchers in 2005, 2003, and 1989 agree, the 2003 result as given in Eq. (201) and Table 19, the uncertainty of which is dominated by the proton NMR line fitting procedure, is taken as the input datum in the 2006 adjustment because of the attention paid to possible systematic effects, both experimental and theoretical.

Based on their HD measurements and related analysis, especially the fact that sample pressure and rotation do not appear to be a problem at the current level of uncertainty, Neronov and Karshenboim (2003) conclude that the result for $\mu_t(HT)/\mu_p(HT)$ reported by Neronov and Barzakh (1977) is reliable but that it should be assigned about the same relative uncertainty as their result for $\mu_p(HD)/\mu_d(HD)$. We therefore include as an input datum in the 2006 adjustment the result for $\mu_t(HT)/\mu_p(HT)$ given in Eq. (202) and Table 19.

Without reliable theoretically calculated values for the shielding correction differences σ_{dp} and σ_{tp}, reliable experimental values for the ratios $\mu_p(HD)/\mu_d(HD)$ and $\mu_t(HT)/\mu_p(HT)$ are of little use. Although Neronov and Barzakh (1977) give theoretical estimates of these quantities based on their own calculations, they do not discuss the uncertainties of their estimates. To address this issue, Neronov and Karshenboim (2003) carefully examined the calculations and concluded that a reasonable estimate of the relative uncertainty is 15 %. This leads to the values for σ_{dp} and σ_{tp} in Eqs. (203) and (204) and Table 19, which we also take as input data for the 2006 adjustment. [For simplicity, we use StPtrsb-03 as the identifier in Table 19 for $\mu_p(HD)/\mu_d(HD)$, $\mu_t(HT)/\mu_p(HT)$, σ_{dp}, and σ_{tp}, because they are directly or indirectly a consequence of the work of Neronov and Karshenboim (2003).]

The equations for the measured moment ratios $\mu_p(HD)/\mu_d(HD)$ and $\mu_t(HT)/\mu_p(HT)$ in terms of the adjusted constants μ_{e^-}/μ_p, μ_d/μ_{e^-}, μ_t/μ_p, σ_{dp}, and σ_{tp} are, from Eqs. (199) and (200),

$$\frac{\mu_p(HD)}{\mu_d(HD)} = [1+\sigma_{dp}]\left(\frac{\mu_{e^-}}{\mu_p}\right)^{-1}\left(\frac{\mu_d}{\mu_{e^-}}\right)^{-1} \quad (207)$$

$$\frac{\mu_t(HT)}{\mu_p(HT)} = [1-\sigma_{tp}]\,\frac{\mu_t}{\mu_p}\ . \quad (208)$$

4. Electron to shielded proton magnetic moment ratio $\mu_{\rm e}/\mu'_{\rm p}$. Based on the measurement of the ratio of the electron moment in the 1S state of hydrogen to the shielded proton moment at 34.7 °C by Phillips *et al.* (1977) at MIT, and temperature-dependence measurements of the shielded proton moment by Petley and Donaldson (1984) at the National Physical Laboratory (NPL), Teddington, UK, we have

$$\frac{\mu_{\rm e^-}(\rm H)}{\mu'_{\rm p}} = -658.215\,9430(72) \quad [1.1\times10^{-8}] \,, \tag{209}$$

where the prime indicates that the protons are in a spherical sample of pure H_2O at 25 °C surrounded by vacuum. Hence

$$\begin{aligned}\frac{\mu_{\rm e^-}}{\mu'_{\rm p}} &= \left(\frac{g_{\rm e^-}(\rm H)}{g_{\rm e^-}}\right)^{-1}\frac{\mu_{\rm e^-}(\rm H)}{\mu'_p} \\ &= -658.227\,5971(72) \quad [1.1\times10^{-8}] \,, \end{aligned} \tag{210}$$

where the bound-state g-factor ratio is given in Table 18. Support for the MIT result in Eq. (210) from measurements at NPL on the helion (see the following section) is discussed in CODATA-02.

5. Shielded helion to shielded proton magnetic moment ratio $\mu'_{\rm h}/\mu'_{\rm p}$. The ratio of the magnetic moment of the helion h, the nucleus of the ^{3}He atom, to the magnetic moment of the proton in H_2O was determined in a high-accuracy experiment at NPL (Flowers *et al.*, 1993) with the result

$$\frac{\mu'_{\rm h}}{\mu'_{\rm p}} = -0.761\,786\,1313(33) \quad [4.3\times10^{-9}] \,. \tag{211}$$

The prime on the symbol for the helion moment indicates that the helion is not free, but is bound in a helium atom. Although the exact shape and temperature of the gaseous ^{3}He sample is unimportant, we assume that it is spherical, at 25 °C, and surrounded by vacuum.

6. Neutron to shielded proton magnetic moment ratio $\mu_{\rm n}/\mu'_{\rm p}$. Based on a measurement carried out at the Institut Max von Laue-Paul Langevin (ILL) in Grenoble, France (Greene *et al.*, 1979, 1977), we have

$$\frac{\mu_{\rm n}}{\mu'_{\rm p}} = -0.684\,996\,94(16) \quad [2.4\times10^{-7}] \,. \tag{212}$$

The observational equations for the measured values of $\mu'_{\rm h}/\mu'_{\rm p}$ and $\mu_{\rm n}/\mu'_{\rm p}$ are simply

$$\mu'_{\rm h}/\mu'_{\rm p} \doteq \mu'_{\rm h}/\mu'_{\rm p} \tag{213}$$

and

$$\mu_{\rm n}/\mu'_{\rm p} \doteq \mu_{\rm n}/\mu'_{\rm p}, \tag{214}$$

while the observational equations for the measured values of $\mu_{\rm e^-}(\rm H)/\mu_{\rm p}(\rm H)$, $\mu_{\rm d}(\rm D)/\mu_{\rm e^-}(\rm D)$, and $\mu_{\rm e^-}(\rm H)/\mu'_{\rm p}$ follow directly from Eqs. (196), (198), and (210), respectively.

2. Muonium transition frequencies, the muon-proton magnetic moment ratio $\mu_\mu/\mu_{\rm p}$, and muon-electron mass ratio $m_\mu/m_{\rm e}$

Measurements of transition frequencies between Zeeman energy levels in muonium (the μ^+e^- atom) yield values of $\mu_\mu/\mu_{\rm p}$ and the muonium ground-state hyperfine splitting $\Delta\nu_{\rm Mu}$ that depend weakly on theory. The relevant expression for the magnetic moment ratio is

$$\frac{\mu_{\mu^+}}{\mu_{\rm p}} = \frac{\Delta\nu^2_{\rm Mu} - \nu^2(f_{\rm p}) + 2s_{\rm e}f_{\rm p}\,\nu(f_{\rm p})}{4s_{\rm e}f_{\rm p}^2 - 2f_{\rm p}\,\nu(f_{\rm p})}\left(\frac{g_{\mu^+}(\rm Mu)}{g_{\mu^+}}\right)^{-1} \,, \tag{215}$$

where $\Delta\nu_{\rm Mu}$ and $\nu(f_{\rm p})$ are the sum and difference of two measured transition frequencies, $f_{\rm p}$ is the free proton NMR reference frequency corresponding to the magnetic flux density used in the experiment, $g_{\mu^+}(\rm Mu)/g_{\mu^+}$ is the bound-state correction for the muon in muonium given in Table 18, and

$$s_{\rm e} = \frac{\mu_{\rm e^-}}{\mu_{\rm p}}\frac{g_{\rm e^-}(\rm Mu)}{g_{\rm e^-}} \,, \tag{216}$$

where $g_{\rm e^-}(\rm Mu)/g_{\rm e^-}$ is the bound-state correction for the electron in muonium given in the same table.

The muon to electron mass ratio $m_\mu/m_{\rm e}$ and the muon to proton magnetic moment ratio $\mu_\mu/\mu_{\rm p}$ are related by

$$\frac{m_\mu}{m_{\rm e}} = \left(\frac{\mu_{\rm e}}{\mu_{\rm p}}\right)\left(\frac{\mu_\mu}{\mu_{\rm p}}\right)^{-1}\left(\frac{g_\mu}{g_{\rm e}}\right) \,. \tag{217}$$

The theoretical expression for the hyperfine splitting $\Delta\nu_{\rm Mu}(\rm th)$ is discussed in the following section and may be written as

$$\begin{aligned}\Delta\nu_{\rm Mu}(\rm th) &= \frac{16}{3}cR_\infty\alpha^2\frac{m_{\rm e}}{m_\mu}\left(1+\frac{m_{\rm e}}{m_\mu}\right)^{-3}\mathcal{F}(\alpha, m_{\rm e}/m_\mu) \\ &= \Delta\nu_{\rm F}\mathcal{F}(\alpha, m_{\rm e}/m_\mu) \,, \end{aligned} \tag{218}$$

where the function $\mathcal{F}$ depends weakly on α and $m_{\rm e}/m_\mu$. By equating this expression to an experimental value of $\Delta\nu_{\rm Mu}$, one can calculate a value of α from a given value of $m_\mu/m_{\rm e}$ or one can calculate a value of $m_\mu/m_{\rm e}$ from a given value of α.

1. Theory of the muonium ground-state hyperfine splitting

This section gives a brief summary of the present theory of $\Delta\nu_{\rm Mu}$, the ground-state hyperfine splitting of muonium (μ^+e^- atom). There has been essentially no change in the theory since the 2002 adjustment. Although complete results of the relevant calculations are given here, references to the original literature included in CODATA-98 or CODATA-02 are generally not repeated.

The hyperfine splitting is given mainly by the Fermi formula:

$$\Delta\nu_{\rm F} = \frac{16}{3}cR_\infty Z^3\alpha^2\frac{m_{\rm e}}{m_\mu}\left[1+\frac{m_{\rm e}}{m_\mu}\right]^{-3} \,. \tag{219}$$

Some of the theoretical expressions correspond to a muon with charge Ze rather than e in order to identify the source of the terms. The theoretical value of the hyperfine splitting is given by

$$\Delta\nu_{\mathrm{Mu}}(\mathrm{th}) = \Delta\nu_{\mathrm{D}} + \Delta\nu_{\mathrm{rad}} + \Delta\nu_{\mathrm{rec}} + \Delta\nu_{\mathrm{r\text{-}r}} + \Delta\nu_{\mathrm{weak}} + \Delta\nu_{\mathrm{had}} \ , \quad (220)$$

where the terms labeled D, rad, rec, r-r, weak, and had account for the Dirac (relativistic), radiative, recoil, radiative-recoil, electroweak, and hadronic (strong interaction) contributions to the hyperfine splitting, respectively.

The contribution $\Delta\nu_{\mathrm{D}}$, given by the Dirac equation, is

$$\Delta\nu_{\mathrm{D}} = \Delta\nu_{\mathrm{F}}(1+a_\mu)\left[1+\tfrac{3}{2}(Z\alpha)^2+\tfrac{17}{8}(Z\alpha)^4+\cdots\right] , \quad (221)$$

where a_μ is the muon magnetic moment anomaly.

The radiative corrections are written as

$$\Delta\nu_{\mathrm{rad}} = \Delta\nu_{\mathrm{F}}(1+a_\mu)\Big[D^{(2)}(Z\alpha)\left(\frac{\alpha}{\pi}\right) + D^{(4)}(Z\alpha)\left(\frac{\alpha}{\pi}\right)^2 + D^{(6)}(Z\alpha)\left(\frac{\alpha}{\pi}\right)^3 + \cdots\Big] , \quad (222)$$

where the functions $D^{(2n)}(Z\alpha)$ are contributions associated with n virtual photons. The leading term is

$$\begin{aligned} D^{(2)}(Z\alpha) = {} & A_1^{(2)} + \left(\ln 2 - \tfrac{5}{2}\right)\pi Z\alpha \\ & + \Big[-\tfrac{2}{3}\ln^2(Z\alpha)^{-2} + \left(\tfrac{281}{360}-\tfrac{8}{3}\ln 2\right)\ln(Z\alpha)^{-2} \\ & +16.9037\ldots\Big](Z\alpha)^2 \\ & + \Big[\left(\tfrac{5}{2}\ln 2 - \tfrac{547}{96}\right)\ln(Z\alpha)^{-2}\Big]\pi(Z\alpha)^3 \\ & + G(Z\alpha)(Z\alpha)^3 \ , \end{aligned} \quad (223)$$

where $A_1^{(2)} = \frac{1}{2}$, as in Eq. (83). The function $G(Z\alpha)$ accounts for all higher-order contributions in powers of $Z\alpha$, and can be divided into parts that correspond to the self-energy or vacuum polarization, $G(Z\alpha) = G_{\mathrm{SE}}(Z\alpha) + G_{\mathrm{VP}}(Z\alpha)$. We adopt the value

$$G_{\mathrm{SE}}(\alpha) = -14(2) \ , \quad (224)$$

which is the simple mean and standard deviation of the three values: $G_{\mathrm{SE}}(\alpha) = -12.0(2.0)$ from Blundell *et al.* (1997); $G_{\mathrm{SE}}(0) = -15.9(1.6)$ from Nio (2001, 2002); and $G_{\mathrm{SE}}(\alpha) = -14.3(1.1)$ from Yerokhin and Shabaev (2001). The vacuum polarization part $G_{\mathrm{VP}}(Z\alpha)$ has been calculated to several orders of $Z\alpha$ by Karshenboim *et al.* (1999, 2000). Their expression yields

$$G_{\mathrm{VP}}(\alpha) = 7.227(9) \ . \quad (225)$$

For $D^{(4)}(Z\alpha)$, as in CODATA-02, we have

$$\begin{aligned} D^{(4)}(Z\alpha) = {} & A_1^{(4)} + 0.7717(4)\pi Z\alpha + \Big[-\tfrac{1}{3}\ln^2(Z\alpha)^{-2} \\ & -0.6390\ldots\times\ln(Z\alpha)^{-2} + 10(2.5)\Big](Z\alpha)^2 \\ & +\cdots \ , \end{aligned} \quad (226)$$

where $A_1^{(4)}$ is given in Eq. (84).

Finally,

$$D^{(6)}(Z\alpha) = A_1^{(6)} + \cdots \ , \quad (227)$$

where only the leading contribution $A_1^{(6)}$ as given in Eq. (85) is known. Higher-order functions $D^{(2n)}(Z\alpha)$ with $n > 3$ are expected to be negligible.

The recoil contribution is given by

$$\begin{aligned} \Delta\nu_{\mathrm{rec}} = {} & \Delta\nu_{\mathrm{F}}\frac{m_{\mathrm{e}}}{m_\mu}\Bigg(-\frac{3}{1-(m_{\mathrm{e}}/m_\mu)^2}\ln\left(\frac{m_\mu}{m_{\mathrm{e}}}\right)\frac{Z\alpha}{\pi} \\ & + \frac{1}{(1+m_{\mathrm{e}}/m_\mu)^2}\Bigg\{\ln(Z\alpha)^{-2} - 8\ln 2 + \frac{65}{18} \\ & + \left[\frac{9}{2\pi^2}\ln^2\left(\frac{m_\mu}{m_{\mathrm{e}}}\right) + \left(\frac{27}{2\pi^2}-1\right)\ln\left(\frac{m_\mu}{m_{\mathrm{e}}}\right)\right. \\ & \left. + \frac{93}{4\pi^2} + \frac{33\zeta(3)}{\pi^2} - \frac{13}{12} - 12\ln 2\right]\frac{m_{\mathrm{e}}}{m_\mu}\Bigg\}(Z\alpha)^2 \\ & + \Bigg\{-\frac{3}{2}\ln\left(\frac{m_\mu}{m_{\mathrm{e}}}\right)\ln(Z\alpha)^{-2} - \frac{1}{6}\ln^2(Z\alpha)^{-2} \\ & + \left(\frac{101}{18} - 10\ln 2\right)\ln(Z\alpha)^{-2} \\ & + 40(10)\Bigg\}\frac{(Z\alpha)^3}{\pi}\Bigg) + \cdots \ , \end{aligned} \quad (228)$$

as discussed in CODATA-02

The radiative-recoil contribution is

$$\begin{aligned} \Delta\nu_{\mathrm{r\text{-}r}} = {} & \nu_{\mathrm{F}}\left(\frac{\alpha}{\pi}\right)^2\frac{m_{\mathrm{e}}}{m_\mu}\Bigg\{\left[-2\ln^2\left(\frac{m_\mu}{m_{\mathrm{e}}}\right) + \frac{13}{12}\ln\left(\frac{m_\mu}{m_{\mathrm{e}}}\right)\right. \\ & \left. + \frac{21}{2}\zeta(3) + \frac{\pi^2}{6} + \frac{35}{9}\right] + \left[\frac{4}{3}\ln^2\alpha^{-2}\right. \\ & \left. + \left(\frac{16}{3}\ln 2 - \frac{341}{180}\right)\ln\alpha^{-2} - 40(10)\right]\pi\alpha \\ & + \left[-\frac{4}{3}\ln^3\left(\frac{m_\mu}{m_{\mathrm{e}}}\right) + \frac{4}{3}\ln^2\left(\frac{m_\mu}{m_{\mathrm{e}}}\right)\right]\frac{\alpha}{\pi}\Bigg\} \\ & -\nu_{\mathrm{F}}\alpha^2\left(\frac{m_{\mathrm{e}}}{m_\mu}\right)^2\left(6\ln 2 + \frac{13}{6}\right) + \cdots \ , \end{aligned} \quad (229)$$

where, for simplicity, the explicit dependence on Z is not shown.

The electroweak contribution due to the exchange of a Z^0 boson is (Eides, 1996)

$$\Delta\nu_{\mathrm{weak}} = -65 \ \mathrm{Hz} \ . \quad (230)$$

For the hadronic vacuum polarization contribution we use the result of Eidelman *et al.* (2002),

$$\Delta\nu_{\mathrm{had}} = 236(4) \ \mathrm{Hz} \ , \quad (231)$$

which takes into account experimental data on the cross section for $e^-e^+ \to \pi^+\pi^-$ and on the ϕ meson leptonic

width. The leading hadronic contribution is 231.2(2.9) and the next order term is 5(2) giving a total of 236(4). The pion and kaon contributions to the hadronic correction have been considered within a chiral unitary approach and found to be in general agreement with (but have a three times larger uncertainty) the corresponding contributions given in earlier studies using data from e^+-e^- scattering (Palomar, 2003).

The standard uncertainty of $\Delta\nu_{\rm Mu}({\rm th})$ was fully discussed in Appendix E of CODATA-02. The four principle sources of uncertainty are the terms $\Delta\nu_{\rm rad}$, $\Delta\nu_{\rm rec}$, $\Delta\nu_{\rm r-r}$, and $\Delta\nu_{\rm had}$ in Eq. (220). Included in the 67 Hz uncertainty of $\Delta\nu_{\rm r-r}$ is a 41 Hz component, based on the partial calculations of Eides *et al.* (2002, 2003); Li *et al.* (1993), to account for a possible uncalculated radiative-recoil contribution of order $\Delta\nu_{\rm F}(m_{\rm e}/(m_\mu)(\alpha/\pi)^3\ln(m_\mu/m_{\rm e})$ and non-logarithmic terms. Since the completion of the 2002 adjustment, the results of additional partial calculations have been published that, if taken at face value, would lead to a small reduction in the 41 Hz estimate (D'Agostino *et al.*, 2005; Eides *et al.*, 2004; Eides and Shelyuto, 2003, 2004, 2007). However, because the calculations are not yet complete and the decrease of the 101 Hz total uncertainty assigned to $\Delta\nu_{\rm Mu}({\rm th})$ for the 2002 adjustment would only be a few percent, the Task Group decided to retain the 101 Hz uncertainty for the 2006 adjustment.

We thus have for the standard uncertainty of the theoretical expression for the muonium hyperfine splitting $\Delta\nu_{\rm Mu}({\rm th})$

$$u[\Delta\nu_{\rm Mu}({\rm th})] = 101\ {\rm Hz}\quad [2.3\times10^{-8}]\ . \tag{232}$$

For the least-squares calculations, we use as the theoretical expression for the hyperfine splitting

$$\Delta\nu_{\rm Mu}\!\left(R_\infty,\alpha,\frac{m_{\rm e}}{m_\mu},\delta_\mu,\delta_{\rm Mu}\right) = \Delta\nu_{\rm Mu}({\rm th}) + \delta_{\rm Mu}\ , \tag{233}$$

where $\delta_{\rm Mu}$ is assigned, a priori, the value

$$\delta_{\rm Mu} = 0(101)\ {\rm Hz} \tag{234}$$

in order to account for the uncertainty of the theoretical expression.

The theory summarized above predicts

$$\Delta\nu_{\rm Mu} = 4\,463\,302\,881(272)\ {\rm Hz}\quad [6.1\times10^{-8}]\ , \tag{235}$$

based on values of the constants obtained from a variation of the 2006 least-squares adjustment that omits as input data the two LAMPF measured values of $\Delta\nu_{\rm Mu}$ discussed in the following section.

The main source of uncertainty in this value is the mass ratio $m_{\rm e}/m_\mu$ that appears in the theoretical expression as an overall factor. See the text following Eq. (D14) of Appendix D of CODATA-98 for an explanation of why the relative uncertainty of the predicted value of $\Delta\nu_{\rm Mu}$ in Eq. (235) is smaller than the relative uncertainty of the electron-muon mass ratio as given in Eq. (243) of Sec. 6.2.2.3.

2. Measurements of muonium transition frequencies and values of $\mu_\mu/\mu_{\rm p}$ and $m_\mu/m_{\rm e}$

The two most precise determinations of muonium Zeeman transition frequencies were carried out at the Clinton P. Anderson Meson Physics Facility at Los Alamos (LAMPF), USA, and were reviewed in detail in CODATA-98. The following three sections and Table 20 give the key results.

1. LAMPF 1982 The results obtained by Mariam (1981); Mariam *et al.* (1982), which we take as input data in the current adjustment as in the two previous adjustments, may be summarized as follows:

$$\Delta\nu_{\rm Mu} = 4\,463\,302.88(16)\ {\rm kHz}\quad [3.6\times10^{-8}] \tag{236}$$

$$\nu(f_{\rm p}) = 627\,994.77(14)\ {\rm kHz}\quad [2.2\times10^{-7}] \tag{237}$$

$$r[\Delta\nu_{\rm Mu},\nu(f_{\rm p})] = 0.23\ , \tag{238}$$

where $f_{\rm p}$ is very nearly 57.972 993 MHz, corresponding to the flux density of about 1.3616 T used in the experiment, and $r[\Delta\nu_{\rm Mu},\nu(f_{\rm p})]$ is the correlation coefficient of $\Delta\nu_{\rm Mu}$ and $\nu(f_{\rm p})$.

2. LAMPF 1999 The results obtained by Liu *et al.* (1999), which we also take as input data in the current adjustment as in the 1998 and 2002 adjustments, may be summarized as follows:

$$\Delta\nu_{\rm Mu} = 4\,463\,302\,765(53)\ {\rm Hz}\quad [1.2\times10^{-8}] \tag{239}$$

$$\nu(f_{\rm p}) = 668\,223\,166(57)\ {\rm Hz}\quad [8.6\times10^{-8}] \tag{240}$$

$$r[\Delta\nu_{\rm Mu},\nu(f_{\rm p})] = 0.19\ , \tag{241}$$

where $f_{\rm p}$ is exactly 72.320 000 MHz, corresponding to the flux density of approximately 1.7 T used in the experiment, and $r[\Delta\nu_{\rm Mu},\nu(f_{\rm p})]$ is the correlation coefficient of $\Delta\nu_{\rm Mu}$ and $\nu(f_{\rm p})$.

3. Combined LAMPF results By carrying out a least-squares adjustment using only the LAMPF 1982 and LAMPF 1999 data, the 2006 recommended values of the quantities R_∞, $\mu_{\rm e}/\mu_{\rm p}$, $g_{\rm e}$, and g_μ, together with Eqs. (215) to (218), we obtain

$$\frac{\mu_{\mu^+}}{\mu_{\rm p}} = 3.183\,345\,24(37)\quad [1.2\times10^{-7}] \tag{242}$$

$$\frac{m_\mu}{m_{\rm e}} = 206.768\,276(24)\quad [1.2\times10^{-7}] \tag{243}$$

$$\alpha^{-1} = 137.036\,0017(80)\quad [5.8\times10^{-8}]\ , \tag{244}$$

TABLE 20 Summary of data related to the hyperfine splitting in muonium and inferred values of μ_μ/μ_p, m_μ/m_e, and α from the combined 1982 and 1999 LAMPF data.

Quantity	Value	Relative standard uncertainty u_r	Identification	Sect. and Eq.
$\Delta\nu_{Mu}$	4 463 302.88(16) kHz	3.6×10^{-8}	LAMPF-82	6.2.2.1 (236)
$\nu(f_p)$	627 994.77(14) kHz	2.2×10^{-7}	LAMPF-82	6.2.2.1 (237)
$\Delta\nu_{Mu}$	4 463 302 765(53) Hz	1.2×10^{-8}	LAMPF-99	6.2.2.2 (239)
$\nu(f_p)$	668 223 166(57) Hz	8.6×10^{-8}	LAMPF-99	6.2.2.2 (240)
μ_μ/μ_p	3.183 345 24(37)	1.2×10^{-7}	LAMPF	6.2.2.3 (242)
m_μ/m_e	206.768 276(24)	1.2×10^{-7}	LAMPF	6.2.2.3 (243)
α^{-1}	137.036 0017(80)	5.8×10^{-8}	LAMPF	6.2.2.3 (244)

where this value of α may be called the muonium value of the fine-structure constant and denoted as $\alpha^{-1}(\Delta\nu_{Mu})$.

It is noteworthy that the uncertainty of the value of the mass ratio m_μ/m_e given in Eq. (243) is about four times the uncertainty of the 2006 recommended value. The reason is that taken together, the experimental value of and theoretical expression for the hyperfine splitting essentially determine only the value of the product $\alpha^2 m_\mu/m_e$, as is evident from Eq. (218). In the full adjustment the value of α is determined by other data with an uncertainty significantly smaller than that of the value in Eq. (244), which in turn determines the value of m_μ/m_e with a smaller uncertainty than that of Eq. (243).

7. ELECTRICAL MEASUREMENTS

This section is devoted to the discussion of quantities that require electrical measurements of the most basic kind for their determination: the gyromagnetic ratios of the shielded proton and helion, the von Klitzing constant R_K, the Josephson constant K_J, the product $K_J^2 R_K$, and the Faraday constant. However, some of the results we discuss were taken as input data for the 2002 adjustment but were not included in the final least-squares adjustment from which the 2002 recommended values were obtained, mainly because of their comparatively large uncertainties and hence low weight. Nevertheless, we take them as input data in the 2006 adjustment because they provide information on the overall consistency of the available data and tests of the exactness of the relations $K_J = 2e/h$ and $R_K = h/e^2$. The lone exception is the low-field measurement of the gyromagnetic ratio of the helion reported by Tarbeev *et al.* (1989). Because of its large uncertainty and strong disagreement with many other data, we no longer consider it—see CODATA-02.

1. Shielded gyromagnetic ratios γ', the fine-structure constant α, and the Planck constant h

The gyromagnetic ratio γ of a bound particle of spin quantum number i and magnetic moment μ is given by

$$\gamma = \frac{2\pi f}{B} = \frac{\omega}{B} = \frac{|\mu|}{i\hbar}\ , \qquad (245)$$

where f is the precession (that is, spin-flip) frequency and ω is the angular precession frequency of the particle in the magnetic flux density B. The SI unit of γ is $\mathrm{s^{-1}\ T^{-1}} = \mathrm{C\ kg^{-1}} = \mathrm{A\ s\ kg^{-1}}$. In this section we summarize measurements of the gyromagnetic ratio of the shielded proton

$$\gamma'_p = \frac{2\mu'_p}{\hbar}\ , \qquad (246)$$

and of the shielded helion

$$\gamma'_h = \frac{2|\mu'_h|}{\hbar}\ , \qquad (247)$$

where, as in previous sections that dealt with magnetic-moment ratios involving these particles, the protons are those in a spherical sample of pure H_2O at 25 °C surrounded by vacuum; and the helions are those in a spherical sample of low-pressure, pure ^{3}He gas at 25 °C surrounded by vacuum.

As discussed in detail in CODATA-98, two methods are used to determine the shielded gyromagnetic ratio γ' of a particle: the low-field method and the high-field method. In either case the measured current I in the experiment can be expressed in terms of the product $K_J R_K$, but B depends on I differently in the two cases. In essence, the low-field experiments determine $\gamma'/K_J R_K$ and the high-field experiments determine $\gamma' K_J R_K$. This leads to the relations

$$\gamma' = \Gamma'_{90}(\mathrm{lo})\frac{K_J\,R_K}{K_{J-90}\,R_{K-90}} \qquad (248)$$

$$\gamma' = \Gamma'_{90}(\mathrm{hi})\frac{K_{J-90}\,R_{K-90}}{K_J\,R_K}\ , \qquad (249)$$

where $\Gamma'_{90}(\text{lo})$ and $\Gamma'_{90}(\text{hi})$ are the experimental values of γ' in SI units that would result from the low- and high-field experiments if K_J and R_K had the exactly known conventional values of $K_{\text{J}-90}$ and $R_{\text{K}-90}$, respectively. The quantities $\Gamma'_{90}(\text{lo})$ and $\Gamma'_{90}(\text{hi})$ are the input data used in the adjustment, but the observational equations take into account the fact that $K_{\text{J}-90} \neq K_\text{J}$ and $R_{\text{K}-90} \neq R_\text{K}$.

Accurate values of $\Gamma'_{90}(\text{lo})$ and $\Gamma'_{90}(\text{hi})$ for the proton and helion are of potential importance because they provide information on the values of α and h. Assuming the validity of the relations $K_\text{J} = 2e/h$ and $R_\text{K} = h/e^2$, the following expressions apply to the four available proton results and one available helion result:

$$\Gamma'_{\text{p}-90}(\text{lo}) = \frac{K_{\text{J}-90}\, R_{\text{K}-90}\, g_{\text{e}^-}}{4\mu_0 R_\infty} \frac{\mu'_\text{p}}{\mu_{\text{e}^-}} \alpha^3 \,, \tag{250}$$

$$\Gamma'_{\text{h}-90}(\text{lo}) = -\frac{K_{\text{J}-90}\, R_{\text{K}-90}\, g_{\text{e}^-}}{4\mu_0 R_\infty} \frac{\mu'_\text{h}}{\mu_{\text{e}^-}} \alpha^3 \,, \tag{251}$$

$$\Gamma'_{\text{p}-90}(\text{hi}) = \frac{c\,\alpha^2 g_{\text{e}^-}}{2K_{\text{J}-90}\, R_{\text{K}-90}\, R_\infty} \frac{\mu'_\text{p}}{\mu_{\text{e}^-}} \frac{1}{h} \,. \tag{252}$$

Since the five experiments, including necessary corrections, were discussed fully in CODATA-98, only a brief summary is given in the following sections. The five results, together with the value of α inferred from each low-field measurement and the value of h inferred from each high-field measurement, are collected in Table 21.

1. Low-field measurements

A number of national metrology institutes have long histories of measuring the gyromagnetic ratio of the shielded proton, motivated, in part, by their need to monitor the stability of their practical unit of current based on groups of standard cells and standard resistors. This was prior to the development of the Josephson and quantum hall effects for the realization of practical electric units.

1. NIST: Low field The most recent National Institute of Standards and Technology (NIST), Gaithersburg, USA, low-field measurement was reported by Williams *et al.* (1989). Their result is

$$\Gamma'_{\text{p}-90}(\text{lo}) = 2.675\,154\,05(30) \times 10^8 \text{ s}^{-1} \text{ T}^{-1} \quad [1.1 \times 10^{-7}] \,, \tag{253}$$

where $\Gamma'_{\text{p}-90}(\text{lo})$ is related to γ'_p by Eq. (248).

The value of α that may be inferred from this result follows from Eq. (250). Using the 2006 recommended values for the other relevant quantities, the uncertainties of which are significantly smaller than the uncertainty of the NIST result (statements that also apply to the following four similar calculations), we obtain

$$\alpha^{-1} = 137.035\,9879(51) \quad [3.7 \times 10^{-8}] \,, \tag{254}$$

where the relative uncertainty is about one-third the relative uncertainty of the NIST value of $\Gamma'_{\text{p}-90}(\text{lo})$ because of the cube-root dependence of α on $\Gamma'_{\text{p}-90}(\text{lo})$.

2. NIM: Low field The latest low-field proton gyromagnetic ratio experiment carried out by researchers at the National Institute of Metrology (NIM), Beijing, PRC, yielded (Liu *et al.*, 1995)

$$\Gamma'_{\text{p}-90}(\text{lo}) = 2.675\,1530(18) \times 10^8 \text{ s}^{-1} \text{ T}^{-1} \quad [6.6 \times 10^{-7}] \,. \tag{255}$$

Based on Eq. (250), the inferred value of α from the NIM result is

$$\alpha^{-1} = 137.036\,006(30) \quad [2.2 \times 10^{-7}] \,. \tag{256}$$

3. KRISS/VNIIM: Low field The determination of γ'_h at the Korea Research Institute of Standards and Science (KRISS), Taedok Science Town, Republic of Korea, was carried out in a collaborative effort with researchers from the Mendeleyev All-Russian Research Institute for Metrology (VNIIM), St. Petersburg, Russian Federation (Kim *et al.*, 1995; Park *et al.*, 1999; Shifrin *et al.*, 1998a,b, 1999). The result of this work can be expressed as

$$\Gamma'_{\text{h}-90}(\text{lo}) = 2.037\,895\,37(37) \times 10^8 \text{ s}^{-1} \text{ T}^{-1} \quad [1.8 \times 10^{-7}] \,, \tag{257}$$

and the value of α that may be inferred from it through Eq. (251) is

$$\alpha^{-1} = 137.035\,9852(82) \quad [6.0 \times 10^{-8}] \,. \tag{258}$$

2. High-field measurements

1. NIM:high field The latest high-field proton gyromagnetic ratio experiment at NIM yielded (Liu *et al.*, 1995)

$$\Gamma'_{\text{p}-90}(\text{hi}) = 2.675\,1525(43) \times 10^8 \text{ s}^{-1} \text{ T}^{-1} \quad [1.6 \times 10^{-6}] \,, \tag{259}$$

where $\Gamma'_{\text{p}-90}(\text{hi})$ is related to γ'_p by Eq. (249). Its correlation coefficient with the NIM low-field result in Eq. (255) is

$$r(\text{lo}, \text{hi}) = -0.014 \,. \tag{260}$$

Based on Eq. (252), the value of h that may be inferred from the NIM high-field result is

$$h = 6.626\,071(11) \times 10^{-34} \text{ J s} \quad [1.6 \times 10^{-6}] \,. \tag{261}$$

TABLE 21 Summary of data related to shielded gyromagnetic ratios of the proton and helion, and inferred values of α and h.

Quantity	Value	Relative standard uncertainty u_r	Identification	Sect. and Eq.
$\Gamma'_{p-90}(\mathrm{lo})$	$2.675\,154\,05(30)\times 10^{8}$ s^{-1} T^{-1}	1.1×10^{-7}	NIST-89	7.1.1.1 (253)
α^{-1}	$137.035\,9879(51)$	3.7×10^{-8}		7.1.1.1 (254)
$\Gamma'_{p-90}(\mathrm{lo})$	$2.675\,1530(18)\times 10^{8}$ s^{-1} T^{-1}	6.6×10^{-7}	NIM-95	7.1.1.2 (255)
α^{-1}	$137.036\,006(30)$	2.2×10^{-7}		7.1.1.2 (256)
$\Gamma'_{h-90}(\mathrm{lo})$	$2.037\,895\,37(37)\times 10^{8}$ s^{-1} T^{-1}	1.8×10^{-7}	KR/VN-98	7.1.1.3 (257)
α^{-1}	$137.035\,9852(82)$	6.0×10^{-8}		7.1.1.3 (258)
$\Gamma'_{p-90}(\mathrm{hi})$	$2.675\,1525(43)\times 10^{8}$ s^{-1} T^{-1}	1.6×10^{-6}	NIM-95	7.1.2.1 (259)
h	$6.626\,071(11)\times 10^{-34}$ J s	1.6×10^{-6}		7.1.2.1 (261)
$\Gamma'_{p-90}(\mathrm{hi})$	$2.675\,1518(27)\times 10^{8}$ s^{-1} T^{-1}	1.0×10^{-6}	NPL-79	7.1.2.2 (262)
h	$6.626\,0729(67)\times 10^{-34}$ J s	1.0×10^{-6}		7.1.2.2 (263)

2. NPL: High field The most accurate high-field γ'_p experiment was carried out at NPL by Kibble and Hunt (1979), with the result

$$\Gamma'_{p-90}(\mathrm{hi}) = 2.675\,1518(27)\times 10^{8}\ \mathrm{s}^{-1}\ \mathrm{T}^{-1}\quad [1.0\times 10^{-6}]\ . \qquad (262)$$

This leads to the inferred value

$$h = 6.626\,0729(67)\times 10^{-34}\ \mathrm{J\ s}\quad [1.0\times 10^{-6}]\ , \qquad (263)$$

based on Eq. (252).

2. von Klitzing constant R_K and α

Since the the quantum Hall effect, the von Klitzing constant R_K associated with it, and the available determinations of R_K are fully discussed in CODATA-98 and CODATA-02, we only outline the main points here.

The quantity R_K is measured by comparing a quantized Hall resistance $R_H(i) = R_K/i$, where i is an integer, to a resistance R whose value is known in terms of the SI unit of resistance Ω. In practice, the latter quantity, the ratio R/Ω, is determined by means of a calculable cross capacitor, a device based on a theorem in electrostatics discovered in the 1950s (Lampard, 1957; Thompson and Lampard, 1956). The theorem allows one to construct a cylindrical capacitor, generally called a Thompson-Lampard calculable capacitor (Thompson, 1959), whose capacitance, to high accuracy, depends only on its length.

As indicated in Sec. 2, if one assumes the validity of the relation $R_K = h/e^2$, then R_K and the fine-structure constant α are related by

$$\alpha = \mu_0 c/2R_K\ . \qquad (264)$$

Hence, the relative uncertainty of the value of α that may be inferred from a particular experimental value of R_K is the same as the relative uncertainty of that value.

The values of R_K we take as input data in the 2006 adjustment and the corresponding inferred values values of α are given in the following sections and are summarized in Table 22.

1. NIST: Calculable capacitor

The result obtained at NIST is (Jeffery *et al.*, 1997) [see also Jeffery *et al.* (1998)]

$$\begin{aligned} R_K &= 25\,812.8\,[1+0.322(24)\times 10^{-6}]\ \Omega \\ &= 25\,812.808\,31(62)\ \Omega\quad [2.4\times 10^{-8}]\ , \end{aligned} \qquad (265)$$

and is viewed as superseding the NIST result reported in 1989 by Cage *et al.* (1989). Work by Jeffery *et al.* (1999) provides additional support for the uncertainty budget of the NIST calculable capacitor.

The value of α that may be inferred from the NIST value of R_K is, from Eq. (264),

$$\alpha^{-1} = 137.036\,0037(33)\quad [2.4\times 10^{-8}]\ . \qquad (266)$$

2. NMI: Calculable capacitor

Based on measurements carried out at the National Metrology Institute (NMI), Lindfield, Australia, from December 1994 to April 1995 and a complete reassessment of uncertainties associated with their calculable capacitor and associated apparatus, Small *et al.* (1997) reported the result

$$\begin{aligned} R_K &= R_{K-90}\,[1+0.4(4.4)\times 10^{-8}] \\ &= 25\,812.8071(11)\ \Omega\quad [4.4\times 10^{-8}]\ . \end{aligned} \qquad (267)$$

The value of α it implies is

$$\alpha^{-1} = 137.035\,9973(61)\quad [4.4\times 10^{-8}]\ . \qquad (268)$$

TABLE 22 Summary of data related to the von Klitzing constant R_{K} and inferred values of α.

Quantity	Value	Relative standard uncertainty u_{r}	Identification	Sect. and Eq.
R_{K}	$25\,812.808\,31(62)\ \Omega$	2.4×10^{-8}	NIST-97	7.2.1 (265)
α^{-1}	$137.036\,0037(33)$	2.4×10^{-8}		7.2.1 (266)
R_{K}	$25\,812.8071(11)\ \Omega$	4.4×10^{-8}	NMI-97	7.2.2 (267)
α^{-1}	$137.035\,9973(61)$	4.4×10^{-8}		7.2.2 (268)
R_{K}	$25\,812.8092(14)\ \Omega$	5.4×10^{-8}	NPL-88	7.2.3 (269)
α^{-1}	$137.036\,0083(73)$	5.4×10^{-8}		7.2.3 (270)
R_{K}	$25\,812.8084(34)\ \Omega$	1.3×10^{-7}	NIM-95	7.2.4 (271)
α^{-1}	$137.036\,004(18)$	1.3×10^{-7}		7.2.4 (272)
R_{K}	$25\,812.8081(14)\ \Omega$	5.3×10^{-8}	LNE-01	7.2.5 (273)
α^{-1}	$137.036\,0023(73)$	5.3×10^{-8}		7.2.5 (274)

Because of problems associated with the 1989 NMI value of R_{K}, only the result reported in 1997 is used in the 2006 adjustment, as was the case in the 1998 and 2002 adjustments.

3. NPL: Calculable capacitor

The NPL calculable capacitor is similar in design to those of NIST and NMI. The result for R_{K} reported by Hartland *et al.* (1988) is

$$\begin{aligned} R_{\mathrm{K}} &= 25\,812.8\,[1 + 0.356(54) \times 10^{-6}]\ \Omega \\ &= 25\,812.8092(14)\ \Omega \quad [5.4 \times 10^{-8}]\,, \end{aligned} \tag{269}$$

and the value of α that one may infer from it is

$$\alpha^{-1} = 137.036\,0083(73) \quad [5.4 \times 10^{-8}]\,. \tag{270}$$

4. NIM: Calculable capacitor

The NIM calculable cross capacitor differs markedly from the version used at NIST, NMI, and NPL. The four bars (electrodes) that comprise the capacitor are horizontal rather than vertical and the length that determines its known capacitance is fixed rather than variable. The NIM result for R_{K}, as reported by Zhang *et al.* (1995), is

$$R_{\mathrm{K}} = 25\,812.8084(34)\ \Omega \quad [1.3 \times 10^{-7}]\,, \tag{271}$$

which implies

$$\alpha^{-1} = 137.036\,004(18) \quad [1.3 \times 10^{-7}]\,. \tag{272}$$

5. LNE: Calculable capacitor

The value of R_{K} obtained at the Laboratoire National d'Essais (LNE), Trappes, France, is (Trapon *et al.*, 2003, 2001)

$$R_{\mathrm{K}} = 25\,812.8081(14)\ \Omega \quad [5.3 \times 10^{-8}]\,, \tag{273}$$

which implies

$$\alpha^{-1} = 137.036\,0023(73) \quad [5.3 \times 10^{-8}]\,. \tag{274}$$

The LNE Thompson-Lampard calculable capacitor is unique among all calculable capacitors in that it consists of five horizontal bars arranged at the corners of a regular pentagon.

3. Josephson constant K_{J} and h

Again, since the Josephson effect, the Josephson constant K_{J} associated with it, and the available determinations of K_{J} are fully discussed in CODATA-98 and CODATA-02, we only outline the main points here.

The quantity K_{J} is measured by comparing a Josephson voltage $U_{\mathrm{J}}(n) = nf/K_{\mathrm{J}}$ to a high voltage U whose value is known in terms of the SI unit of voltage V. Here, n is an integer and f is the frequency of the microwave radiation applied to the Josephson device. In practice, the latter quantity, the ratio U/V, is determined by counterbalancing an electrostatic force arising from the voltage U with a known gravitational force.

A measurement of K_{J} can also provide a value of h. If, as discussed in Sec. 2, we assume the validity of the relation $K_{\mathrm{J}} = 2e/h$ and recall that $\alpha = e^2/4\pi\epsilon_0\hbar = \mu_0 c e^2/2h$, we have

$$h = \frac{8\alpha}{\mu_0 c K_{\mathrm{J}}^2}\ . \tag{275}$$

Since u_{r} of the fine-structure constant is significantly smaller than u_{r} of the measured values of K_{J}, the u_{r} of h derived from Eq. (275) will be essentially twice the u_{r} of K_{J}.

The values of $K_{\rm J}$ we take as input data in the 2006 adjustment, and the corresponding inferred values of h, are given in the following two sections and are summarized in Table 23. Also summarized in that table are the measured values of the product $K_{\rm J}^2R_{\rm K}$ and the quantity $\mathcal{F}_{90}$ related to the Faraday constant F, together with their corresponding inferred values of h. These results are discussed below in Secs. 7.4 and 7.5.

1. NMI: Hg electrometer

The determination of $K_{\rm J}$ at NMI, carried out using an apparatus called a liquid-mercury electrometer, yielded the result (Clothier *et al.*, 1989)

$$\begin{aligned} K_{\rm J} &= 483\,594\left[1+8.087(269)\times10^{-6}\right]\ \text{GHz/V} \\ &= 483\,597.91(13)\ \text{GHz/V} \quad [2.7\times10^{-7}]\ . \end{aligned} \quad (276)$$

Equation (275), the NMI value of $K_{\rm J}$, and the 2006 recommended value of α, which has a much smaller $u_{\rm r}$, yields an inferred value for the Planck constant of

$$h = 6.626\,0684(36)\times10^{-34}\ \text{J s} \quad [5.4\times10^{-7}]\ . \quad (277)$$

2. PTB: Capacitor voltage balance

The determination of $K_{\rm J}$ at PTB was carried out by using a voltage balance consisting of two coaxial cylindrical electrodes (Funck and Sienknecht, 1991; Sienknecht and Funck, 1985, 1986). Taking into account the correction associated with the reference capacitor used in the PTB experiment as described in CODATA-98, the result of the PTB determination is

$$K_{\rm J} = 483\,597.96(15)\ \text{GHz/V} \quad [3.1\times10^{-7}]\ , \quad (278)$$

from which we infer, using Eq. (275),

$$h = 6.626\,0670(42)\times10^{-34}\ \text{J s} \quad [6.3\times10^{-7}]\ . \quad (279)$$

4. Product $K_{\rm J}^2R_{\rm K}$ and h

A value of the product $K_{\rm J}^2R_{\rm K}$ is of importance to the determination of the Planck constant h, because if one assumes that the relations $K_{\rm J}=2e/h$ and $R_{\rm K}=h/e^2$ are valid, then

$$h = \frac{4}{K_{\rm J}^2R_{\rm K}}\ . \quad (280)$$

The product $K_{\rm J}^2R_{\rm K}$ is determined by comparing electrical power known in terms of a Josephson voltage and quantized Hall resistance to the equivalent mechanical power known in the SI unit W $=$ m^2 kg s^{-3}. The comparison is carried out using an apparatus known as a moving-coil watt balance first proposed by Kibble (1975) at NPL. To date two laboratories, NPL and NIST, have determined $K_{\rm J}^2R_{\rm K}$ using this method.

1. NPL: Watt balance

Shortly after Kibble's original proposal in 1975, Kibble and Robinson (1977) carried out a feasibility study of the idea based on experience with the NPL apparatus that was used to determine $\gamma_{\rm p}'$ by the high-field method (Kibble and Hunt, 1979). The work continued and led to the publication in 1990 by Kibble *et al.* (1990) of a result with an uncertainty of about 2 parts in 10^7. This result, discussed in detail in CODATA-98 and which was taken as an input datum in the 1998 and 2002 adjustments, and which we also take as an input datum in the 2006 adjustment, may be expressed as

$$\begin{aligned} K_{\rm J}^2R_{\rm K} &= K_{\rm J-NPL}^2R_{\rm K-NPL}[1+16.14(20)\times10^{-6}] \\ &= 6.036\,7625(12)\times10^{33}\ \text{J}^{-1}\ \text{s}^{-1} \\ &\qquad [2.0\times10^{-7}]\ , \end{aligned} \quad (281)$$

where $K_{\rm J-NPL}=483\,594$ GHz/V and $R_{\rm K-NPL}=25\,812.809\,2\ \Omega$. The value of h that may be inferred from the NPL result is, according to Eq. (280),

$$h = 6.626\,0682(13)\times10^{-34}\ \text{J s} \quad [2.0\times10^{-7}]\ . \quad (282)$$

Based on the experience gained in this experiment, NPL researchers designed and constructed what is essentially a completely new apparatus, called the NPL Mark II watt balance, that could possibly achieve a result for $K_{\rm J}^2R_{\rm K}$ with an uncertainty of a few parts in 10^8 (Kibble and Robinson, 2003; Robinson and Kibble, 1997). Although the balance itself employs the same balance beam as the previous NPL watt balance, little else from that experiment is retained in the new experiment.

Over 1000 measurements in vacuum were carried out with the MK II between January 2000 and November 2001. Many were made in an effort to identify the cause of an observed fractional change in the value of $K_{\rm J}^2R_{\rm K}$ of about 3×10^{-7} that occurred in mid-April 2000 (Robinson and Kibble, 2002). A change in the alignment of the apparatus was suspected of contributing to the shift.

Significant improvements were subsequently made in the experiment and very recently, based on measurements carried out from October 2006 to March 2007, the initial result from MK II, $h=6.626\,070\,95(44)$ J s $[6.6\times10^{-8}]$, was reported by Robinson and Kibble (2007) assuming the validity of Eq. (280). Although this result became available much too late to be considered for the 2006 adjustment, we do note that it lies between the value of h inferred from the 2007 NIST result for $K_{\rm J}^2R_{\rm K}$ discussed in Sec. 7.4.2.2, and that inferred from the measurement of the molar volume of silicon $V_{\rm m}(\text{Si})$ discussed in Sec. 8.2. The NPL work is continuing and a result with a smaller uncertainty is anticipated (Robinson and Kibble, 2007).

2. NIST: Watt balance

TABLE 23 Summary of data related to the Josephson constant K_J, the product $K_J^2 R_K$, and the Faraday constant F, and inferred values of h.

Quantity	Value	Relative standard uncertainty u_r	Identification	Sect. and Eq.
K_J	$483\,597.91(13)$ GHz V^{-1}	2.7×10^{-7}	NMI-89	7.3.1 (276)
h	$6.626\,0684(36) \times 10^{-34}$ J s	5.4×10^{-7}		7.3.1 (277)
K_J	$483\,597.96(15)$ GHz V^{-1}	3.1×10^{-7}	PTB-91	7.3.2 (278)
h	$6.626\,0670(42) \times 10^{-34}$ J s	6.3×10^{-7}		7.3.2 (279)
$K_J^2 R_K$	$6.036\,7625(12) \times 10^{33}$ J^{-1} s^{-1}	2.0×10^{-7}	NPL-90	7.4.1 (281)
h	$6.626\,0682(13) \times 10^{-34}$ J s	2.0×10^{-7}		7.4.1 (282)
$K_J^2 R_K$	$6.036\,761\,85(53) \times 10^{33}$ J^{-1} s^{-1}	8.7×10^{-8}	NIST-98	7.4.2.1 (283)
h	$6.626\,068\,91(58) \times 10^{-34}$ J s	8.7×10^{-8}		7.4.2.1 (284)
$K_J^2 R_K$	$6.036\,761\,85(22) \times 10^{33}$ J^{-1} s^{-1}	3.6×10^{-8}	NIST-07	7.4.2.2 (287)
h	$6.626\,068\,91(24) \times 10^{-34}$ J s	3.6×10^{-8}		7.4.2.2 (288)
$\mathcal{F}_{90}$	$96\,485.39(13)$ C mol^{-1}	1.3×10^{-6}	NIST-80	7.5.1 (295)
h	$6.626\,0657(88) \times 10^{-34}$ J s	1.3×10^{-6}		7.5.1 (296)

1. 1998 measurement Work on a moving-coil watt balance at NIST began shortly after Kibble made his 1975 proposal. A first result with $u_r = 1.3 \times 10^{-6}$ was reported by NIST researchers in 1989 (Cage *et al.*, 1989). Significant improvements were then made to the apparatus and the final result from this phase of the NIST effort was reported in 1998 by Williams *et al.* (1998):

$$\begin{aligned} K_J^2 R_K &= K_{J-90}^2 R_{K-90}[1 - 8(87) \times 10^{-9}] \\ &= 6.036\,761\,85(53) \times 10^{33}\ \mathrm{J^{-1}\ s^{-1}} \\ &\qquad [8.7 \times 10^{-8}]\ . \end{aligned} \tag{283}$$

A lengthy paper giving the details of the NIST 1998 watt balance experiment was published in 2005 by Steiner *et al.* (2005a). This was the NIST result taken as an input datum in the 1998 and 2002 adjustments; although the 1989 result was consistent with that of 1998, its uncertainty was about 15 times larger. The value of h implied by the 1998 NIST result for $K_J^2 R_K$ is

$$h = 6.626\,068\,91(58) \times 10^{-34}\ \mathrm{J\ s} \quad [8.7 \times 10^{-8}]\ . \tag{284}$$

2. 2007 measurement Based on the lessons learned in the decade-long effort with a watt balance operating in air that led to their 1998 result for $K_J^2 R_K$, the NIST watt-balance researchers initiated a new program with the goal of measuring $K_J^2 R_K$ with $u_r \approx 10^{-8}$. The experiment was completely disassembled and renovations to the research facility were made to improve vibration isolation, reduce electromagnetic interference, and incorporate a multilayer temperature control system. A new watt balance with major changes and improvements was constructed with little remaining of the earlier apparatus except the superconducting magnet used to generate the required radial magnetic flux density and the wheel used as the balance.

The most notable change in the experiment is that in the new apparatus, the entire balance mechanism and moving coil are in vacuum, which eliminates the uncertainties of the corrections in the previous experiment for the index of refraction of air in the laser position measurements ($u_r = 43 \times 10^{-9}$) and for the buoyancy force exerted on the mass standard ($u_r = 23 \times 10^{-9}$). Alignment uncertainties were reduced by over a factor of four by (i) incorporating a more comprehensive understanding of all degrees of freedom involving the moving coil; and (ii) the application of precise alignment techniques for all degrees of freedom involving the moving coil, the superconducting magnet, and the velocity measuring interferometers. Hysteresis effects were reduced by a factor of four by using a diamond-like carbon coated knife edge and flat (Schwarz *et al.*, 2001), employing a hysteresis erasure procedure, and reducing the balance deflections during mass exchanges with improved control systems. A programmable Josephson array voltage standard (Benz *et al.*, 1997) was connected directly to the experiment, eliminating two voltage transfers required in the old experiment and reducing the voltage traceability uncertainty by a factor of 15.

A total of 6023 individual values of W_{90}/W were obtained over the two year period from March 2003 to February 2005 as part of the effort to develop and improve the new experiment. The results are converted to the notation used here by the relation $W_{90}/\mathrm{W} = K_{J-90}^2 R_{K-90}/K_J^2 R_K$ discussed in CODATA-98. The initial result from that work was reported in 2005 by Steiner

et al. (2005b):

$$\begin{aligned} K_{\rm J}^2 R_{\rm K} &= K_{\rm J-90}^2 R_{\rm K-90}[1 - 24(52)\times 10^{-9}] \\ &= 6.036\,761\,75(31)\times 10^{33}\ \mathrm{J^{-1}\ s^{-1}} \\ &\qquad [5.2\times 10^{-8}]\ . \end{aligned} \tag{285}$$

This yields a value for the Planck constant of

$$h = 6.626\,069\,01(34)\times 10^{-34}\ \mathrm{J\ s} \quad [5.2\times 10^{-8}]\ . \tag{286}$$

This result for $K_{\rm J}^2 R_{\rm K}$ was obtained from data spanning the final 7 months of the 2 year period. It is based on the weighted mean of 48 W_{90}/W measurement sets using a Au mass standard and 174 sets using a PtIr mass standard, where a typical measurement set consists of 12 to 15 individual values of W_{90}/W. The 2005 NIST result is consistent with the 1998 NIST result but its uncertainty has been reduced by a factor of 1.7.

Following this initial effort with the new apparatus, further improvements were made to it in order to reduce the uncertainties from various systematic effects, the most notable reductions being in the determination of the local acceleration due to gravity g (a factor of 2.5), the effect of balance wheel surface roughness (a factor of 10), and the effect of the magnetic susceptibility of the mass standard (a factor of 1.6). An improved result was then obtained based on 2183 values of W_{90}/W recorded in 134 measurement sets from January 2006 to June 2006. Due to a wear problem with the gold mass standard, only a PtIr mass standard was used in these measurements. The result, first reported at a conference in 2006 and subsequently published in the proceedings of the conference in 2007 by Steiner *et al.* (2007), is

$$\begin{aligned} K_{\rm J}^2 R_{\rm K} &= K_{\rm J-90}^2 R_{\rm K-90}[1 - 8(36)\times 10^{-9}] \\ &= 6.036\,761\,85(22)\times 10^{33}\ \mathrm{J^{-1}\ s^{-1}} \\ &\qquad [3.6\times 10^{-8}]\ . \end{aligned} \tag{287}$$

The value of h that may be inferred from this value of $K_{\rm J}^2 R_{\rm K}$ is

$$h = 6.626\,068\,91(24)\times 10^{-34}\ \mathrm{J\ s} \quad [3.6\times 10^{-8}]\ . \tag{288}$$

The 2007 NIST result for $K_{\rm J}^2 R_{\rm K}$ is consistent with and has an uncertainty smaller by a factor of 1.4 than the uncertainty of the 2005 NIST result. However, because the two results are from essentially the same experiment and hence are highly correlated, we take only the 2007 result as an input datum in the 2006 adjustment.

On the other hand, the experiment on which the NIST 2007 result is based is only slightly dependent on the experiment on which the NIST 1998 result is based, as can be seen from the above discussions. Thus, in keeping with our practice in similar cases, most notably the 1982 and 1999 LAMPF measurements of muonium Zeeman transition frequencies (see Sec. 6.2.2), we also take the NIST 1998 result in Eq. (283) as an input datum in the 2006 adjustment. But to ensure that we do not give undue weight to the NIST work, an analysis of the uncertainty budgets of the 1998 and 2007 NIST results was performed to determine the level of correlation. Of the relative uncertainty components listed in Table II of Williams *et al.* (1998) and in Table 2 of Steiner *et al.* (2005b) but as updated in Table 1 of Steiner *et al.* (2007), the largest common relative uncertainty components were from the magnetic flux profile fit due to the use of the same analysis routine (16×10^{-9}); leakage resistance and electrical grounding since the same current supply was used in both experiments (10×10^{-9}); and the determination of the local gravitational acceleration g due to the use of the same absolute gravimeter (7×10^{-9}). The correlation coefficient was thus determined to be

$$r(K_{\rm J}^2\, R_{\rm k}\text{-}98, K_{\rm J}^2\, R_{\rm k}\text{-}07) = 0.14\ , \tag{289}$$

which we take into account in our calculations as appropriate.

3. Other values

Although there is no competitive published value of $K_{\rm J}^2 R_{\rm K}$ other than those from NPL and NIST discussed above, it is worth noting that at least three additional laboratories have watt-balance experiments in progress: the Swiss Federal Office of Metrology and Accreditation (METAS), Bern-Wabern, Switzerland, the LNE, and the BIPM. Descriptions of these efforts may be found in the papers by Beer *et al.* (2003), Genevès *et al.* (2005), and Picard *et al.* (2007), respectively.

4. Inferred value of $K_{\rm J}$

It is of interest to note that a value of $K_{\rm J}$ with an uncertainty significantly smaller than those of the directly measured values discussed in Sec. 7.3 can be obtained from the directly measured watt-balance values of $K_{\rm J}^2 R_{\rm K}$, together with the directly measured calculable-capacitor values of $R_{\rm K}$, without assuming the validity of the relations $K_{\rm J} = 2e/h$ and $R_{\rm K} = h/e^2$. The relevant expression is simply $K_{\rm J} = [(K_{\rm J}^2 R_{\rm K})_{\rm W}/(R_{\rm K})_{\rm C}]^{1/2}$, where $(K_{\rm J}^2 R_{\rm K})_{\rm W}$ is from the watt-balance, and $(R_{\rm K})_{\rm C}$ is from the calculable capacitor.

Using the weighted mean of the three watt-balance results for $K_{\rm J}^2 R_{\rm K}$ discussed in this section and the weighted mean of the five calculable-capacitor results for $R_{\rm K}$ discussed in Sec 7.2, we have

$$\begin{aligned} K_{\rm J} &= K_{\rm J-90}[1 - 2.8(1.9)\times 10^{-8}] \\ &= 483\,597.8865(94)\ \mathrm{GHz/V} \quad [1.9\times 10^{-8}]\ , \end{aligned} \tag{290}$$

which is consistent with the directly measured values but has an uncertainty that is smaller by more than an order of magnitude. This result is implicitly included in the least-squares adjustment, even though the explicit value for $K_{\rm J}$ obtained here is not used as an input datum.

5. Faraday constant F and h

The Faraday constant F is equal to the Avogadro constant $N_{\rm A}$ times the elementary charge e, $F = N_{\rm A}e$; its SI unit is coulomb per mol, C mol^{-1} = A s mol^{-1}. It determines the amount of substance $n(X)$ of an entity X that is deposited or dissolved during electrolysis by the passage of a quantity of electricity, or charge, $Q = It$, due to the flow of a current I in a time t. In particular, the Faraday constant F is related to the molar mass $M(X)$ and valence z of entity X by

$$\frac{F = ItM(X)}{zm_{\rm d}(X)\ ,} \qquad (291)$$

where $m_{\rm d}(X)$ is the mass of entity X dissolved as the result of transfer of charge $Q = It$ during the electrolysis. It follows from the relations $F = N_{\rm A}e$, $e^2 = 2\alpha h/\mu_0 c$, $m_{\rm e} = 2R_\infty h/c\alpha^2$, and $N_{\rm A} = A_{\rm r}({\rm e})M_{\rm u}/m_{\rm e}$, where $M_{\rm u} = 10^{-3}$ kg mol^{-1}, that

$$F = \frac{A_{\rm r}({\rm e})M_{\rm u}}{R_\infty}\left(\frac{c}{2\mu_0}\frac{\alpha^5}{h}\right)^{1/2} \ . \qquad (292)$$

Since, according to Eq. (291), F is proportional to the current I, and I is inversely proportional to the product $K_{\rm J}R_{\rm K}$ if the current is determined in terms of the Josephson and quantum Hall effects, we may write

$$\mathcal{F}_{90} = \frac{K_{\rm J}R_{\rm K}}{K_{\rm J-90}R_{\rm K-90}}\frac{A_{\rm r}({\rm e})M_{\rm u}}{R_\infty}\left(\frac{c}{2\mu_0}\frac{\alpha^5}{h}\right)^{1/2} \ , \qquad (293)$$

where $\mathcal{F}_{90}$ is the experimental value of F in SI units that would result from the Faraday experiment if $K_{\rm J} = K_{\rm J-90}$ and $R_{\rm K} = R_{\rm K-90}$. The quantity $\mathcal{F}_{90}$ is the input datum used in the adjustment, but the observational equation accounts for the fact that $K_{\rm J-90} \neq K_{\rm J}$ and $R_{\rm K-90} \neq R_{\rm K}$. If one assumes the validity of the expressions $K_{\rm J} = 2e/h$ and $R_{\rm K} = h/e^2$, then in terms of adjusted constants, Eq. (293) can be written as

$$\mathcal{F}_{90} = \frac{cM_{\rm u}}{K_{\rm J-90}R_{\rm K-90}}\frac{A_{\rm r}({\rm e})\alpha^2}{R_\infty h} \ . \qquad (294)$$

1. NIST: Ag coulometer

There is one high-accuracy experimental value of $\mathcal{F}_{90}$ available, that from NIST (Bower and Davis, 1980). The NIST experiment used a silver dissolution coulometer based on the anodic dissolution by electrolysis of silver, which is monovalent, into a solution of perchloric acid containing a small amount of silver perchlorate. The basic chemical reaction is $\rm Ag \rightarrow Ag^+ + e^-$ and occurs at the anode, which in the NIST work was a highly purified silver bar.

As discussed in detail in CODATA-98, the NIST experiment leads to

$$\mathcal{F}_{90} = 96\,485.39(13)\ {\rm C\ mol^{-1}} \quad [1.3\times10^{-6}]\ . \qquad (295)$$

[Note that the new AME2003 values of $A_{\rm r}(^{107}{\rm Ag})$ and $A_{\rm r}(^{109}{\rm Ag})$ in Table 2 have no effect on this result.]

The value of h that may be inferred from the NIST result, Eq. (294), and the 2006 recommended values for the other quantities is

$$h = 6.626\,0657(88)\times10^{-34}\ {\rm J\ s} \quad [1.3\times10^{-6}]\ , \qquad (296)$$

where the uncertainties of the other quantities are negligible compared to the uncertainty of $\mathcal{F}_{90}$.

8. MEASUREMENTS INVOLVING SILICON CRYSTALS

Here we discuss experiments relevant to the 2006 adjustment that use highly pure, nearly crystallographically perfect, single crystals of silicon. However, because one such experiment determines the quotient $h/m_{\rm n}$, where $m_{\rm n}$ is the mass of the neutron, for convenience and because any experiment that determines the ratio of the Planck constant to the mass of a fundamental particle or atom provides a value of the fine-structure constant α, we also discuss in this section two silicon-independent experiments: the 2002 Stanford University, Stanford, USA, measurement of $h/m(^{133}{\rm Cs})$ and the 2006 Laboratoire Kastler-Brossel or LKB measurement of $h/m(^{87}{\rm Rb})$.

In this section, W4.2a, NR3, W04 and NR4 are shortened forms of the full crystal designations WASO 4.2a, NRLM3, WASO 04, and NRLM4, respectively, for use in quantity symbols. No distinction is made between different crystals taken from the same ingot. As we use the current laboratory name to identify a result rather than the laboratory name at the time the measurement was carried out, we have replaced IMGC and NRLM with INRIM and NMIJ—see the glossary in CODATA-98.

1. {220} lattice spacing of silicon d_{220}

A value of the {220} lattice spacing of a silicon crystal in meters is relevant to the 2006 adjustment not only because of its role in determining α from $h/m_{\rm n}$ (see Sec. 8.4.1), but also because of its role in determining the relative atomic mass of the neutron $A_{\rm r}({\rm n})$ (see Sec.8.3). Further, together with the measured value of the molar volume of silicon $V_{\rm m}({\rm Si})$, it can provide a competitive value of h (see Sec. 8.2).

Various aspects of silicon and its crystal plane spacings of interest here are reviewed in CODATA-98 and CODATA-02. [See also the reviews of Becker (2003), Mana (2001), and Becker (2001)]. Some points worth noting are that silicon is a cubic crystal with $n = 8$ atoms per face-centered cubic unit cell of edge length (or lattice parameter) $a = 543$ pm with $d_{220} = a/\sqrt{8}$. The three naturally occurring isotopes of Si are ^{28}Si, ^{29}Si, and ^{30}Si, and the amount-of-substance fractions $x(^{28}{\rm Si})$, $x(^{29}{\rm Si})$, and $x(^{30}{\rm Si})$ of natural silicon are approximately 0.92, 0.05, and 0.03, respectively.

Although the $\{220\}$ lattice spacing of Si is not a fundamental constant in the usual sense, for practical purposes one can consider a, and hence d_{220}, of an impurity-free, crystallographically perfect or "ideal" silicon crystal under specified conditions, principally of temperature, pressure, and isotopic composition, to be an invariant of nature. The reference temperature and pressure currently adopted are $t_{90} = 22.5$ °C and $p = 0$ (that is, vacuum), where t_{90} is Celsius temperature on the International Temperature Scale of 1990 (ITS-90) (Preston-Thomas, 1990a,b). However, no reference values for $x(^{A}\mathrm{Si})$ have yet been adopted, because the variation of a due to the variation of the isotopic composition of the crystals used in high-accuracy experiments is taken to be negligible at the current level of experimental uncertainty in a. A much larger effect on a is the impurities that the silicon crystal contains—mainly carbon (C), oxygen (O), and nitrogen (N)—and corrections must be applied to convert the $\{220\}$ lattice spacing $d_{220}(X)$ of a real crystal X to the $\{220\}$ lattice spacing d_{220} of an "ideal" crystal.

Nevertheless, we account for the possible variation in the lattice spacing of different samples taken from the same ingot by including an additional component (or components) of relative standard uncertainty in the uncertainty of any measurement result involving a silicon lattice spacing (or spacings). This additional component is typically $\sqrt{2} \times 10^{-8}$ for each crystal, but it can be larger, for example, $(3/2)\sqrt{2} \times 10^{-8}$ in the case of crystal MO* discussed below, because it is known to contain a comparatively large amount of carbon; see Secs. III.A.c and III.I of CODATA-98 for details. For simplicity, we do not explicitly mention our inclusion of such components in the following discussion.

Further, because of this component and the use of the same samples in different experiments, and because of the existence of other common components of uncertainty in the uncertainty budgets of different experimental results involving silicon crystals, many of the input data discussed in the following sections are correlated. In most cases we do not explicity give the relevant correlation coefficients in the text; instead Table 31 in Sec. 12 provides all the non-negligible correlation coefficients of the input data listed in Table 30.

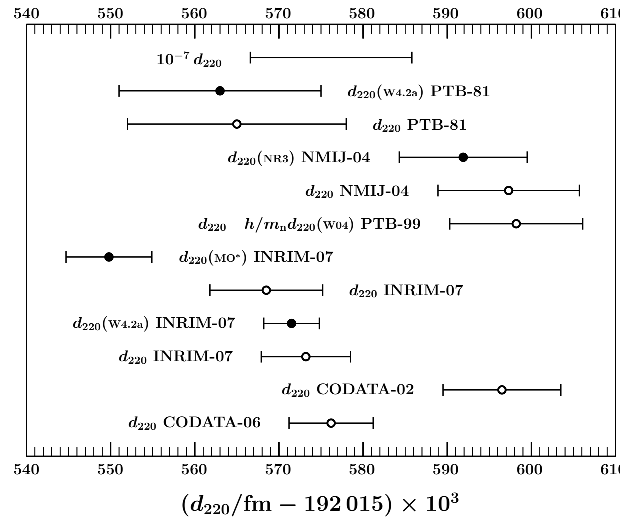

FIG. 1 Inferred values (open circles) of d_{220} from various measurements (solid circles) of $d_{220}(X)$. For comparison, the 2002 and 2006 CODATA recommended values of d_{220} are also shown.

1. X-ray/optical interferometer measurements of $d_{220}(X)$

High accuracy measurements of $d_{220}(X)$, where X denotes any one of various crystals, are carried out using a combined x-ray and optical interferometer (XROI) fabricated from a single crystal of silicon taken from one of several well-characterized single crystal ingots or boules. As discussed in CODATA-98, an XROI is a device that enables x-ray fringes of unknown period $d_{220}(X)$ to be compared with optical fringes of known period by moving one of the crystal plates of the XROI, called the analyzer. Also discussed there are the XROI measurements of $d_{220}(\mathrm{W4.2a})$, $d_{220}(\mathrm{MO^*})$, and $d_{220}(\mathrm{NR3})$, which were carried out at the PTB in Germany (Becker *et al.*, 1981), the Istituto Nazionale di Ricerca Metrologica, Torino, Italy (INRIM) (Basile *et al.*, 1994), and the National Metrology Institute of Japan (NMIJ), Tsukuba, Japan (Nakayama and Fujimoto, 1997), respectively.

For the reasons discussed in CODATA-02 and subsequently documented by Cavagnero *et al.* (2004a,b), only the NMIJ 1997 result was taken as an input datum in the 2002 adjustment. However, further work, published in the Erratum to that paper, showed that the results obtained at INRIM given in the paper were in error. After the error was discovered, additional work was carried out at INRIM to fully understand and correct it. New results were then reported at a conference in 2006 and published in the conference proceedings (Becker *et al.*, 2007). Thus, as summarized in Table 24 and compared in Fig. 1, we take as input data the four absolute $\{220\}$ lattice spacing values determined in three different laboratories, as discussed in the following three sections. The last value in the table, which is not an XROI result, is discussed in Sec. 8.4.1.

We point out that not only do we take the $\{220\}$ lattice spacings of the crystals WASO 4.2a, NRLM3, and MO* as adjusted constants, but also the $\{220\}$ lattice spacings of the crystals N, WASO 17, ILL, WASO 04, and NRLM4, because they too were involved in various experiments, including the d_{220} lattice spacing fractional difference measurements discussed in Sec 8.1.2.

1. PTB measurement of $d_{220}(\mathrm{W4.2a})$ The following value, identified as PTB-81 in Table 24 and Fig. 1, is the origi-

TABLE 24 Summary of measurements of the absolute {220} lattice spacing of various silicon crystals and inferred values of d_{220}.

Quantity	Value	Relative standard uncertainty u_r	Identification	Sect. and Eq.
$d_{220}(\text{W4.2a})$	192 015.563(12) fm	6.2×10^{-8}	PTB-81	8.1.1.1 (297)
d_{220}	192 015.565(13) fm	6.5×10^{-8}		
$d_{220}(\text{NR3})$	192 015.5919(76) fm	4.0×10^{-8}	NMIJ-04	8.1.1.2 (298)
d_{220}	192 015.5973(84) fm	4.4×10^{-8}		
$d_{220}(\text{W4.2a})$	192 015.5715(33) fm	1.7×10^{-8}	INRIM-07	8.1.1.3 (299)
d_{220}	192 015.5732(53) fm	2.8×10^{-8}		
$d_{220}(\text{MO}^*)$	192 015.5498(51) fm	2.6×10^{-8}	INRIM-07	8.1.1.3 (300)
d_{220}	192 015.5685(67) fm	3.5×10^{-8}		
$h/m_n d_{220}(\text{W04})$	2060.267 004(84) m s^{-1}	4.1×10^{-8}	PTB-99	8.4.1 (322)
d_{220}	192 015.5982(79) fm	4.1×10^{-8}		8.4.1 (325)

nal result obtained at PTB as reported by Becker *et al.* (1981) and discussed in CODATA-98:

$$d_{220}(\text{W4.2a}) = 192\,015.563(12)\ \text{fm} \quad [6.2 \times 10^{-8}]\ . \tag{297}$$

2. NMIJ measurement of $d_{220}(\text{NR3})$ The following value, identified as NMIJ-04 in Table 24 and Fig. 1, reflects the NMIJ efforts in the early and mid-1990s as well as the work carried out in the early 2000s:

$$d_{220}(\text{NR3}) = 192\,015.5919(76)\ \text{fm} \quad [4.0 \times 10^{-8}]\ . \tag{298}$$

This value, reported by Cavagnero *et al.* (2004a,b), is the weighted mean of the 1997 NIMJ result of Nakayama and Fujimoto (1997) discussed in CODATA-98 and CODATA-02, and the result from a new series of measurements performed at NMIJ from December 2002 to February 2003 with nearly the same apparatus. One of the principle differences from the earlier experiment was the much improved temperature control system of the room in which the NMIJ XROI was located; the new system provided a temperature stability of about 1 mK/d and allowed the temperature of the XROI to be set to within 20 mK of 22.5 °C.

The result for $d_{220}(\text{NR3})$ from the 2002-2003 measurements is based on 61 raw data. In each measurement, the phases of the x-ray and optical fringes (optical orders) were compared at the 0th, 100th, and 201st optical orders, and then with the analyzer moving in the reverse direction, at the 201st, 100th, and 0th orders. The n/m ratio was calculated from the phase of the x-ray fringe at the 0th and 201st orders, where n is the number of x-ray fringes in m optical fringes (optical orders) of period $\lambda/2$, where λ is the wavelength of the laser beam used in the optical interferometer and $d_{220}(\text{NR3}) = (\lambda/2)/(n/m)$.

In the new work, the fractional corrections to $d_{220}(\text{NR3})$, in parts in 10^9, total 181(35), the largest by far being the correction 173(33) for laser beam diffraction. The next largest is 5.0(7.1) for laser beam alignment. The statistical uncertainty is 33 (Type A).

Before calculating the weighted mean of the new and 1997 results for $d_{220}(\text{NR3})$, Cavagnero *et al.* (2004a,b) revised the 1997 value based on a reanalysis of the old experiment, taking into account what was learned in the new experiment. Not only did the reanalysis result in a reduction of the statistical uncertainty from (again, in parts in 10^9) 50 to 1.8 due to a better understanding of the undulation of n/m values as a function of time, but also in more reliable estimates of the corrections for laser beam diffraction and laser beam alignment. Indeed, the fractional corrections for the revised 1997 NMIJ value of $d_{220}(\text{NR3})$ total 190(38) compared to the original total of 173(14), and the final uncertainty of the revised 1997 value is $u_r = 3.8 \times 10^{-9}$ compared to $u_r = 4.8 \times 10^{-9}$ of the new value.

For completeness, we note that two possible corrections to the NMIJ result have been discussed in the literature. In the Erratum to Cavagnero *et al.* (2004a,b), it is estimated that a fractional correction to the value of $d_{220}(\text{NR3})$ in Eq. (298) of -1.3×10^{-8} may be required to account for the contamination of the NMIJ laser by a parasitic component of laser radiation as in the case of the INRIM laser discussed in the next section. However, it is not applied, because of its comparatively small size and the fact that no measurements of $d_{220}(\text{NR3})$ have yet been made at NMIJ (or INRIM) with a problem-free laser that confirm the correction, as has been done at INRIM for the crystals WASO 4.2a and MO*.

In Fujimoto *et al.* (2007), it is estimated, based on a Monte Carlo simulation, that the fractional correction to $d_{220}(\text{NR3})$ labeled "Fresnel diffraction" in Table I of Nakayama and Fujimoto (1997) and equal to $16.0(8) \times 10^{-8}$ should be $10(3) \times 10^{-8}$. The change arises from taking into account the misalignment of the interfering beams in the laser interferometer. Because this additional diffraction effect was present in both the 1997 and 2002-2003 measurements but was not considered in the reanalysis of the 1997 result nor in the analysis of the 2002-2003 data, it implies that the weighted mean

value for $d_{220}(\text{NR3})$ in Eq. (298) should be reduced by this amount and its u_r increased from 4.0×10^{-8} to 5.0×10^{-8}. However, because the data required for the calculation were not precisely known (they were not logged in the laboratory notebooks because the experimenters were unaware of their importance), the correction is viewed as somewhat conjectural and thus that applying it would not be justified (Mana and Massa, 2006).

3. INRIM measurement of $d_{220}(\text{W4.2a})$ and $d_{220}(\text{MO}^*)$ The following two new INRIM values, with identifier INRIM-07, were reported by Becker *et al.* (2007):

$$d_{220}(\text{W4.2a}) = 192\,015.5715(33)\text{ fm} \quad [1.7\times10^{-8}] \tag{299}$$

$$d_{220}(\text{MO}^*) = 192\,015.5498(51)\text{ fm} \quad [2.6\times10^{-8}]\,. \tag{300}$$

The correlation coefficient of these values is 0.057, based on the detailed uncertainty budget for $d_{220}(\text{MO}^*)$ in Cavagnero *et al.* (2004a,b) and the similar uncertainty budget for $d_{220}(\text{W4.2a})$ provided by Fujimoto *et al.* (2006). Although the 2007 result for $d_{220}(\text{MO}^*)$ of Becker *et al.* (2007) in Eq. (300) agrees with the 1994 INRIM result of Basile *et al.* (1994), which was used as an input datum in the 1998 adjustment, because of the many advances incorporated in the new work, we no longer consider the old result.

In addition to the determination, described in the previous section, of the {220} lattice spacing of crystal NRLM3 carried out at NMIJ in 2002-2003 using the NMIJ NRLM3 x-ray interferometer and associated NMIJ apparatus, Cavagnero *et al.* (2004a,b) reported the results of measurements carried out at INRIM of the {220} lattice spacings of crystals MO* and NRLM3, where in the latter case it was an INRIM-NMIJ joint effort that used the NIMJ NRLM3 x-ray interferometer but the INRIM associated apparatus. But as indicated above, both results were subsequently found to be in error: the optical laser beam used to measure the displacement of the x-ray interferometer's analyzer crystal was contaminated by a parasitic component with a frequency that differed by about 1.1 GHz from the frequency assigned the laser beam.

After eliminating the error by replacing the problem laser with a 633 nm He-Ne external-cavity diode laser locked to a $^{127}I_2$ stabilized laser, the INRIM researchers repeated the measurements they had previously carried out with the INRIM MO* x-ray interferometer and with the refurbished PTB WASO 4.2a x-ray interferometer originally used in the PTB experiment that led to the 1981 value of $d_{220}(\text{W4.2a})$ in Eq. (297). The PTB WASO 4.2a x-ray interferometer was refurbished at PTB through remachining, but the result for $d_{220}(\text{W4.2a})$ obtained at INRIM with the contaminated laser was not included in Cavagnero *et al.* (2004a,b). The values of $d_{220}(\text{W4.2a})$ and $d_{220}(\text{MO}^*)$ in Eqs. 299 and 300 resulted from the repeated measurements (Becker *et al.*, 2007).

In principle, based on the experimentally observed shifts in the measured values of $d_{220}(\text{W4.2a})$ and $d_{220}(\text{MO}^*)$ obtained with the malfunctioning laser and the properly functioning laser, the value of $d_{220}(\text{NR3})$ obtained in the INRIM-NMIJ joint effort using the malfunctioning laser mentioned above, and the value of $d_{220}(\text{WS5C})$ also obtained with this laser, could be corrected and taken as input data. WS5C is an XROI manufactured by INRIM from a WASO 04 sample, but the value of $d_{220}(\text{WS5C})$ obtained using the contaminated laser was also not included in Cavagnero *et al.* (2004a,b). However, because of the somewhat erratic history of silicon lattice spacing measurements, the Task Group decided to use only data obtained with a laser known to be functioning properly.

The improvements in the INRIM XROI apparatus since the 1994 $d_{220}(\text{MO}^*)$ measurement of Basile *et al.* (1994) include (i) a new two-axis "tip-tilt" platform for the XROI that is electronically controlled to compensate for parasitic rotations and straightness error of the guiding system that moves the platform; (ii) imaging the x-ray interference pattern formed by the x-ray beam transmitted through the moving analyzer in such a way that detailed information concerning lattice distortion and analyzer pitch can be extracted on line from the analysis of the phases of the x-ray fringes; and (iii) an upgraded computer-aided system for combined interferometer displacement and control, x-ray and optical fringe scanning, signal digitization and sampling, environmental monitoring, and data analysis.

The values of $d_{220}(\text{W4.2a})$ and $d_{220}(\text{MO}^*)$ in Eqs. (299) and (300) are the means of tens of individual values, with each value being the average of about ten data points collected in 1 h measurement cycles during which the analyzer was translated back and forth by 300 optical orders. For the two crystals, respectively, the statistical uncertainties in parts in 10^9 are 3.5 and 11.6, and the various corrections and their uncertainties are laser beam wavelength, −0.8(4), −0.8(4); laser beam diffraction, 12.0(2.2), 12.0(2.2); laser beam alignment, 2.5(3.5), 2.5(3.5); Abbe error, 0.0(2.8), 0.0(3.7); trajectory error, 0.0(1.4), 0.0(3.6); analyzer temperature, 1.0(5.2), 1.0(7.9); and abberations, 0.0(5.0), 0.0(2.0). The total uncertainties are 9.6 and 15.7.

2. d_{220} difference measurements

To relate the lattice spacings of crystals used in various experiments, highly accurate measurements are made of the fractional difference $[d_{220}(X) - d_{220}(\text{ref})]\,/d_{220}(\text{ref})$ of the {220} lattice spacing of a sample of a single crystal ingot X and that of a reference crystal "ref". Both NIST and PTB have carried out such measurements, and the fractional differences from these two laboratories that we take as input data in the 2006 adjustment are given in the following two sections and are summarized in Table 25. For details concerning these measurements, see CODATA-98 and CODATA-02.

1. NIST difference measurements The following fractional difference involving a crystal denoted simply as "N" was obtained as part of the NIST effort to measure the wavelengths in meters of the Kα_1 x-ray lines of Cu, Mo, and W; see Sec. 11.1.

$$\frac{d_{220}(\mathrm{W17}) - d_{220}(\mathrm{N})}{d_{220}(\mathrm{W17})} = 7(22)\times 10^{-9} \,. \tag{301}$$

The following three fractional differences involving crystals from the four crystals denoted ILL, WASO 17, MO*, and NRLM3 were obtained as part of the NIST effort, discussed in Sec. 8.3, to determine the relative atomic mass of the neutron $A_{\mathrm{r}}(\mathrm{n})$ (Kessler *et al.*, 1999):

$$\frac{d_{220}(\mathrm{ILL}) - d_{220}(\mathrm{W17})}{d_{220}(\mathrm{ILL})} = -8(22)\times 10^{-9} \tag{302}$$

$$\frac{d_{220}(\mathrm{ILL}) - d_{220}(\mathrm{MO^*})}{d_{220}(\mathrm{ILL})} = 86(27)\times 10^{-9} \tag{303}$$

$$\frac{d_{220}(\mathrm{ILL}) - d_{220}(\mathrm{NR3})}{d_{220}(\mathrm{ILL})} = 34(22)\times 10^{-9} \,. \tag{304}$$

The following more recent NIST difference measurements, which we also take as input data in the 2006 adjustment, were provided by Kessler (2006) of NIST and are updates of the results reported by Hanke and Kessler (2005):

$$\frac{d_{220}(\mathrm{NR3}) - d_{220}(\mathrm{W04})}{d_{220}(\mathrm{W04})} = -11(21)\times 10^{-9} \tag{305}$$

$$\frac{d_{220}(\mathrm{NR4}) - d_{220}(\mathrm{W04})}{d_{220}(\mathrm{W04})} = 25(21)\times 10^{-9} \tag{306}$$

$$\frac{d_{220}(\mathrm{W17}) - d_{220}(\mathrm{W04})}{d_{220}(\mathrm{W04})} = 11(21)\times 10^{-9} \,. \tag{307}$$

The full designations of the two new crystals involved in these comparisons are WASO 04 and NRLM4. The measurements benefited significantly from the relocation of the NIST lattice comparator to a new laboratory where the temperature varied by only about 5 mK in several weeks compared to the previous laboratory where the temperature varied by about 40 mK in one day (Hanke and Kessler, 2005).

2. PTB difference measurements Results for the {220} lattice-spacing fractional differences of various crystals that we also take as input data in the 2006 adjustment have been obtained at the PTB (Martin *et al.*, 1998):

$$\frac{d_{220}(\mathrm{W4.2a}) - d_{220}(\mathrm{W04})}{d_{220}(\mathrm{W04})} = -1(21)\times 10^{-9} \tag{308}$$

$$\frac{d_{220}(\mathrm{W17}) - d_{220}(\mathrm{W04})}{d_{220}(\mathrm{W04})} = 22(22)\times 10^{-9} \tag{309}$$

$$\frac{d_{220}(\mathrm{MO^*}) - d_{220}(\mathrm{W04})}{d_{220}(\mathrm{W04})} = -103(28)\times 10^{-9} \tag{310}$$

$$\frac{d_{220}(\mathrm{NR3}) - d_{220}(\mathrm{W04})}{d_{220}(\mathrm{W04})} = -23(21)\times 10^{-9} \,. \tag{311}$$

To relate $d_{220}(\mathrm{W04})$ to the {220} lattice spacing d_{220} of an "ideal" silicon crystal, we take as an input datum

$$\frac{d_{220} - d_{220}(\mathrm{W04})}{d_{220}(\mathrm{W04})} = 10(11)\times 10^{-9} \tag{312}$$

given by Becker *et al.* (2003), who obtained it by taking into account the known carbon, oxygen, and nitrogen impurities in WASO 04. However, following what was done in the 1998 and 2002 adjustments, we have included an additional component of uncertainty of 1×10^{-8} to account for the possibility that, even after correction for C, O, and N impurities, the crystal WASO 04, although very well characterized as to its purity and crystallographic perfection, does not meet all of the criteria for an ideal crystal. Indeed, in general, we prefer to use experimentally measured fractional lattice spacing differences rather than differences implied by the C, O, and N impurity content of the crystals in order to avoid the need to assume that all crystals of interest meet these criteria.

In order to include this fractional difference in the 2002 adjustment, the quantity d_{220} is also taken as an adjusted constant.

2. Molar volume of silicon $V_{\mathrm{m}}(\mathrm{Si})$ and the Avogadro constant N_{A}

The definition of the molar volume of silicon $V_{\mathrm{m}}(\mathrm{Si})$ and its relationship to the Avogadro constant N_{A} and Planck constant h as well as other constants is discussed in CODATA-98 and summarized in CODATA-02. In brief we have

$$m(\mathrm{Si}) = \rho(\mathrm{Si})\frac{a^3}{n} \,, \tag{313}$$

$$V_{\mathrm{m}}(\mathrm{Si}) = \frac{M(\mathrm{Si})}{\rho(\mathrm{Si})} = \frac{A_{\mathrm{r}}(\mathrm{Si})M_{\mathrm{u}}}{\rho(\mathrm{Si})} \,, \tag{314}$$

$$N_{\mathrm{A}} = \frac{V_{\mathrm{m}}(\mathrm{Si})}{a^3/n} = \frac{A_{\mathrm{r}}(\mathrm{Si})M_{\mathrm{u}}}{\sqrt{8}\,d_{220}^3\,\rho(\mathrm{Si})} \,, \tag{315}$$

$$V_{\mathrm{m}}(\mathrm{Si}) = \frac{\sqrt{2}\,cM_{\mathrm{u}}A_{\mathrm{r}}(\mathrm{e})\alpha^2 d_{220}^3}{R_\infty h} \,, \tag{316}$$

which are to be understood in the context of an impurity free, crystallographically perfect, "ideal" silicon crystal at the reference conditions $t_{90} = 22.5$ °C and $p = 0$, and of isotopic composition in the range normally observed for crystals used in high-accuracy experiments. Thus $m(\mathrm{Si})$, $V_{\mathrm{m}}(\mathrm{Si})$, $M(\mathrm{Si})$, and $A_{\mathrm{r}}(\mathrm{Si})$ are the mean mass, mean molar volume, mean molar mass, and mean relative atomic mass of the silicon atoms in such a crystal, respectively, and $\rho(\mathrm{Si})$ is the crystal's macroscopic mass density. Equation (316) is the observational equation for a measured value of $V_{\mathrm{m}}(\mathrm{Si})$.

It follows from Eq. (314) that the experimental determination of $V_{\mathrm{m}}(\mathrm{Si})$ requires (i) measurement of the amount-of-substance ratios $n(^{29}\mathrm{Si})/n(^{28}\mathrm{Si})$ and $n(^{30}\mathrm{Si})/n(^{28}\mathrm{Si})$ of a nearly perfect silicon crystal—and

TABLE 25 Summary of measurements of the relative {220} lattice spacings of silicon crystals.

Quantity	Value	Identification	Sect. and Eq.
$1 - d_{220}(\textsc{w17})/d_{220}(\textsc{ill})$	$-8(22) \times 10^{-9}$	NIST-99	8.1.2.1 (302)
$1 - d_{220}(\textsc{mo}^*)/d_{220}(\textsc{ill})$	$86(27) \times 10^{-9}$	NIST-99	8.1.2.1 (303)
$1 - d_{220}(\textsc{nr3})/d_{220}(\textsc{ill})$	$34(22) \times 10^{-9}$	NIST-99	8.1.2.1 (304)
$1 - d_{220}(\textsc{n})/d_{220}(\textsc{w17})$	$7(22) \times 10^{-9}$	NIST-97	8.1.2.1 (301)
$d_{220}(\textsc{nr3})/d_{220}(\textsc{w04}) - 1$	$-11(21) \times 10^{-9}$	NIST-06	8.1.2.1 (305)
$d_{220}(\textsc{nr4})/d_{220}(\textsc{w04}) - 1$	$25(21) \times 10^{-9}$	NIST-06	8.1.2.1 (306)
$d_{220}(\textsc{w17})/d_{220}(\textsc{w04}) - 1$	$11(21) \times 10^{-9}$	NIST-06	8.1.2.1 (307)
$d_{220}(\textsc{w4.2a})/d_{220}(\textsc{w04}) - 1$	$-1(21) \times 10^{-9}$	PTB-98	8.1.2.2 (308)
$d_{220}(\textsc{w17})/d_{220}(\textsc{w04}) - 1$	$22(22) \times 10^{-9}$	PTB-98	8.1.2.2 (309)
$d_{220}(\textsc{mo}^*)/d_{220}(\textsc{w04}) - 1$	$-103(28) \times 10^{-9}$	PTB-98	8.1.2.2 (310)
$d_{220}(\textsc{nr3})/d_{220}(\textsc{w04}) - 1$	$-23(21) \times 10^{-9}$	PTB-98	8.1.2.2 (311)
$d_{220}/d_{220}(\textsc{w04}) - 1$	$10(11) \times 10^{-9}$	PTB-03	8.1.2.2 (312)

hence amount of substance fractions $x(^A\mathrm{Si})$—and then calculation of $A_\mathrm{r}(\mathrm{Si})$ from the well-known values of $A_\mathrm{r}(^A\mathrm{Si})$; and (ii) measurement of the macroscopic mass density $\rho(\mathrm{Si})$ of the crystal. Determining N_A from Eq. (315) by measuring $V_\mathrm{m}(\mathrm{Si})$ in this way and d_{220} using x rays is called the x-ray-crystal-density (XRCD) method.

An extensive international effort has been under way since the early 1990s to determine N_A using this technique with the smallest possible uncertainty. The effort is being coordinated by the Working Group on the Avogadro Constant (WGAC) of the Consultative Committee for Mass and Related Quantities (CCM) of the CIPM. The WGAC, which has representatives from all major research groups working in areas relevant to the determination of N_A, is currently chaired by P. Becker of PTB.

As discussed at length in CODATA-02, the value of $V_\mathrm{m}(\mathrm{Si})$ used as an input datum in the 2002 adjustment was provided to the CODATA Task Group by the WGAC and was a consensus value based on independent measurements of $\rho(\mathrm{Si})$ at NMIJ and PTB using a number of different silicon crystals, and measurements of their molar masses $M(\mathrm{Si})$ using isotopic mass spectrometry at the Institute for Reference Materials and Measurements (IRMM), European Commission, Geel, Belgium. This value, identified as N/P/I-03 in recognition of the work done by researchers at NMIJ, PTB, and IRMM, is $V_\mathrm{m}(\mathrm{Si}) = 12.058\,8257(36) \times 10^{-6}\ \mathrm{m^3\ mol^{-1}}$ $[3.0 \times 10^{-7}]$. Since then, the data used to obtain it were reanalyzed by the WGAC, resulting in the slightly revised value (Fujii *et al.*, 2005)

$$V_\mathrm{m}(\mathrm{Si}) = 12.058\,8254(34) \times 10^{-6}\ \mathrm{m^3\ mol^{-1}} \quad [2.8 \times 10^{-7}]\ , \quad (317)$$

which we take as an input datum in the 2006 adjustment and identify as N/P/I-05. The slight shift in value and reduction in uncertainty is due to the fact that the effect of nitrogen impurities in the silicon crystals used in the NMIJ measurements was taken into account in the reanalysis (Fujii *et al.*, 2005). Note that the new value of $A_\mathrm{r}(^{29}\mathrm{Si})$ in Table 4 has no effect on this result.

Based on Eq. (316) and the 2006 recommended values of $A_\mathrm{r}(\mathrm{e})$, α, d_{220}, and R_∞, the value of h implied by this result is

$$h = 6.626\,0745(19) \times 10^{-34}\ \mathrm{J\ s} \quad [2.9 \times 10^{-7}]\ . \quad (318)$$

A comparison of this value of h with those in Tables 21 and 23 shows that it is generally not in good agreement with the most accurate of the other values.

In this regard, two relatively recent publications, the first describing work performed in China (Ding *et al.*, 2005) and the second describing work performed in Switzerland (Reynolds *et al.*, 2006), reported results which, if taken at face value, seem to call into question the uncertainty with which the molar mass of naturally occurring silicon is currently known. [See also Valkiers *et al.* (2005).] These results highlight the importance of the current WGAC project to measure $V_\mathrm{m}(\mathrm{Si})$ using highly enriched silicon crystals with $x(^{28}\mathrm{Si}) > 0.99985$ (Becker *et al.*, 2006), which should simplify the determination of the molar mass of such crystals.

3. Gamma-ray determination of the neutron relative atomic mass $A_\mathrm{r}(\mathrm{n})$

Although the value of $A_\mathrm{r}(\mathrm{n})$ listed in Table 2 is a result of AME2003, it is not used in the 2006 adjustment. Instead, $A_\mathrm{r}(\mathrm{n})$ is obtained as discussed in this section in order to ensure that its recommended value is consistent with the best current information on the {220} lattice spacing of silicon.

The value of $A_r(\mathrm{n})$ can be obtained by measuring the wavelength of the 2.2 MeV γ ray in the reaction n + p $\rightarrow$ d + γ in terms of the d_{220} lattice spacing of a particular silicon crystal corrected to the commonly used reference conditions $t_{90} = 22.5$ °C and $p = 0$. The result for the wavelength-to-lattice spacing ratio, obtained from Bragg-angle measurements carried out in 1995 and 1998 using a flat crystal spectrometer of the GAMS4 diffraction facility at the high-flux reactor of the Institut Max von Laue-Paul Langevin (ILL), Grenoble, France, in a NIST and ILL collaboration, is (Kessler *et al.*, 1999)

$$\frac{\lambda_{\mathrm{meas}}}{d_{220}(\textsc{ill})} = 0.002\,904\,302\,46(50) \qquad [1.7 \times 10^{-7}] \,, \tag{319}$$

where $d_{220}(\textsc{ill})$ is the $\{220\}$ lattice spacing of the silicon crystals of the ILL GAMS4 spectrometer at $t_{90} = 22.5$ °C and $p = 0$. Relativistic kinematics of the reaction yields the equation

$$\frac{\lambda_{\mathrm{meas}}}{d_{220}(\textsc{ill})} = \frac{\alpha^2 A_r(\mathrm{e})}{R_\infty d_{220}(\textsc{ill})} \frac{A_r(\mathrm{n}) + A_r(\mathrm{p})}{[A_r(\mathrm{n}) + A_r(\mathrm{p})]^2 - A_r^2(\mathrm{d})} \,, \tag{320}$$

where all seven quantities on the right-hand side are adjusted constants.

Recently, Dewey *et al.* (2006); Rainville *et al.* (2005) reported determinations of the wavelengths of the gamma rays emitted in the cascade from the neutron capture state to the ground state in the reactions n + ^{28}Si $\rightarrow$ ^{29}Si + 2γ, n + ^{32}S $\rightarrow$ ^{33}Si + 3γ, and n + ^{35}Cl $\rightarrow$ ^{36}Cl + 2γ. The gamma-ray energies are 3.5 MeV and 4.9 MeV for the Si reaction, 5.4 MeV, 2.4 MeV, and 0.8 MeV for the S reaction, and 6.1 MeV, 0.5 MeV, and 2.0 MeV for the Cl reaction. While these data together with the relevant relative atomic masses are potentially an additional source of information on the neutron relative atomic mass, the uncertainties are too large for this purpose; the inferred value of $A_r(\mathrm{n})$ has an uncertainty nearly an order of magnitude larger than that obtained from Eq. (320). Instead, this work is viewed as the most accurate test of $E = mc^2$ to date (Rainville *et al.*, 2005).

4. Quotient of Planck constant and particle mass $h/m(X)$ and α

The relation $R_\infty = \alpha^2 m_e c/2h$ leads to

$$\alpha = \left[\frac{2R_\infty}{c}\frac{A_r(X)}{A_r(\mathrm{e})}\frac{h}{m(X)}\right]^{1/2} \,, \tag{321}$$

where $A_r(X)$ is the relative atomic mass of particle X with mass $m(X)$ and $A_r(\mathrm{e})$ is the relative atomic mass of the electron. Because c is exactly known, u_r of R_∞ and $A_r(\mathrm{e})$ are less than 7×10^{-12} and 5×10^{-10}, respectively, and u_r of $A_r(X)$ for many particles and atoms is less than that of $A_r(\mathrm{e})$, Eq. (321) can provide a value of α with a competitive uncertainty if $h/m(X)$ is determined with a sufficiently small uncertainty. Here, we discuss the determination of $h/m(X)$ for the neutron n, the ^{133}Cs atom, and the ^{87}Rb atom. The results, including the inferred values of α, are summarized in Table 26.

1. Quotient h/m_n

The PTB determination of h/m_n was carried out at the ILL high-flux reactor. The de Broglie relation $p = m_\mathrm{n} v = h/\lambda$ was used to determine $h/m_\mathrm{n} = \lambda v$ for the neutron by measuring both its de Broglie wavelength λ and corresponding velocity v. More specifically, the de Broglie wavelength, $\lambda \approx 0.25$ nm, of slow neutrons was determined using back reflection from a silicon crystal, and the velocity, $v \approx 1600$ m/s, of the neutrons was determined by a special time-of-flight method. The final result of the experiment is (Krüger *et al.*, 1999)

$$\frac{h}{m_\mathrm{n} d_{220}(\textsc{w04})} = 2060.267\,004(84)\ \mathrm{m\ s^{-1}} \qquad [4.1 \times 10^{-8}] \,, \tag{322}$$

where as before, $d_{220}(\textsc{w04})$ is the $\{220\}$ lattice spacing of the crystal WASO 04 at $t_{90} = 22.5$ °C in vacuum. This result is correlated with the PTB fractional lattice-spacing differences given in Eqs. (308) to (311)—the correlation coefficients are about 0.2.

The equation for the PTB result, which follows from Eq. (321), is

$$\frac{h}{m_\mathrm{n} d_{220}(\textsc{w04})} = \frac{A_r(\mathrm{e})}{A_r(\mathrm{n})} \frac{c\alpha^2}{2R_\infty d_{220}(\textsc{w04})} \,. \tag{323}$$

The value of α that can be inferred from this relation and the PTB value of $h/m_\mathrm{n} d_{220}(\textsc{w04})$, the 2006 recommended values of R_∞, $A_r(\mathrm{e})$, and $A_r(\mathrm{n})$, the NIST and PTB fractional lattice-spacing-differences in Table 25, and the four XROI values of $d_{220}(\textsc{x})$ in Table 24 for crystals WASO 4.2a, NRLM3, and MO*, is

$$\alpha^{-1} = 137.036\,0077(28) \quad [2.1 \times 10^{-8}] \,. \tag{324}$$

This value is included in Table 26 as the first entry; it disagrees with the α values from the two other h/m results.

It is also of interest to calculate the value of d_{220} implied by the PTB result for $h/m_\mathrm{n} d_{220}(\textsc{w04})$. Based on Eq. (323), the 2006 recommended values of R_∞, $A_r(\mathrm{e})$, $A_r(\mathrm{p})$, $A_r(\mathrm{d})$, α, the NIST and PTB fractional lattice-spacing-differences in Table 25, and the value of $\lambda_{\mathrm{meas}}/d_{220}(\textsc{ill})$ given in Eq. (319), we find

$$d_{220} = 192\,015.5982(79)\ \mathrm{fm} \quad [4.1 \times 10^{-8}] \,. \tag{325}$$

This result is included in Table 24 as the last entry; it agrees with the NMIJ value, but disagrees with the PTB and INRIM values.

TABLE 26 Summary of data related to the quotients $h/m_{\rm n}d_{220}({\rm W04})$, $h/m({\rm Cs})$, and $h/m({\rm Rb})$, together with inferred values of α.

Quantity	Value	Relative standard uncertainty $u_{\rm r}$	Identification	Sect. and Eq.
$h/m_{\rm n}d_{220}({\rm W04})$	$2060.267\,004(84)$ m s^{-1}	4.1×10^{-8}	PTB-99	8.4.1 (322)
α^{-1}	$137.036\,0077(28)$	2.1×10^{-8}		8.4.1 (324)
$h/m({\rm Cs})$	$3.002\,369\,432(46)\times10^{-9}$ m^2 s^{-1}	1.5×10^{-8}	StanfU-02	8.4.2 (329)
α^{-1}	$137.036\,0000(11)$	7.7×10^{-9}		8.4.2 (331)
$h/m({\rm Rb})$	$4.591\,359\,287(61)\times10^{-9}$ m^2 s^{-1}	1.3×10^{-8}	LKB-06	8.4.2 (332)
α^{-1}	$137.035\,998\,83(91)$	6.7×10^{-9}		8.4.2 (334)

2. Quotient $h/m(^{133}{\rm Cs})$

The Stanford University atom interferometry experiment to measure the atomic recoil frequency shift of photons absorbed and emitted by ^{133}Cs atoms, $\Delta\nu_{\rm Cs}$, in order to determine the quotient $h/m(^{133}{\rm Cs})$ is described in CODATA-02. As discussed there, the expression applicable to the Stanford experiment is

$$\frac{h}{m(^{133}{\rm Cs})} = \frac{c^2\,\Delta\nu_{\rm Cs}}{2\,\nu_{\rm eff}^2}\,, \tag{326}$$

where the frequency $\nu_{\rm eff}$ corresponds to the sum of the energy difference between the ground-state hyperfine level with $F=3$ and the $6{\rm P}_{1/2}$ state $F=3$ hyperfine level and the energy difference between the ground-state hyperfine level with $F=4$ and the same $6{\rm P}_{1/2}$ hyperfine level. The result for $\Delta\nu_{\rm Cs}/2$ reported in 2002 by the Stanford researchers is (Wicht *et al.*, 2002)

$$\frac{\Delta\nu_{\rm Cs}}{2} = 15\,006.276\,88(23)\ {\rm Hz} \qquad [1.5\times10^{-8}]\,. \tag{327}$$

The Stanford effort included an extensive study of corrections due to possible systematic effects. The largest component of uncertainty by far contributing to the uncertainty of the final result for $\Delta\nu_{\rm Cs}$, $u_{\rm r}=14\times10^{-9}$ (Type B), arises from the possible deviation from 1 of the index of refraction of the dilute background gas of cold cesium atoms that move with the signal atoms. This component, estimated experimentally, places a lower limit on the relative uncertainty of the inferred value of α from Eq. (321) of $u_{\rm r}=7\times10^{-9}$. Without it, $u_{\rm r}$ of α would be about 3 to 4 parts in 10^9.

In the 2002 adjustment, the value $\nu_{\rm eff} = 670\,231\,933\,044(81)$ kHz $[1.2\times10^{-10}]$, based on the measured frequencies of ^{133}Cs D$_1$-line transitions reported by Udem *et al.* (1999), was used to obtain the ratio $h/m(^{133}{\rm Cs})$ from the Stanford value of $\Delta\nu_{\rm Cs}/2$. Recently, using a femtosecond laser frequency comb and a narrow-linewidth diode laser, and eliminating Doppler shift by orienting the laser beam perpendicular to the ^{133}Cs atomic beam to within 5 μrad, Gerginov *et al.* (2006) remeasured the frequencies of the required transitions and obtained a value of $\nu_{\rm eff}$ that agrees with the value used in 2002 but which has a $u_{\rm r}$ 15 times smaller:

$$\nu_{\rm eff} = 670\,231\,932\,889.9(4.8)\ {\rm kHz} \quad [7.2\times10^{-12}]\,. \tag{328}$$

Evaluation of Eq. (326) with this result for $\nu_{\rm eff}$ and the value of $\Delta\nu_{\rm Cs}/2$ in Eq. (327) yields

$$\frac{h}{m(^{133}{\rm Cs})} = 3.002\,369\,432(46)\times10^{-9}\ {\rm m^2\,s^{-1}} \quad [1.5\times10^{-8}]\,, \tag{329}$$

which we take as an input datum in the 2006 adjustment. The observational equation for this datum is, from Eq. (321),

$$\frac{h}{m(^{133}{\rm Cs})} = \frac{A_{\rm r}({\rm e})}{A_{\rm r}(^{133}{\rm Cs})}\frac{c\,\alpha^2}{2R_\infty}\,. \tag{330}$$

The value of α that may be inferred from this expression, the Stanford result for $h/m(^{133}{\rm Cs})$ in Eq. (329), the 2006 recommended values of R_∞ and $A_{\rm r}({\rm e})$, and the ASME2003 value of $A_{\rm r}(^{133}{\rm Cs})$ in Table 2, the uncertainties of which are inconsequential in this application, is

$$\alpha^{-1} = 137.036\,0000(11) \qquad [7.7\times10^{-9}]\,, \tag{331}$$

where the dominant component of uncertainty arises from the measured value of the recoil frequency shift, in particular, the component of uncertainty due to a possible index of refraction effect.

In this regard, we note that Campbell *et al.* (2005) have experimentally demonstrated the reality of one aspect of such an effect with a two-pulse light grating interferometer and have shown that it can have a significant impact on precision measurements with atom interferometers. However, theoretical calculations based on simulations of the Stanford interferometer by Sarajlic *et al.* (2006), although incomplete, suggest that the experimentally based uncertainty component $u_{\rm r}=14\times10^{-9}$ assigned by Wicht *et al.* (2002) to account for this effect is reasonable. We also note that Wicht *et al.* (2005) have developed an improved theory of momentum transfer when localized atoms and localized optical fields interact. The details of such interactions are relevant to

precision atom interferometry. When Wicht *et al.* (2005) applied the theory to the Stanford experiment to evaluate possible systematic errors arising from wave-front curvature and distortion, as well as the Gouy phase shift of gaussian beams, they found that such errors do not limit the uncertainty of the value of α that can be obtained from the experiment at the level of a few parts in 10^9, but will play an important role in future precision atom-interferometer photon-recoil experiments to measure α with $u_r \approx 5 \times 10^{-10}$, such as is currently underway at Stanford (Müller *et al.*, 2006).

3. Quotient $h/m(^{87}\mathrm{Rb})$

In the LKB experiment (Cladé *et al.*, 2006a,b), the quotient $h/m(^{87}\mathrm{Rb})$, and hence α, is determined by accurately measuring the rubidium recoil velocity $v_r = \hbar k/m(^{87}\mathrm{Rb})$ when a rubidium atom absorbs or emits a photon of wave vector $k = 2\pi/\lambda$, where λ is the wavelength of the photon and $\nu = c/\lambda$ is its frequency. The measurements are based on Bloch oscillations in a vertical accelerated optical lattice.

The basic principle of the experiment is to precisely measure the variation of the atomic velocity induced by an accelerated standing wave using velocity selective Raman transitions between two ground-state hyperfine levels. A Raman π pulse of two counter-propagating laser beams selects an initial narrow atomic velocity class. After the acceleration process, the final atomic velocity distribution is probed using a second Raman π pulse of two counter-propagating laser beams.

The coherent acceleration of the rubidium atoms arises from a succession of stimulated two photon transitions also using two counter-propagating laser beams. Each transition modifies the atomic velocity by $2v_r$ leaving the internal state unchanged. The Doppler shift is compensated by linearly sweeping the frequency difference of the two lasers. This acceleration can conveniently be interpreted in terms of Bloch oscillations in the fundamental energy band of an optical lattice created by the standing wave, because the interference of the two laser beams leads to a periodic light shift of the atomic energy levels and hence to the atoms experiencing a periodic potential (Ben Dahan *et al.*, 1996; Peik *et al.*, 1997).

An atom's momentum evolves by steps of $2\hbar k$, each one corresponding to a Bloch oscillation. After N oscillations, the optical lattice is adiabatically released and the final velocity distribution, which is the initial distribution shifted by $2Nv_r$, is measured. Due to the high efficiency of Bloch oscillations, for an acceleration of 2000 m s^{-2}, 900 recoil momenta can be transferred to a rubidium atom in 3 ms with an efficiency of 99.97 % per recoil.

The atoms are alternately accelerated upwards and downwards by reversing the direction of the Bloch acceleration laser beams, keeping the same delay between the selection and the measurement Raman π pulses. The resulting differential measurement is independent of gravity. In addition, the contribution of some systematic effects changes sign when the direction of the selection and measuring Raman beams is exchanged. Hence, for each up and down trajectory, the selection and measuring Raman beams are reversed and two velocity spectra are taken. The mean value of these two measurements is free from systematic errors to first order. Thus each determination of $h/m(^{87}\mathrm{Rb})$ is obtained from four velocity spectra, each requiring 5 minutes of integration time, two from reversing the Raman beams when the acceleration is in the up direction and two when in the down direction. The Raman and Bloch lasers are stabilized by means of an ultrastable Fabry-Pérot cavity and the frequency of the cavity is checked several times during the 20 minute measurement against a well-known two-photon transition in $^{85}\mathrm{Rb}$.

Taking into account a $(-9.2 \pm 4) \times 10^{-10}$ correction to $h/m(^{87}\mathrm{Rb})$ not included in the value reported by Cladé *et al.* (2006a) due to a nonzero force gradient arising from a difference in the radius of curvature of the up and down accelerating beams, the result derived from 72 measurements of $h/m(^{87}\mathrm{Rb})$ acquired over 4 days, which we take as an input datum in the 2006 adjustment, is (Cladé *et al.*, 2006b)

$$\frac{h}{m(^{87}\mathrm{Rb})} = 4.591\,359\,287(61) \times 10^{-9}\ \mathrm{m^2\ s^{-1}} \quad [1.3 \times 10^{-8}]\ , \tag{332}$$

where the quoted u_r contains a statistical component from the 72 measurements of 8.8×10^{-9}.

Cladé *et al.* (2006b) examined many possible sources of systematic error, both theoretically and experimentally, in this rather complex, sophisticated experiment in order to ensure that their result was correct. These include light shifts, index of refraction effects, and the effect of a gravity gradient, for which the corrections and their uncertainties are in fact comparatively small. More significant are the fractional corrections of $(16.8 \pm 8) \times 10^{-9}$ for wave front curvature and Guoy phase, $(-13.2 \pm 4) \times 10^{-9}$ for second order Zeeman effect, and $4(4) \times 10^{-9}$ for the alignment of the Raman and Bloch beams. The total of all corrections is given as $10.98(10.0) \times 10^{-9}$.

From Eq. (321), the observational equation for the LKB value of $h/m(^{87}\mathrm{Rb})$ in Eq (332) is

$$\frac{h}{m(^{87}\mathrm{Rb})} = \frac{A_r(\mathrm{e})}{A_r(^{87}\mathrm{Rb})}\frac{c\,\alpha^2}{2R_\infty}\ . \tag{333}$$

Evaluation of this expression with the LKB result and the 2006 recommended values of R_∞ and $A_r(\mathrm{e})$, and the value of $A_r(^{87}\mathrm{Rb})$ resulting from the final least-squares adjustment on which the 2006 recommended values are based, all of whose uncertainties are negligible in this context, yields

$$\alpha^{-1} = 137.035\,998\,83(91) \qquad [6.7 \times 10^{-9}]\ , \tag{334}$$

which is included in Table 26. The uncertainty of this value of α^{-1} is smaller than the uncertainty of any other

value except those in Table 14 deduced from the measurement of a_e, exceeding the smallest uncertainty of the two values of $\alpha^{-1}[a_e]$ in that table by a factor of ten.

9. THERMAL PHYSICAL QUANTITIES

The following sections discuss the molar gas constant, Boltzmann constant, and Stefan-Boltzmann constant—constants associated with phenomena in the fields of thermodynamics and/or statistical mechanics.

1. Molar gas constant R

The square of the speed of sound $c_a^2(p,T)$ of a real gas at pressure p and thermodynamic temperature T can be written as (Colclough, 1973)

$$\begin{aligned} c_a^2(p,T) &= A_0(T) + A_1(T)p \\ &\quad + A_2(T)p^2 + A_3(T)p^3 + \cdots \ , \end{aligned} \tag{335}$$

where $A_1(T)$ is the first acoustic virial coefficient, $A_2(T)$ is the second, *etc.* In the limit $p \to 0$, Eq. (335) yields

$$c_a^2(0,T) = A_0(T) = \frac{\gamma_0 R T}{A_r(X) M_u} \ , \tag{336}$$

where the expression on the right-hand side is the square of the speed of sound for an unbounded ideal gas, and where $\gamma_0 = c_p/c_V$ is the ratio of the specific heat capacity of the gas at constant pressure to that at constant volume, $A_r(X)$ is the relative atomic mass of the atoms or molecules of the gas, and $M_u = 10^{-3}$ kg mol^{-1}. For a monatomic ideal gas, $\gamma_0 = 5/3$.

The 2006 recommended value of R, like the 2002 and 1998 values, is based on measurements of the speed of sound in argon carried out in two independent experiments, one done in the 1970s at NPL and the other done in the 1980s at NIST. Values of $c_a^2(p, T_{\rm TPW})$, where $T_{\rm TPW} = 273.16$ K is the triple point of water, were obtained at various pressures and extrapolated to $p = 0$ in order to determine $A_0(T_{\rm TPW}) = c_a^2(0, T_{\rm TPW})$, and hence R, from the relation

$$R = \frac{c_a^2(0, T_{\rm TPW}) A_r({\rm Ar}) M_u}{\gamma_0 T_{\rm TPW}} \ , \tag{337}$$

which follows from Eq. (336).

Because the work of both NIST and NPL is reviewed in CODATA-98 and CODATA-02 and nothing has occurred in the last 4 years that would change the values of R implied by their reported values of $c_a^2(0, T_{\rm TPW})$, we give only a brief summary here. Changes in these values due to the new values of $A_r(^A{\rm Ar})$ resulting from the 2003 atomic mass evaluation as given in Table 2, or the new IUPAC compilation of atomic weights of the elements given by Wieser (2006), are negligible.

Since R cannot be expressed as a function of any other of the 2006 adjusted constants, R itself is taken as an adjusted constant for the NIST and NPL measurements.

1. NIST: speed of sound in argon

In the NIST experiment of Moldover *et al.* (1988), a spherical acoustic resonator at a temperature $T = T_{\rm TPW}$ filled with argon was used to determine $c_a^2(p, T_{\rm TPW})$. The final NIST result for the molar gas constant is

$$R = 8.314\,471(15) \ {\rm J\ mol^{-1}\ K^{-1}} \quad [1.8 \times 10^{-6}] \ . \tag{338}$$

The mercury employed to determine the volume of the spherical resonator was traceable to the mercury whose density was measured by Cook (1961) [see also Cook and Stone (1957)]. The mercury employed in the NMI Hg electrometer determination of K_J (see 7.3.1) was also traceable to the same mercury. Consequently, the NIST value of R and the NMI value of K_J are correlated with the non-negligible correlation coefficient 0.068.

2. NPL: speed of sound in argon

In contrast to the dimensionally fixed resonator used in the NIST experiment, the NPL experiment employed a variable path length fixed-frequency cylindrical acoustic interferometer to measure $c_a^2(p, T_{\rm TPW})$. The final NPL result for the molar gas constant is (Colclough *et al.*, 1979)

$$R = 8.314\,504(70) \ {\rm J\ mol^{-1}\ K^{-1}} \quad [8.4 \times 10^{-6}] \ . \tag{339}$$

Although both the NIST and NPL values of R are based on the same values of $A_r(^{40}{\rm Ar})$, $A_r(^{38}{\rm Ar})$, and $A_r(^{36}{\rm Ar})$, the uncertainties of these relative atomic masses are sufficiently small that the covariance of the two values of R is negligible.

3. Other values

The most important of the historical values of R have been reviewed by Colclough (1984) [see also (Quinn *et al.*, 1976) and CODATA-98]. However, because of the large uncertainties of these early values, they were not considered for use in either the 1986, 1998, or 2002 CODATA adjustments, and we do not consider them for the 2006 adjustment as well.

Also because of its non-competitive uncertainty ($u_r = 36 \times 10^{-6}$), we exclude from consideration in the 2006 adjustment, as in the 2002 adjustment, the value of R obtained from measurements of the speed of sound in argon reported by He and Liu (2002) at the Xián Jiaotong University, Xián, China (People's Republic of).

2. Boltzmann constant k

The Boltzmann constant is related to the molar gas constant R and other adjusted constants by

$$k = \frac{2R_\infty h}{c A_r({\rm e}) M_u \alpha^2} R = \frac{R}{N_A} \ . \tag{340}$$

No competitive directly measured value of k was available for the 1998 or 2002 adjustments, and the situation remains unchanged for the present adjustment. Thus, the 2006 recommended value with $u_r = 1.7 \times 10^{-6}$ is obtained from this relation, as were the 1998 and 2002 recommended values. However, a number of experiments are currently underway that might lead to competitive values of k (or R) in the future; see Fellmuth *et al.* (2006) for a recent review.

Indeed, one such experiment underway at the PTB based on dielectric constant gas thermometry (DCGT) was discussed in both CODATA-98 and CODATA-02, but no experimental result for A_ϵ/R, where A_ϵ is the molar polarizability of the ^{4}He atom, other than that considered in these two reports, has been published by the PTB group [see also Fellmuth *et al.* (2006) and Luther *et al.* (1996)]. However, the relative uncertainty of the theoretical value of the static electric dipole polarizability of the ground state of the ^{4}He atom, which is required to calculate k from A_ϵ/R, has been lowered by more than a factor of ten to below 2×10^{-7} (Łach *et al.*, 2004). Nevertheless, the change in its value is negligible at the level of uncertainty of the PTB result for A_ϵ/R; hence, the value $k = 1.380\,65(4) \times 10^{-23}$ J K^{-1} $[30 \times 10^{-6}]$ from the PTB experiment given in CODATA-02 is unchanged.

In addition, preliminary results from two other ongoing experiments, the first being carried out at NIST by Schmidt *et al.* (2007) and the second at the University of Paris by Daussy *et al.* (2007), have recently been published.

Schmidt *et al.* (2007) report $R = 8.314\,487(76)$ J mol K^{-1} $[9.1 \times 10^{-6}]$, obtained from measurements of the index of refraction $n(p,T)$ of ^{4}He gas as a function of p and T by measuring the difference in the resonant frequencies of a quasispherical microwave resonator when filled with ^{4}He at a given pressure and when evacuated (that is, at $p = 0$). This experiment has some similarities to the PTB DCGT experiment in that it determines the quantity A_ϵ/R and hence k. However, in DCGT one measures the difference in capacitance of a capacitor when filled with ^{4}He at a given pressure and at $p = 0$, and hence one determines the dielectric constant of the ^{4}He gas rather than its index of refraction. Because ^{4}He is slightly diamagnetic, this means that to obtain A_ϵ/R in the NIST experiment, a value for A_μ/R is required, where $A_\mu = 4\pi\chi_0/3$ and χ_0 is the diamagnetic susceptibility of a ^{4}He atom.

Daussy *et al.* (2007) report $k = 1.380\,65(26) \times 10^{-23}$ J K^{-1} $[190 \times 10^{-6}]$, obtained from measurements as a function of pressure of the Doppler profile at $T = 273.15$ K (the ice point) of a well-isolated rovibrational line in the ν_2 band of the ammonium molecule, $^{14}NH_3$, and extrapolation to $p = 0$. The experiment actually measures $R = kN_A$, because the mass of the ammonium molecule in kilograms is required but can only be obtained with the requisite accuracy from the molar masses of ^{14}N and ^{1}H, thereby introducing N_A.

It is encouraging that the preliminary values of k and R resulting from these three experiments are consistent with the 2006 recommended values.

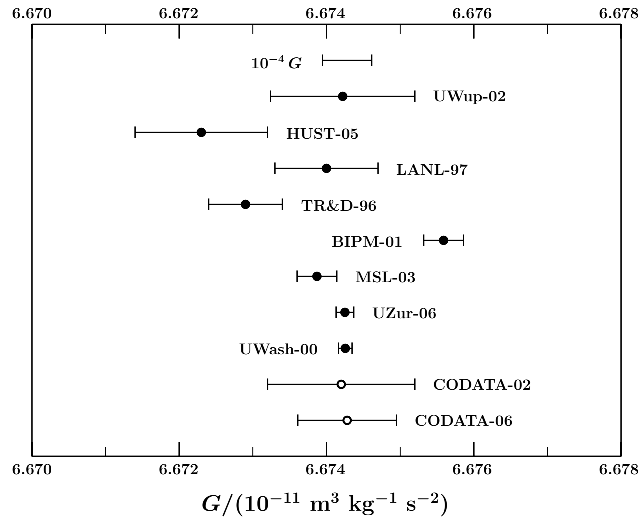

FIG. 2 Values of the Newtonian constant of gravitation G.

3. Stefan-Boltzmann constant σ

The Stefan-Boltzmann constant is related to c, h, and the Boltzmann constant k by

$$\sigma = \frac{2\pi^5 k^4}{15 h^3 c^2}\ , \tag{341}$$

which, with the aid of Eq. (340), can be expressed in terms of the molar gas constant and other adjusted constants as

$$\sigma = \frac{32\pi^5 h}{15 c^6}\left(\frac{R_\infty R}{A_r(\mathrm{e}) M_u \alpha^2}\right)^4\ . \tag{342}$$

No competitive directly measured value of σ was available for the 1998 or 2002 adjustments, and the situation remains unchanged for the 2006 adjustment. Thus, the 2006 recommended value with $u_r = 7.0 \times 10^{-6}$ is obtained from this relation, as were the 1998 and 2002 recommended values. For a concise summary of experiments that might provide a competiive value of σ, see the review by Fellmuth *et al.* (2006).

10. NEWTONIAN CONSTANT OF GRAVITATION G

Because there is no known quantitative theoretical relationship between the Newtonian constant of gravitation G and other fundamental constants, and because the currently available experimental values of G are independent of all of the other data relevant to the 2006 adjustment,

TABLE 27 Summary of the results of measurements of the Newtonian constant of gravitation relevant to the 2006 adjustment together with the 2002 and 2006 CODATA recommended values.

Item	Source	Identification[a]	Method	$\frac{10^{11}\,G}{\mathrm{m^3\ kg^{-1}\ s^{-2}}}$	Rel. stand. uncert u_r
	2002 CODATA Adjustment	CODATA-02		6.6742(10)	1.5×10^{-4}
a.	(Karagioz and Izmailov, 1996)	TR&D-96	Fiber torsion balance, dynamic mode	6.672 9(5)	7.5×10^{-5}
b.	(Bagley and Luther, 1997)	LANL-97	Fiber torsion balance, dynamic mode	6.674 0(7)	1.0×10^{-4}
c.	(Gundlach and Merkowitz, 2000, 2002)	UWash-00	Fiber torsion balance, dynamic compensation	6.674 255(92)	1.4×10^{-5}
d.	(Quinn *et al.*, 2001)	BIPM-01	Strip torsion balance, compensation mode, static deflection	6.675 59(27)	4.0×10^{-5}
e.	(Kleinevoß, 2002; Kleinvoß *et al.*, 2002)	UWup-02	Suspended body, displacement	6.674 22(98)	1.5×10^{-4}
f.	(Armstrong and Fitzgerald, 2003)	MSL-03	Strip torsion balance, compensation mode	6.673 87(27)	4.0×10^{-5}
g.	(Hu *et al.*, 2005)	HUST-05	Fiber torsion balance, dynamic mode	6.672 3(9)	1.3×10^{-4}
h.	(Schlamminger *et al.*, 2006)	UZur-06	Stationary body, weight change	6.674 25(12)	1.9×10^{-5}
	2006 CODATA Adjustment	CODATA-06		6.674 28(67)	1.0×10^{-4}

[a]TR&D: Tribotech Research and Development Company, Moscow, Russian Federation; LANL: Los Alamos National Laboratory, Los Alamos, New Mexico, USA; UWash: University of Washington, Seattle, Washington, USA; BIPM: International Bureau of Weights and Measures, Sèvres, France; UWup: University of Wuppertal, Wuppertal, Germany; MSL: Measurement Standards Laboratory, Lower Hutt, New Zealand; HUST: Huazhong University of Science and Technology, Wuhan, PRC; UZur: University of Zurich, Zurich, Switzerland.

these experimental values contribute only to the determination of the 2006 recommended value of G and can be considered independently from the other data.

The historic difficulty of determining G, as demonstrated by the inconsistencies among different measurements, is described in detail in CODATA-86, CODATA-98, and CODATA-02. Although no new competitive independent result for G has become available in the last 4 years, adjustments to two existing results considered in 2002 have been made by researchers involved in the original work. One of the two results that has changed is from the Huazhong University of Science and Technology (HUST) and is now identified as HUST-05; the other is from the University of Zurich (UZur) and is now identified as UZur-06. These revised results are discussed in some detail below.

Table 27 summarizes the various values of G considered here, which are the same as in 2002 with the exception of these two revised results, and Fig. 2 compares them graphically. For reference purposes, both the table and figure include the 2002 and 2006 CODATA recommended values. The result now identified as TR&D-96 was previously identified as TR&D-98. The change is because a 1996 reference, (Karagioz and Izmailov, 1996), was found that reports the same result as does the 1998 reference (Karagioz *et al.*, 1998).

For simplicity, in the following text, we write G as a numerical factor multiplying G_0, where

$$G_0 = 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}\ . \tag{343}$$

1. Updated values

1. Huazhong University of Science and Technology

The HUST group, which determines G by the time-of-swing method using a high-Q torsion pendulum with two horizontal, 6.25 kg stainless steel cylindrical source masses labeled A and B positioned on either side of the test mass, has reported a fractional correction of $+360 \times 10^{-6}$ to their original result given by Luo *et al.* (1999). It arises in part from recently discovered density inhomogeneities in the source masses, the result of which is a displacement of the center of mass of each source mass from its geometrical center (Hu *et al.*, 2005). Using a "weighbridge" with a commercial electronic balance—a method developed by Davis (1995) to locate the center of mass of a test object with micrometers precision—Hu *et al.* (2005) found that the axial eccentricities e_A and e_B of the two source masses were $(10.3\ \pm\ 2.6)$ μm and $(6.3\ \pm\ 3.7)$ μm, with the result that the equivalent displacements between the test mass and the source masses are larger than the values used by (Luo *et al.*, 1999). Assuming a linear axial density distribution, the calculated

fractional correction to the previous result is $+210 \times 10^{-6}$ with an additional component of relative standard uncertainty of 78×10^{-6} due to the uncertainties of the eccentricities.

The remaining 150×10^{-6} portion of the 360×10^{-6} fractional correction is also discussed by Hu *et al.* (2005) and arises as follows. In the HUST experiment, G is determined by comparing the period of the torsion pendulum with and without the source masses. When the source masses are removed, they are replaced by air. Since the masses of the source masses used by Luo *et al.* (1999) are the vacuum masses, a correction for the air, first suggested by R. S. Davis and T. J. Quinn of the BIPM, is required. This correction was privately communicated to the Task Group by the HUST researchers in 2003 and included in the HUST value of G used in the 2002 adjustment.

The HUST revised value of G, including the additional component of uncertainty due to the measurement of the eccentricities e_{A} and e_{B}, is item g in Table 27.

2. University of Zurich

The University of Zurich result for G discussed in CODATA-02 and used in the 2002 adjustment, $G = 6.674\,07(22)\,G_0$ $[3.3 \times 10^{-5}]$, was reported by Schlamminger *et al.* (2002). It was based on the weighted mean of three highly consistent values obtained from three series of measurements carried out at the Paul Scherrer Institute (PSI), Villigen, Switzerland, in 2001 and 2002 and denoted Cu, Ta I, and Ta II. The designation Cu means that the test masses were gold plated copper, and the designation Ta means that they were tantalum. Following the publication of Schlamminger *et al.* (2002), an extensive reanalysis of the original data was carried out by these authors together with other University of Zurich researchers, the result being the value of G in Table 27, item h, as given in the detailed final report on the experiment (Schlamminger *et al.*, 2006).

In the University of Zurich approach to determining G, a modified commercial single-pan balance is used to measure the change in the difference in weight of two cylindrical test masses when the relative position of two source masses is changed. The quantity measured is the 800 μg difference signal obtained at many different working points in the balance calibration range using two sets of 16 individual wire weights, allowing an *in situ* measurement of the balance non-linearity over the entire 0.2 g balance calibration interval. A more rigorous analysis using a fitting method with Legendre polynomials has now allowed the relative standard uncertainty contribution to G from balance nonlinearity to be reduced from 18×10^{-6} to 6.1×10^{-6} based on the Cu test-mass data. Various problems with the mass handler for the wire weights that did not allow the application of the Legendre polynomial fitting procedure occurred during the Ta test-mass measurements, resulting in large systematic errors. Therefore, the researchers decided to include only the Cu data in their final analysis (Schlamminger *et al.*, 2006).

Each source mass consisted of a cylindrical tank filled with 7.5×10^3 kg of mercury. Since the mercury represented approximately 94 % of the total mass, special care was taken in determining its mass and density. These measurements were further used to obtain more accurate values for the key tank dimensions and Hg mass. This was done by minimizing a χ^2 function that depended on the tank dimensions and the Hg mass and density, and using the dependence of the density on these dimensions and the Hg mass as a constraint. Calculation of the mass integration constant with these improved values reduced the u_{r} of this critical quantity from 20.6×10^{-6} to 6.7×10^{-6}.

Although the analysis of Schlamminger *et al.* (2002) assumed a linear temporal drift of the balance zero point, a careful examination by Schlamminger *et al.* (2006) found that the drift was significantly nonlinear and was influenced by the previous load history of the balance. A series of Legendre polynomials and a sawtooth function, respectively, were therefore used to describe the slow and rapid variations of the observed balance zero-point with time.

The 2002 value of G obtained from the Cu data was $6.674\,03\,G_0$, consistent with the Ta I, and Ta II values of $6.674\,09\,G_0$ and $6.674\,10\,G_0$ (Schlamminger *et al.*, 2002), whereas the value from the present Cu data analysis is $6.674\,25(12)\,G_0$, with the 3.3×10^{-5} fractional increase being due primarily to the application of the nonlinear zero point drift correction. A minor contributor to the difference is the inclusion of the very first Cu data set that was omitted in the 2002 analysis due to a large start-up zero-point drift that is now correctable with the new Legendre polynomial-sawtooth function analysis technique, and the exclusion of a data set that had a temperature stabilization system failure that went undetected by the old data analysis method (Schlamminger, 2007).

2. Determination of 2006 recommended value of G

The overall agreement of the eight values of G in Table 27 (items a to h) has improved somewhat since the 2002 adjustment, but the situation is still far from satisfactory. Their weighted mean is $G = 6.674\,275(68)\,G_0$ with $\chi^2 = 38.6$ for degrees of freedom $\nu = N - M = 8 - 1 = 7$, Birge ratio $R_{\mathrm{B}} = \sqrt{\chi^2/\nu} = 2.35$, and normalized residuals r_i of -2.75, -0.39, -0.22, 4.87, -0.56, -1.50, -2.19, and -0.19, respectively (see Appendix E of CODATA-98). The BIPM-01 value with $r_i = 4.87$ is clearly the most problematic. For comparison, the 2002 weighted mean was $G = 6.674\,232(75)\,G_0$ with $\chi^2 = 57.7$ for $\nu = 7$ and $R_{\mathrm{B}} = 2.87$.

If the BIPM value is deleted, the weighted mean is reduced by 1.3 standard uncertainties to $G = 6.674\,187(70)\,G_0$, and $\chi^2 = 13.3$, $\nu = 6$, and $R_{\mathrm{B}} = 1.49$. In this case, the two remaining data with significant nor-

malized residuals are the the TR&D-96 and the HUST-05 results with $r_i = -2.57$ and -2.10, respectively. If these two data, which agree with each other, are deleted, the weighted mean is $G = 6.674\,225(71)\,G_0$ with $\chi^2 = 2.0$, $\nu = 4$, $R_{\rm B} = 0.70$, and with all normalized residuals less than one except $r_i = -1.31$ for datum MSL-03. Finally, if the UWash-00 and UZur-06 data, which have the smallest assigned uncertainties of the initial eight values and which are in excellent agreement with each other, are deleted from the initial group of eight data, the weighted mean of the remaining six data is $G = 6.674\,384(167)\,G_0$ with $\chi^2 = 38.1$, $\nu = 6$, and $R_{\rm B} = 2.76$. The normalized residuals for these six data, TR&D-96, LANL-97, BIPM-01, UWup-02, MSL-03, and HUST-05, are -2.97, -0.55, 4.46, -0.17, -1.91 and -2.32, respectively.

Finally, if the uncertainties of each of the eight values of G are multiplied by the Birge ratio associated with their weighted mean, $R_{\rm B} = 2.35$, so that χ^2 of their weighted mean becomes equal to its expected value of $\nu = 7$ and $R_{\rm B} = 1$, the normalized residual of the datum BIPM-01 would still be larger than two.

Based on the results of the above calculations, the historical difficulty of determining G, the fact that all eight values of G in Table 27 are credible, and that the two results with the smallest uncertainties, UWash-00 and UZur-06, are highly consistent with one another, the Task Group decided to take as the 2006 CODATA recommended value of G the weighted mean of all of the data, but with an uncertainty of $0.000\,67\,G_0$, corresponding to $u_{\rm r} = 1.0 \times 10^{-4}$:

$$G = 6.674\,28(67) \times 10^{-11}\ {\rm m^3\ kg^{-1}\ s^{-2}}\ . \quad [1.0 \times 10^{-4}] \tag{344}$$

This value exceeds the 2002 recommended value by the fractional amount 1.2×10^{-5}, which is less than one tenth of the uncertainty $u_{\rm r} = 1.5 \times 10^{-4}$ of the 2002 value. Further, the uncertainty of the 2006 value, $u_{\rm r} = 1.0 \times 10^{-4}$, is two thirds that of the 2002 value.

In assigning this uncertainty to the 2006 recommended value of G, the Task Group recognized that if the uncertainty was smaller than really justified by the data, taking into account the history of measurements of G, it might discourage the initiation of new research efforts to determine G, if not the continuation of some of the research efforts already underway. Such efforts need to be encouraged in order to provide a more solid and redundant data set upon which to base future recommended values. On the other hand, if the uncertainty was too large, for example, if the uncertainty of the 2002 recommended value had been retained for the 2006 value, then the recommended value would not have reflected the fact that we now have two data that are in excellent agreement, have $u_{\rm r}$ less than 2×10^{-5}, and are the two most accurate values available.

3. Prospective values

New techniques to measure G using atom interferometry are currently under development in at least two laboratories—the Università de Firenze in Italy and Stanford University in the United States. This comes as no surprise since atom interferometry is also being developed to measure the local acceleration due to gravity g (see the last paragraph of Sec. 2). Recent proof of principle experiments combine two vertically separated atomic clouds forming an atom-interferometer-gravity-gradiometer that measures the change in the gravity gradient when a well characterized source mass is displaced. Measuring the change in the gravity gradient allows the rejection of many possible systematic errors. Bertoldi *et al.* (2006) at the Università de Firenze used a Rb fountain and a fast launch juggling sequence of two atomic clouds to measure G to 1 %, obtaining the value $6.64(6)\,G_0$; they hope to reach a final uncertainty of 1 part in 10^4. Fixler *et al.* (2007) at Stanford used two separate Cs atom interferometer gravimeters to measure G and obtained the value $6.693(34)\,G_0$. The two largest uncertainties from systematic effects were the determination of the initial atom velocity and the initial atom position. The Stanford researchers also hope to achieve a final uncertainty of 1 part in 10^4. Although neither of these results is significant for the current analysis of G, future results could be of considerable interest.

11. X-RAY AND ELECTROWEAK QUANTITIES

1. X-ray units

Historically, units that have been used to express the wavelengths of x-ray lines are the copper $\rm K\alpha_1$ x unit, symbol $\rm xu(CuK\alpha_1)$, the molybdenum $\rm K\alpha_1$ x unit, symbol $\rm xu(MoK\alpha_1)$, and the ångstrom star, symbol Å*. They are defined by assigning an exact, conventional value to the wavelength of the $\rm CuK\alpha_1$, $\rm MoK\alpha_1$, and $\rm WK\alpha_1$ x-ray lines when each is expressed in its corresponding unit:

$$\lambda({\rm CuK\alpha_1}) = 1\,537.400\ {\rm xu(CuK\alpha_1)} \tag{345}$$
$$\lambda({\rm MoK\alpha_1}) = 707.831\ {\rm xu(MoK\alpha_1)} \tag{346}$$
$$\lambda({\rm WK\alpha_1}) = 0.209\,010\,0\ {\rm Å^*}\ . \tag{347}$$

The experimental work that determines the best values of these three units was reviewed in CODATA-98, and the relevant data may be summarized as follows:

$$\frac{\lambda({\rm CuK\alpha_1})}{d_{220}({\rm W4.2a})} = 0.802\,327\,11(24) \quad [3.0 \times 10^{-7}] \tag{348}$$
$$\frac{\lambda({\rm WK\alpha_1})}{d_{220}({\rm N})} = 0.108\,852\,175(98) \quad [9.0 \times 10^{-7}] \tag{349}$$
$$\frac{\lambda({\rm MoK\alpha_1})}{d_{220}({\rm N})} = 0.369\,406\,04(19) \quad [5.3 \times 10^{-7}] \tag{350}$$
$$\frac{\lambda({\rm CuK\alpha_1})}{d_{220}({\rm N})} = 0.802\,328\,04(77) \quad [9.6 \times 10^{-7}]\ , \tag{351}$$

where $d_{220}(\mathrm{W4.2a})$ and $d_{220}(\mathrm{N})$ denote the $\{220\}$ lattice spacings, at the standard reference conditions $p = 0$ and $t_{90} = 22.5$ °C, of particular silicon crystals used in the measurements. The result in Eq. (348) is from a collaboration between researchers from Friedrich-Schiller University (FSU), Jena, Germany and the PTB (Härtwig *et al.*, 1991). The lattice spacing $d_{220}(\mathrm{N})$ is related to crystals of known lattice spacing through Eq. (301).

In order to obtain best values in the least-squares sense for $\mathrm{xu}(\mathrm{CuK}\alpha_1)$, $\mathrm{xu}(\mathrm{MoK}\alpha_1)$, and Å*, we take these units to be adjusted constants. Thus, the observational equations for the data of Eqs. (348) to (351) are

$$\frac{\lambda(\mathrm{CuK}\alpha_1)}{d_{220}(\mathrm{W4.2a})} \doteq \frac{1\,537.400\ \mathrm{xu}(\mathrm{CuK}\alpha_1)}{d_{220}(\mathrm{W4.2a})} \tag{352}$$

$$\frac{\lambda(\mathrm{WK}\alpha_1)}{d_{220}(\mathrm{N})} \doteq \frac{0.209\,010\,0\ \text{Å}^*}{d_{220}(\mathrm{N})} \tag{353}$$

$$\frac{\lambda(\mathrm{MoK}\alpha_1)}{d_{220}(\mathrm{N})} \doteq \frac{707.831\ \mathrm{xu}(\mathrm{MoK}\alpha_1)}{d_{220}(\mathrm{N})} \tag{354}$$

$$\frac{\lambda(\mathrm{CuK}\alpha_1)}{d_{220}(\mathrm{N})} \doteq \frac{1\,537.400\ \mathrm{xu}(\mathrm{CuK}\alpha_1)}{d_{220}(\mathrm{N})}\,, \tag{355}$$

where $d_{220}(\mathrm{N})$ is taken to be an adjusted constant and $d_{220}(\mathrm{W17})$ and $d_{220}(\mathrm{W4.2a})$ are adjusted constants as well.

2. Particle Data Group input

There are a few cases in the 2006 adjustment where an inexact constant that is used in the analysis of input data is not treated as an adjusted quantity, because the adjustment has a negligible effect on its value. Three such constants, used in the calculation of the theoretical expressions for the electron and muon magnetic moment anomalies a_e and a_μ, are the mass of the tau lepton m_τ, the Fermi coupling constant G_F, and sine squared of the weak mixing angle $\sin^2\theta_\mathrm{W}$, and are obtained from the most recent report of the Particle Data Group (Yao *et al.*, 2006):

$$m_\tau c^2 = 1776.99(29)\ \mathrm{MeV} \qquad [1.6\times10^{-4}] \tag{356}$$

$$\frac{G_\mathrm{F}}{(\hbar c)^3} = 1.166\,37(1)\times10^{-5}\ \mathrm{GeV}^{-2} \quad [8.6\times10^{-6}] \tag{357}$$

$$\sin^2\theta_\mathrm{W} = 0.222\,55(56) \qquad [2.5\times10^{-3}]\,. \tag{358}$$

To facilitate the calculations, the uncertainty of $m_\tau c^2$ is symmetrized and taken to be 0.29 MeV rather than +0.29 MeV, −0.26 MeV. We use the definition $\sin^2\theta_\mathrm{W} = 1 - (m_\mathrm{W}/m_\mathrm{Z})^2$, where m_W and m_Z are, respectively, the masses of the $\mathrm{W}^\pm$ and Z^0 bosons, because it is employed in the calculation of the electroweak contributions to a_e and a_μ (Czarnecki *et al.*, 1996). The Particle Data Group's recommended value for the mass ratio of these bosons is $m_\mathrm{W}/m_\mathrm{Z} = 0.881\,73(32)$, which leads to the value of $\sin^2\theta_\mathrm{W}$ given above.

12. ANALYSIS OF DATA

The previously discussed input data are examined in this section for their mutual compatibility and their potential role in determining the 2006 recommended values of the constants. Based on this analysis, the data are selected for the final least-squares adjustment from which the recommended values are obtained. Because the data on the Newtonian constant of gravitation G are independent of the other data and are analyzed in Sec. 10, they are not examined further. The consistency of the input data is evaluated by directly comparing different measurements of the same quantity, and by directly comparing the values of a single fundamental constant inferred from measurements of different quantities. As noted in the outline section of this paper, the inferred value is for comparison purposes only; the datum from which it is obtained, not the inferred value, is the input datum in the adjustment. The potential role of a particular input datum is gauged by carrying out a least-squares adjustment using all of the initially considered data. A particular measurement of a quantity is included in the final adjustment if its uncertainty is not more than about ten times the uncertainty of the value of that quantity provided by other data in the adjustment. The measure we use is the "self sensitivity coefficient" of an input datum S_c (see CODATA-98), which must be greater than 0.01 in order for the datum to be included.

The input data are given in Tables 28, 30, and 32 and their covariances are given as correlation coefficients in Tables 29, 31, and 33. The δs given in Tables 28, 30, and 32 are quantities added to corresponding theoretical expressions to account for the uncertainties of those expressions, as previously discussed (see, for example, Sec. 4.1.1.12). Note that the value of the Rydberg constant R_∞ depends only weakly on changes, at the level of the uncertainties, of the data in Tables 30 and 32.

TABLE 28 Summary of principal input data for the determination of the 2006 recommended value of the Rydberg constant R_∞. [The notation for the additive corrections $\delta_X(n\mathrm{L}_j)$ in this table has the same meaning as the notation $\delta^X_{n\mathrm{L}j}$ in Sec. 4.1.1.12.]

Item number	Input datum	Value	Relative standard uncertainty[a] u_r	Identification	Sec.
$A1$	$\delta_\mathrm{H}(1\mathrm{S}_{1/2})$	0.0(3.7) kHz	$[1.1 \times 10^{-12}]$	theory	4.1.1.12
$A2$	$\delta_\mathrm{H}(2\mathrm{S}_{1/2})$	0.00(46) kHz	$[5.6 \times 10^{-13}]$	theory	4.1.1.12
$A3$	$\delta_\mathrm{H}(3\mathrm{S}_{1/2})$	0.00(14) kHz	$[3.7 \times 10^{-13}]$	theory	4.1.1.12
$A4$	$\delta_\mathrm{H}(4\mathrm{S}_{1/2})$	0.000(58) kHz	$[2.8 \times 10^{-13}]$	theory	4.1.1.12
$A5$	$\delta_\mathrm{H}(6\mathrm{S}_{1/2})$	0.000(20) kHz	$[2.1 \times 10^{-13}]$	theory	4.1.1.12
$A6$	$\delta_\mathrm{H}(8\mathrm{S}_{1/2})$	0.0000(82) kHz	$[1.6 \times 10^{-13}]$	theory	4.1.1.12
$A7$	$\delta_\mathrm{H}(2\mathrm{P}_{1/2})$	0.000(69) kHz	$[8.4 \times 10^{-14}]$	theory	4.1.1.12
$A8$	$\delta_\mathrm{H}(4\mathrm{P}_{1/2})$	0.0000(87) kHz	$[4.2 \times 10^{-14}]$	theory	4.1.1.12
$A9$	$\delta_\mathrm{H}(2\mathrm{P}_{3/2})$	0.000(69) kHz	$[8.4 \times 10^{-14}]$	theory	4.1.1.12
$A10$	$\delta_\mathrm{H}(4\mathrm{P}_{3/2})$	0.0000(87) kHz	$[4.2 \times 10^{-14}]$	theory	4.1.1.12
$A11$	$\delta_\mathrm{H}(8\mathrm{D}_{3/2})$	0.000 00(48) kHz	$[9.3 \times 10^{-15}]$	theory	4.1.1.12
$A12$	$\delta_\mathrm{H}(12\mathrm{D}_{3/2})$	0.000 00(15) kHz	$[6.6 \times 10^{-15}]$	theory	4.1.1.12
$A13$	$\delta_\mathrm{H}(4\mathrm{D}_{5/2})$	0.0000(38) kHz	$[1.9 \times 10^{-14}]$	theory	4.1.1.12
$A14$	$\delta_\mathrm{H}(6\mathrm{D}_{5/2})$	0.0000(11) kHz	$[1.2 \times 10^{-14}]$	theory	4.1.1.12
$A15$	$\delta_\mathrm{H}(8\mathrm{D}_{5/2})$	0.000 00(48) kHz	$[9.3 \times 10^{-15}]$	theory	4.1.1.12
$A16$	$\delta_\mathrm{H}(12\mathrm{D}_{5/2})$	0.000 00(16) kHz	$[7.0 \times 10^{-15}]$	theory	4.1.1.12
$A17$	$\delta_\mathrm{D}(1\mathrm{S}_{1/2})$	0.0(3.6) kHz	$[1.1 \times 10^{-12}]$	theory	4.1.1.12
$A18$	$\delta_\mathrm{D}(2\mathrm{S}_{1/2})$	0.00(45) kHz	$[5.4 \times 10^{-13}]$	theory	4.1.1.12
$A19$	$\delta_\mathrm{D}(4\mathrm{S}_{1/2})$	0.000(56) kHz	$[2.7 \times 10^{-13}]$	theory	4.1.1.12
$A20$	$\delta_\mathrm{D}(8\mathrm{S}_{1/2})$	0.0000(80) kHz	$[1.6 \times 10^{-13}]$	theory	4.1.1.12
$A21$	$\delta_\mathrm{D}(8\mathrm{D}_{3/2})$	0.000 00(48) kHz	$[9.3 \times 10^{-15}]$	theory	4.1.1.12
$A22$	$\delta_\mathrm{D}(12\mathrm{D}_{3/2})$	0.000 00(15) kHz	$[6.6 \times 10^{-15}]$	theory	4.1.1.12
$A23$	$\delta_\mathrm{D}(4\mathrm{D}_{5/2})$	0.0000(38) kHz	$[1.9 \times 10^{-14}]$	theory	4.1.1.12
$A24$	$\delta_\mathrm{D}(8\mathrm{D}_{5/2})$	0.000 00(48) kHz	$[9.3 \times 10^{-15}]$	theory	4.1.1.12
$A25$	$\delta_\mathrm{D}(12\mathrm{D}_{5/2})$	0.000 00(16) kHz	$[7.0 \times 10^{-15}]$	theory	4.1.1.12
$A26$	$\nu_\mathrm{H}(1\mathrm{S}_{1/2} - 2\mathrm{S}_{1/2})$	2 466 061 413 187.074(34) kHz	1.4×10^{-14}	MPQ-04	4.1.2
$A27$	$\nu_\mathrm{H}(2\mathrm{S}_{1/2} - 8\mathrm{S}_{1/2})$	770 649 350 012.0(8.6) kHz	1.1×10^{-11}	LK/SY-97	4.1.2
$A28$	$\nu_\mathrm{H}(2\mathrm{S}_{1/2} - 8\mathrm{D}_{3/2})$	770 649 504 450.0(8.3) kHz	1.1×10^{-11}	LK/SY-97	4.1.2
$A29$	$\nu_\mathrm{H}(2\mathrm{S}_{1/2} - 8\mathrm{D}_{5/2})$	770 649 561 584.2(6.4) kHz	8.3×10^{-12}	LK/SY-97	4.1.2
$A30$	$\nu_\mathrm{H}(2\mathrm{S}_{1/2} - 12\mathrm{D}_{3/2})$	799 191 710 472.7(9.4) kHz	1.2×10^{-11}	LK/SY-98	4.1.2
$A31$	$\nu_\mathrm{H}(2\mathrm{S}_{1/2} - 12\mathrm{D}_{5/2})$	799 191 727 403.7(7.0) kHz	8.7×10^{-12}	LK/SY-98	4.1.2
$A32$	$\nu_\mathrm{H}(2\mathrm{S}_{1/2} - 4\mathrm{S}_{1/2}) - \frac{1}{4}\nu_\mathrm{H}(1\mathrm{S}_{1/2} - 2\mathrm{S}_{1/2})$	4 797 338(10) kHz	2.1×10^{-6}	MPQ-95	4.1.2
$A33$	$\nu_\mathrm{H}(2\mathrm{S}_{1/2} - 4\mathrm{D}_{5/2}) - \frac{1}{4}\nu_\mathrm{H}(1\mathrm{S}_{1/2} - 2\mathrm{S}_{1/2})$	6 490 144(24) kHz	3.7×10^{-6}	MPQ-95	4.1.2
$A34$	$\nu_\mathrm{H}(2\mathrm{S}_{1/2} - 6\mathrm{S}_{1/2}) - \frac{1}{4}\nu_\mathrm{H}(1\mathrm{S}_{1/2} - 3\mathrm{S}_{1/2})$	4 197 604(21) kHz	4.9×10^{-6}	LKB-96	4.1.2
$A35$	$\nu_\mathrm{H}(2\mathrm{S}_{1/2} - 6\mathrm{D}_{5/2}) - \frac{1}{4}\nu_\mathrm{H}(1\mathrm{S}_{1/2} - 3\mathrm{S}_{1/2})$	4 699 099(10) kHz	2.2×10^{-6}	LKB-96	4.1.2
$A36$	$\nu_\mathrm{H}(2\mathrm{S}_{1/2} - 4\mathrm{P}_{1/2}) - \frac{1}{4}\nu_\mathrm{H}(1\mathrm{S}_{1/2} - 2\mathrm{S}_{1/2})$	4 664 269(15) kHz	3.2×10^{-6}	YaleU-95	4.1.2
$A37$	$\nu_\mathrm{H}(2\mathrm{S}_{1/2} - 4\mathrm{P}_{3/2}) - \frac{1}{4}\nu_\mathrm{H}(1\mathrm{S}_{1/2} - 2\mathrm{S}_{1/2})$	6 035 373(10) kHz	1.7×10^{-6}	YaleU-95	4.1.2
$A38$	$\nu_\mathrm{H}(2\mathrm{S}_{1/2} - 2\mathrm{P}_{3/2})$	9 911 200(12) kHz	1.2×10^{-6}	HarvU-94	4.1.2
$A39.1$	$\nu_\mathrm{H}(2\mathrm{P}_{1/2} - 2\mathrm{S}_{1/2})$	1 057 845.0(9.0) kHz	8.5×10^{-6}	HarvU-86	4.1.2
$A39.2$	$\nu_\mathrm{H}(2\mathrm{P}_{1/2} - 2\mathrm{S}_{1/2})$	1 057 862(20) kHz	1.9×10^{-5}	USus-79	4.1.2
$A40$	$\nu_\mathrm{D}(2\mathrm{S}_{1/2} - 8\mathrm{S}_{1/2})$	770 859 041 245.7(6.9) kHz	8.9×10^{-12}	LK/SY-97	4.1.2
$A41$	$\nu_\mathrm{D}(2\mathrm{S}_{1/2} - 8\mathrm{D}_{3/2})$	770 859 195 701.8(6.3) kHz	8.2×10^{-12}	LK/SY-97	4.1.2
$A42$	$\nu_\mathrm{D}(2\mathrm{S}_{1/2} - 8\mathrm{D}_{5/2})$	770 859 252 849.5(5.9) kHz	7.7×10^{-12}	LK/SY-97	4.1.2
$A43$	$\nu_\mathrm{D}(2\mathrm{S}_{1/2} - 12\mathrm{D}_{3/2})$	799 409 168 038.0(8.6) kHz	1.1×10^{-11}	LK/SY-98	4.1.2
$A44$	$\nu_\mathrm{D}(2\mathrm{S}_{1/2} - 12\mathrm{D}_{5/2})$	799 409 184 966.8(6.8) kHz	8.5×10^{-12}	LK/SY-98	4.1.2
$A45$	$\nu_\mathrm{D}(2\mathrm{S}_{1/2} - 4\mathrm{S}_{1/2}) - \frac{1}{4}\nu_\mathrm{D}(1\mathrm{S}_{1/2} - 2\mathrm{S}_{1/2})$	4 801 693(20) kHz	4.2×10^{-6}	MPQ-95	4.1.2
$A46$	$\nu_\mathrm{D}(2\mathrm{S}_{1/2} - 4\mathrm{D}_{5/2}) - \frac{1}{4}\nu_\mathrm{D}(1\mathrm{S}_{1/2} - 2\mathrm{S}_{1/2})$	6 494 841(41) kHz	6.3×10^{-6}	MPQ-95	4.1.2
$A47$	$\nu_\mathrm{D}(1\mathrm{S}_{1/2} - 2\mathrm{S}_{1/2}) - \nu_\mathrm{H}(1\mathrm{S}_{1/2} - 2\mathrm{S}_{1/2})$	670 994 334.64(15) kHz	2.2×10^{-10}	MPQ-98	4.1.2
$A48$	R_p	0.895(18) fm	2.0×10^{-2}	Rp-03	4.1.3
$A49$	R_d	2.130(10) fm	4.7×10^{-3}	Rd-98	4.1.3

[a]The values in brackets are relative to the frequency equivalent of the binding energy of the indicated level.

TABLE 29 Correlation coefficients $r(x_i, x_j) \geq 0.0001$ of the input data related to R_∞ in Table 28. For simplicity, the two items of data to which a particular correlation coefficient corresponds are identified by their item numbers in Table 28.

$r(A1,\,A2) = 0.9958$	$r(A6,\,A19) = 0.8599$	$r(A27,\,A28) = 0.3478$	$r(A30,\,A44) = 0.1136$
$r(A1,\,A3) = 0.9955$	$r(A6,\,A20) = 0.9913$	$r(A27,\,A29) = 0.4532$	$r(A31,\,A34) = 0.0278$
$r(A1,\,A4) = 0.9943$	$r(A7,\,A8) = 0.0043$	$r(A27,\,A30) = 0.0899$	$r(A31,\,A35) = 0.0553$
$r(A1,\,A5) = 0.8720$	$r(A9,\,A10) = 0.0043$	$r(A27,\,A31) = 0.1206$	$r(A31,\,A40) = 0.1512$
$r(A1,\,A6) = 0.8711$	$r(A11,\,A12) = 0.0005$	$r(A27,\,A34) = 0.0225$	$r(A31,\,A41) = 0.1647$
$r(A1,\,A17) = 0.9887$	$r(A11,\,A21) = 0.9999$	$r(A27,\,A35) = 0.0448$	$r(A31,\,A42) = 0.1750$
$r(A1,\,A18) = 0.9846$	$r(A11,\,A22) = 0.0003$	$r(A27,\,A40) = 0.1225$	$r(A31,\,A43) = 0.1209$
$r(A1,\,A19) = 0.9830$	$r(A12,\,A21) = 0.0003$	$r(A27,\,A41) = 0.1335$	$r(A31,\,A44) = 0.1524$
$r(A1,\,A20) = 0.8544$	$r(A12,\,A22) = 0.9999$	$r(A27,\,A42) = 0.1419$	$r(A32,\,A33) = 0.1049$
$r(A2,\,A3) = 0.9954$	$r(A13,\,A14) = 0.0005$	$r(A27,\,A43) = 0.0980$	$r(A32,\,A45) = 0.2095$
$r(A2,\,A4) = 0.9942$	$r(A13,\,A15) = 0.0005$	$r(A27,\,A44) = 0.1235$	$r(A32,\,A46) = 0.0404$
$r(A2,\,A5) = 0.8719$	$r(A13,\,A16) = 0.0004$	$r(A28,\,A29) = 0.4696$	$r(A33,\,A45) = 0.0271$
$r(A2,\,A6) = 0.8710$	$r(A13,\,A23) = 0.9999$	$r(A28,\,A30) = 0.0934$	$r(A33,\,A46) = 0.0467$
$r(A2,\,A17) = 0.9846$	$r(A13,\,A24) = 0.0002$	$r(A28,\,A31) = 0.1253$	$r(A34,\,A35) = 0.1412$
$r(A2,\,A18) = 0.9887$	$r(A13,\,A25) = 0.0002$	$r(A28,\,A34) = 0.0234$	$r(A34,\,A40) = 0.0282$
$r(A2,\,A19) = 0.9829$	$r(A14,\,A15) = 0.0005$	$r(A28,\,A35) = 0.0466$	$r(A34,\,A41) = 0.0307$
$r(A2,\,A20) = 0.8543$	$r(A14,\,A16) = 0.0005$	$r(A28,\,A40) = 0.1273$	$r(A34,\,A42) = 0.0327$
$r(A3,\,A4) = 0.9939$	$r(A14,\,A23) = 0.0002$	$r(A28,\,A41) = 0.1387$	$r(A34,\,A43) = 0.0226$
$r(A3,\,A5) = 0.8717$	$r(A14,\,A24) = 0.0003$	$r(A28,\,A42) = 0.1475$	$r(A34,\,A44) = 0.0284$
$r(A3,\,A6) = 0.8708$	$r(A14,\,A25) = 0.0002$	$r(A28,\,A43) = 0.1019$	$r(A35,\,A40) = 0.0561$
$r(A3,\,A17) = 0.9843$	$r(A15,\,A16) = 0.0005$	$r(A28,\,A44) = 0.1284$	$r(A35,\,A41) = 0.0612$
$r(A3,\,A18) = 0.9842$	$r(A15,\,A23) = 0.0002$	$r(A29,\,A30) = 0.1209$	$r(A35,\,A42) = 0.0650$
$r(A3,\,A19) = 0.9827$	$r(A15,\,A24) = 0.9999$	$r(A29,\,A31) = 0.1622$	$r(A35,\,A43) = 0.0449$
$r(A3,\,A20) = 0.8541$	$r(A15,\,A25) = 0.0002$	$r(A29,\,A34) = 0.0303$	$r(A35,\,A44) = 0.0566$
$r(A4,\,A5) = 0.8706$	$r(A16,\,A23) = 0.0002$	$r(A29,\,A35) = 0.0602$	$r(A36,\,A37) = 0.0834$
$r(A4,\,A6) = 0.8698$	$r(A16,\,A24) = 0.0002$	$r(A29,\,A40) = 0.1648$	$r(A40,\,A41) = 0.5699$
$r(A4,\,A17) = 0.9831$	$r(A16,\,A25) = 0.9999$	$r(A29,\,A41) = 0.1795$	$r(A40,\,A42) = 0.6117$
$r(A4,\,A18) = 0.9830$	$r(A17,\,A18) = 0.9958$	$r(A29,\,A42) = 0.1908$	$r(A40,\,A43) = 0.1229$
$r(A4,\,A19) = 0.9888$	$r(A17,\,A19) = 0.9942$	$r(A29,\,A43) = 0.1319$	$r(A40,\,A44) = 0.1548$
$r(A4,\,A20) = 0.8530$	$r(A17,\,A20) = 0.8641$	$r(A29,\,A44) = 0.1662$	$r(A41,\,A42) = 0.6667$
$r(A5,\,A6) = 0.7628$	$r(A18,\,A19) = 0.9941$	$r(A30,\,A31) = 0.4750$	$r(A41,\,A43) = 0.1339$
$r(A5,\,A17) = 0.8622$	$r(A18,\,A20) = 0.8640$	$r(A30,\,A34) = 0.0207$	$r(A41,\,A44) = 0.1687$
$r(A5,\,A18) = 0.8621$	$r(A19,\,A20) = 0.8627$	$r(A30,\,A35) = 0.0412$	$r(A42,\,A43) = 0.1423$
$r(A5,\,A19) = 0.8607$	$r(A21,\,A22) = 0.0001$	$r(A30,\,A40) = 0.1127$	$r(A42,\,A44) = 0.1793$
$r(A5,\,A20) = 0.7481$	$r(A23,\,A24) = 0.0001$	$r(A30,\,A41) = 0.1228$	$r(A43,\,A44) = 0.5224$
$r(A6,\,A17) = 0.8613$	$r(A23,\,A25) = 0.0001$	$r(A30,\,A42) = 0.1305$	$r(A45,\,A46) = 0.0110$
$r(A6,\,A18) = 0.8612$	$r(A24,\,A25) = 0.0001$	$r(A30,\,A43) = 0.0901$	

TABLE 30: Summary of principal input data for the determination of the 2006 recommended values of the fundamental constants (R_∞ and G excepted).

Item number	Input datum	Value	Relative standard uncertainty[a] u_r	Identification	Sec. and Eq.
$B1$	$A_r(^1\mathrm{H})$	1.007 825 032 07(10)	1.0×10^{-10}	AMDC-03	3.1
$B2.1$	$A_r(^2\mathrm{H})$	2.014 101 777 85(36)	1.8×10^{-10}	AMDC-03	3.1
$B2.2$	$A_r(^2\mathrm{H})$	2.014 101 778 040(80)	4.0×10^{-11}	UWash-06	3.1
$B3$	$A_r(^3\mathrm{H})$	3.016 049 2787(25)	4.0×10^{-11}	MSL-06	3.1
$B4$	$A_r(^3\mathrm{He})$	3.016 029 3217(26)	8.6×10^{-10}	MSL-06	3.1
$B5$	$A_r(^4\mathrm{He})$	4.002 603 254 131(62)	1.5×10^{-11}	UWash-06	3.1
$B6$	$A_r(^{16}\mathrm{O})$	15.994 914 619 57(18)	1.1×10^{-11}	UWash-06	3.1
$B7$	$A_r(^{87}\mathrm{Rb})$	86.909 180 526(12)	1.4×10^{-10}	AMDC-03	3.1
$B8^b$	$A_r(^{133}\mathrm{Cs})$	132.905 451 932(24)	1.8×10^{-10}	AMDC-03	3.1
$B9$	$A_r(\mathrm{e})$	0.000 548 579 9111(12)	2.1×10^{-9}	UWash-95	3.3 (5)
$B10$	δ_e	$0.00(27) \times 10^{-12}$	$[2.4 \times 10^{-10}]$	theory	5.1.1 (101)
$B11.1$	a_e	$1.159\,652\,1883(42) \times 10^{-3}$	3.7×10^{-9}	UWash-87	5.1.2.1 (102)
$B11.2$	a_e	$1.159\,652\,180\,85(76) \times 10^{-3}$	6.6×10^{-10}	HarvU-06	5.1.2.2 (103)
$B12$	δ_μ	$0.0(2.1) \times 10^{-9}$	$[1.8 \times 10^{-6}]$	theory	5.2.1 (126)
$B13$	$\overline{R}$	0.003 707 2064(20)	5.4×10^{-7}	BNL-06	5.2.2 (128)
$B14$	δ_C	$0.00(27) \times 10^{-10}$	$[1.4 \times 10^{-11}]$	theory	5.3.1 (169)
$B15$	δ_O	$0.0(1.1) \times 10^{-10}$	$[5.3 \times 10^{-11}]$	theory	5.3.1 (172)
$B16$	$f_\mathrm{s}(^{12}\mathrm{C}^{5+})/f_\mathrm{c}(^{12}\mathrm{C}^{5+})$	4376.210 4989(23)	5.2×10^{-10}	GSI-02	5.3.2.1 (175)
$B17$	$f_\mathrm{s}(^{16}\mathrm{O}^{7+})/f_\mathrm{c}(^{16}\mathrm{O}^{7+})$	4164.376 1837(32)	7.6×10^{-10}	GSI-02	5.3.2.2 (178)
$B18$	$\mu_{\mathrm{e}^-}(\mathrm{H})/\mu_\mathrm{p}(\mathrm{H})$	−658.210 7058(66)	1.0×10^{-8}	MIT-72	6.1.2.1 (195)
$B19$	$\mu_\mathrm{d}(\mathrm{D})/\mu_{\mathrm{e}^-}(\mathrm{D})$	$-4.664\,345\,392(50) \times 10^{-4}$	1.1×10^{-8}	MIT-84	6.1.2.2 (197)
$B20$	$\mu_\mathrm{p}(\mathrm{HD})/\mu_\mathrm{d}(\mathrm{HD})$	3.257 199 531(29)	8.9×10^{-9}	StPtrsb-03	6.1.2.3 (201)
$B21$	σ_dp	$15(2) \times 10^{-9}$		StPtrsb-03	6.1.2.3 (203)
$B22$	$\mu_\mathrm{t}(\mathrm{HT})/\mu_\mathrm{p}(\mathrm{HT})$	1.066 639 887(10)	9.4×10^{-9}	StPtrsb-03	6.1.2.3 (202)
$B23$	σ_tp	$20(3) \times 10^{-9}$		StPtrsb-03	6.1.2.3 (204)
$B24$	$\mu_{\mathrm{e}^-}(\mathrm{H})/\mu'_\mathrm{p}$	−658.215 9430(72)	1.1×10^{-8}	MIT-77	6.1.2.4 (209)
$B25$	$\mu'_\mathrm{h}/\mu'_\mathrm{p}$	−0.761 786 1313(33)	4.3×10^{-9}	NPL-93	6.1.2.5 (211)
$B26$	$\mu_\mathrm{n}/\mu'_\mathrm{p}$	−0.684 996 94(16)	2.4×10^{-7}	ILL-79	6.1.2.6 (212)
$B27$	δ_Mu	0(101) Hz	$[2.3 \times 10^{-8}]$	theory	6.2.1 (234)
$B28.1$	$\Delta\nu_\mathrm{Mu}$	4 463 302.88(16) kHz	3.6×10^{-8}	LAMPF-82	6.2.2.1 (236)
$B28.2$	$\Delta\nu_\mathrm{Mu}$	4 463 302 765(53) Hz	1.2×10^{-8}	LAMPF-99	6.2.2.2 (239)
$B29$	$\nu(58\ \mathrm{MHz})$	627 994.77(14) kHz	2.2×10^{-7}	LAMPF-82	6.2.2.1 (237)
$B30$	$\nu(72\ \mathrm{MHz})$	668 223 166(57) Hz	8.6×10^{-8}	LAMPF-99	6.2.2.2 (240)
$B31.1^b$	$\Gamma'_{\mathrm{p}-90}(\mathrm{lo})$	$2.675\,154\,05(30) \times 10^8\ \mathrm{s^{-1}\ T^{-1}}$	1.1×10^{-7}	NIST-89	7.1.1.1 (253)
$B31.2^b$	$\Gamma'_{\mathrm{p}-90}(\mathrm{lo})$	$2.675\,1530(18) \times 10^8\ \mathrm{s^{-1}\ T^{-1}}$	6.6×10^{-7}	NIM-95	7.1.1.2 (255)
$B32^b$	$\Gamma'_{\mathrm{h}-90}(\mathrm{lo})$	$2.037\,895\,37(37) \times 10^8\ \mathrm{s^{-1}\ T^{-1}}$	1.8×10^{-7}	KR/VN-98	7.1.1.3 (257)
$B33.1^b$	$\Gamma'_{\mathrm{p}-90}(\mathrm{hi})$	$2.675\,1525(43) \times 10^8\ \mathrm{s^{-1}\ T^{-1}}$	1.6×10^{-6}	NIM-95	7.1.2.1 (259)
$B33.2^b$	$\Gamma'_{\mathrm{p}-90}(\mathrm{hi})$	$2.675\,1518(27) \times 10^8\ \mathrm{s^{-1}\ T^{-1}}$	1.0×10^{-6}	NPL-79	7.1.2.2 (262)
$B34.1^b$	R_K	25 812.808 31(62) Ω	2.4×10^{-8}	NIST-97	7.2.1 (265)
$B34.2^b$	R_K	25 812.8071(11) Ω	4.4×10^{-8}	NMI-97	7.2.2 (267)
$B34.3^b$	R_K	25 812.8092(14) Ω	5.4×10^{-8}	NPL-88	7.2.3 (269)
$B34.4^b$	R_K	25 812.8084(34) Ω	1.3×10^{-7}	NIM-95	7.2.4 (271)
$B34.5^b$	R_K	25 812.8081(14) Ω	5.3×10^{-8}	LNE-01	7.2.5 (273)
$B35.1^b$	K_J	$483\,597.91(13)\ \mathrm{GHz\ V^{-1}}$	2.7×10^{-7}	NMI-89	7.3.1 (276)
$B35.2^b$	K_J	$483\,597.96(15)\ \mathrm{GHz\ V^{-1}}$	3.1×10^{-7}	PTB-91	7.3.2 (278)
$B36.1^c$	$K_\mathrm{J}^2 R_\mathrm{K}$	$6.036\,7625(12) \times 10^{33}\ \mathrm{J^{-1}\ s^{-1}}$	2.0×10^{-7}	NPL-90	7.4.1 (281)
$B36.2^c$	$K_\mathrm{J}^2 R_\mathrm{K}$	$6.036\,761\,85(53) \times 10^{33}\ \mathrm{J^{-1}\ s^{-1}}$	8.7×10^{-8}	NIST-98	7.4.2.1 (283)
$B36.3^c$	$K_\mathrm{J}^2 R_\mathrm{K}$	$6.036\,761\,85(22) \times 10^{33}\ \mathrm{J^{-1}\ s^{-1}}$	3.6×10^{-8}	NIST-07	7.4.2.2 (287)
$B37^b$	$\mathcal{F}_{90}$	$96\,485.39(13)\ \mathrm{C\ mol^{-1}}$	1.3×10^{-6}	NIST-80	7.5.1 (295)

TABLE 30: *(Continued).* Summary of principal input data for the determination of the 2006 recommended values of the fundamental constants (R_∞ and G excepted).

Item number	Input datum	Value	Relative standard uncertainty[a] u_r	Identification	Sec. and Eq.
$B38.1^c$	$d_{220}(\text{W4.2a})$	192 015.563(12) fm	6.2×10^{-8}	PTB-81	8.1.1.1 (297)
$B38.2^c$	$d_{220}(\text{W4.2a})$	192 015.5715(33) fm	1.7×10^{-8}	INRIM-07	8.1.1.3 (299)
$B39^c$	$d_{220}(\text{NR3})$	192 015.5919(76) fm	4.0×10^{-8}	NMIJ-04	8.1.1.2 (298)
$B40^c$	$d_{220}(\text{MO}^*)$	192 015.5498(51) fm	2.6×10^{-8}	INRIM-07	8.1.1.3 (300)
$B41$	$1 - d_{220}(\text{N})/d_{220}(\text{W17})$	$7(22) \times 10^{-9}$		NIST-97	8.1.2.1 (301)
$B42$	$1 - d_{220}(\text{W17})/d_{220}(\text{ILL})$	$-8(22) \times 10^{-9}$		NIST-99	8.1.2.1 (302)
$B43$	$1 - d_{220}(\text{MO}^*)/d_{220}(\text{ILL})$	$86(27) \times 10^{-9}$		NIST-99	8.1.2.1 (303)
$B44$	$1 - d_{220}(\text{NR3})/d_{220}(\text{ILL})$	$34(22) \times 10^{-9}$		NIST-99	8.1.2.1 (304)
$B45$	$d_{220}(\text{NR3})/d_{220}(\text{W04}) - 1$	$-11(21) \times 10^{-9}$		NIST-06	8.1.2.1 (305)
$B46$	$d_{220}(\text{NR4})/d_{220}(\text{W04}) - 1$	$25(21) \times 10^{-9}$		NIST-06	8.1.2.1 (306)
$B47$	$d_{220}(\text{W17})/d_{220}(\text{W04}) - 1$	$11(21) \times 10^{-9}$		NIST-06	8.1.2.1 (307)
$B48$	$d_{220}(\text{W4.2a})/d_{220}(\text{W04}) - 1$	$-1(21) \times 10^{-9}$		PTB-98	8.1.2.2 (308)
$B49$	$d_{220}(\text{W17})/d_{220}(\text{W04}) - 1$	$22(22) \times 10^{-9}$		PTB-98	8.1.2.2 (309)
$B50$	$d_{220}(\text{MO}^*)/d_{220}(\text{W04}) - 1$	$-103(28) \times 10^{-9}$		PTB-98	8.1.2.2 (310)
$B51$	$d_{220}(\text{NR3})/d_{220}(\text{W04}) - 1$	$-23(21) \times 10^{-9}$		PTB-98	8.1.2.2 (311)
$B52$	$d_{220}/d_{220}(\text{W04}) - 1$	$10(11) \times 10^{-9}$		PTB-03	8.1.2.2 (312)
$B53^c$	$V_\text{m}(\text{Si})$	$12.058\,8254(34) \times 10^{-6}$ m^3 mol^{-1}	2.8×10^{-7}	N/P/I-05	8.2 (317)
$B54$	$\lambda_\text{meas}/d_{220}(\text{ILL})$	0.002 904 302 46(50) m s^{-1}	1.7×10^{-7}	NIST-99	8.3 (319)
$B55^c$	$h/m_\text{n}d_{220}(\text{W04})$	2060.267 004(84) m s^{-1}	4.1×10^{-8}	PTB-99	8.4.1 (322)
$B56^b$	$h/m(^{133}\text{Cs})$	$3.002\,369\,432(46) \times 10^{-9}$ m^2 s^{-1}	1.5×10^{-8}	StanfU-02	8.4.2 (329)
$B57$	$h/m(^{87}\text{Rb})$	$4.591\,359\,287(61) \times 10^{-9}$ m^2 s^{-1}	1.3×10^{-8}	LKB-06	8.4.3 (332)
$B58.1$	R	8.314 471(15) J mol^{-1} K^{-1}	1.8×10^{-6}	NIST-88	9.1.1 (338)
$B58.2$	R	8.314 504(70) J mol^{-1} K^{-1}	8.4×10^{-6}	NPL-79	9.1.2 (339)
$B59$	$\lambda(\text{CuK}\alpha_1)/d_{220}(\text{W4.2a})$	0.802 327 11(24)	3.0×10^{-7}	FSU/PTB-91	11.1 (348)
$B60$	$\lambda(\text{WK}\alpha_1)/d_{220}(\text{N})$	0.108 852 175(98)	9.0×10^{-7}	NIST-79	11.1 (349)
$B61$	$\lambda(\text{MoK}\alpha_1)/d_{220}(\text{N})$	0.369 406 04(19)	5.3×10^{-7}	NIST-73	11.1 (350)
$B62$	$\lambda(\text{CuK}\alpha_1)/d_{220}(\text{N})$	0.802 328 04(77)	9.6×10^{-7}	NIST-73	11.1 (351)

[a]The values in brackets are relative to the quantities a_e, a_μ, $g_{\text{e}^-}(^{12}\text{C}^{5+})$, $g_{\text{e}^-}(^{16}\text{O}^{7+})$, or $\Delta\nu_\text{Mu}$ as appropriate.
[b]Datum not included in the final least-squares adjustment that provides the recommended values of the constants.
[c]Datum included in the final least-squares adjustment with an expanded uncertainty.

1. Comparison of data

The classic Lamb shift is the only quantity among the Rydberg constant data with more than one measured value, but there are ten different quantities with more than one measured value among the other data. The item numbers given in Tables 28 and 30 for the members of such groups of data ($A39$, $B2$, $B11$, $B28$, $B31$, $B33$-$B36$, $B38$, and $B58$) have a decimal point with an additional digit to label each member.

In fact, all of the data for which there is more than one measurement were directly compared in either the 1998 or 2002 adjustments except the following new data: the University of Washington result for $A_\text{r}(^2\text{H})$, item $B2.2$, the Harvard University result for a_e, item $B11.2$, the NIST watt-balance result for $K_\text{J}^2 R_\text{K}$ item $B36.3$, and the INRIM result for $d_{220}(\text{W4.2a})$, item $B38.2$. The two values of $A_\text{r}(^2\text{H})$ agree well—they differ by only $0.5u_\text{diff}$; the two values of a_e are in acceptable agreement—they differ by $1.7u_\text{diff}$; the two values of $d_{220}(\text{W4.2a})$ also agree well–they differ by $0.7u_\text{diff}$; and the three values of $K_\text{J}^2 R_\text{K}$ are highly consistent—their mean and implied value of h are

$$K_\text{J}^2 R_\text{K} = 6.036\,761\,87(21) \times 10^{33}\ \text{J}^{-1}\ \text{s}^{-1} \quad (359)$$
$$h = 6.626\,068\,89(23) \times 10^{-34}\ \text{J s} \quad (360)$$

with $\chi^2 = 0.27$ for $\nu = N - M = 2$ degrees of freedom, where N is the number of measurements and M is the number of unknowns, and with Birge ratio $R_\text{B} = \sqrt{\chi^2/\nu} = 0.37$ (see Appendix E of CODATA-98). The normalized residuals for the three values are 0.52, −0.04, and −0.09, and their weights in the calculation of the weighted mean are 0.03, 0.10, and 0.87.

Data for quantities with more than one directly measured value used in earlier adjustments are consistent, with the exception of the VNIIM 1989 result for $\Gamma'_{\text{h}-90}(\text{lo})$, which is not included in the present adjustment (see Sec. 7). We also note that none of these data has a weight of less than 0.02 in the weighted mean of measurements of the same quantity.

TABLE 31 Non-negligible correlation coefficients $r(x_i, x_j)$ of the input data in Table 30. For simplicity, the two items of data to which a particular correlation coefficient corresponds are identified by their item numbers in Table 30.

$r(B1,\ B2.1) = 0.073$	$r(B38.1,\ B38.2) = 0.191$	$r(B42,\ B46) = 0.065$	$r(B46,\ B47) = 0.509$
$r(B2.2,\ B5) = 0.127$	$r(B38.2,\ B40) = 0.057$	$r(B42,\ B47) = -0.367$	$r(B48,\ B49) = 0.469$
$r(B2.2,\ B6) = 0.089$	$r(B41,\ B42) = -0.288$	$r(B43,\ B44) = 0.421$	$r(B48,\ B50) = 0.372$
$r(B5,\ B6) = 0.181$	$r(B41,\ B43) = 0.096$	$r(B43,\ B45) = 0.053$	$r(B48,\ B51) = 0.502$
$r(B14,\ B15) = 0.919$	$r(B41,\ B44) = 0.117$	$r(B43,\ B46) = 0.053$	$r(B48,\ B55) = 0.258$
$r(B16,\ B17) = 0.082$	$r(B41,\ B45) = 0.066$	$r(B43,\ B47) = 0.053$	$r(B49,\ B50) = 0.347$
$r(B28.1,\ B29) = 0.227$	$r(B41,\ B46) = 0.066$	$r(B44,\ B45) = -0.367$	$r(B49,\ B51) = 0.469$
$r(B28.2,\ B30) = 0.195$	$r(B41,\ B47) = 0.504$	$r(B44,\ B46) = 0.065$	$r(B49,\ B55) = 0.241$
$r(B31.2,\ B33.1) = -0.014$	$r(B42,\ B43) = 0.421$	$r(B44,\ B47) = 0.065$	$r(B50,\ B51) = 0.372$
$r(B35.1,\ B58.1) = 0.068$	$r(B42,\ B44) = 0.516$	$r(B45,\ B46) = 0.509$	$r(B50,\ B55) = 0.192$
$r(B36.2,\ B36.3) = 0.140$	$r(B42,\ B45) = 0.065$	$r(B45,\ B47) = 0.509$	$r(B51,\ B55) = 0.258$

TABLE 32 Summary of principal input data for the determination of the relative atomic mass of the electron from antiprotonic helium transitions. The numbers in parentheses $(n, l : n', l')$ denote the transition $(n, l) \rightarrow (n', l')$.

Item number	Input Datum	Value	Relative standard uncertainty[a] u_r	Identification	Sec.
$C1$	$\delta_{\bar{p}^4He^+}(32,31:31,30)$	0.00(82) MHz	$[7.3\times10^{-10}]$	JINR-06	4.2
$C2$	$\delta_{\bar{p}^4He^+}(35,33:34,32)$	0.0(1.0) MHz	$[1.3\times10^{-9}]$	JINR-06	4.2
$C3$	$\delta_{\bar{p}^4He^+}(36,34:35,33)$	0.0(1.2) MHz	$[1.6\times10^{-9}]$	JINR-06	4.2
$C4$	$\delta_{\bar{p}^4He^+}(39,35:38,34)$	0.0(1.1) MHz	$[1.8\times10^{-9}]$	JINR-06	4.2
$C5$	$\delta_{\bar{p}^4He^+}(40,35:39,34)$	0.0(1.2) MHz	$[2.4\times10^{-9}]$	JINR-06	4.2
$C6$	$\delta_{\bar{p}^4He^+}(32,31:31,30)$	0.0(1.3) MHz	$[2.9\times10^{-9}]$	JINR-06	4.2
$C7$	$\delta_{\bar{p}^4He^+}(37,35:38,34)$	0.0(1.8) MHz	$[4.4\times10^{-9}]$	JINR-06	4.2
$C8$	$\delta_{\bar{p}^3He^+}(32,31:31,30)$	0.00(91) MHz	$[8.7\times10^{-10}]$	JINR-06	4.2
$C9$	$\delta_{\bar{p}^3He^+}(34,32:33,31)$	0.0(1.1) MHz	$[1.4\times10^{-9}]$	JINR-06	4.2
$C10$	$\delta_{\bar{p}^3He^+}(36,33:35,32)$	0.0(1.2) MHz	$[1.8\times10^{-9}]$	JINR-06	4.2
$C11$	$\delta_{\bar{p}^3He^+}(38,34:37,33)$	0.0(1.1) MHz	$[2.3\times10^{-9}]$	JINR-06	4.2
$C12$	$\delta_{\bar{p}^3He^+}(36,34:37,33)$	0.0(1.8) MHz	$[4.4\times10^{-9}]$	JINR-06	4.2
$C13$	$\nu_{\bar{p}^4He^+}(32,31:31,30)$	1 132 609 209(15) MHz	1.4×10^{-8}	CERN-06	4.2
$C14$	$\nu_{\bar{p}^4He^+}(35,33:34,32)$	804 633 059.0(8.2) MHz	1.0×10^{-8}	CERN-06	4.2
$C15$	$\nu_{\bar{p}^4He^+}(36,34:35,33)$	717 474 004(10) MHz	1.4×10^{-8}	CERN-06	4.2
$C16$	$\nu_{\bar{p}^4He^+}(39,35:38,34)$	636 878 139.4(7.7) MHz	1.2×10^{-8}	CERN-06	4.2
$C17$	$\nu_{\bar{p}^4He^+}(40,35:39,34)$	501 948 751.6(4.4) MHz	8.8×10^{-9}	CERN-06	4.2
$C18$	$\nu_{\bar{p}^4He^+}(32,31:31,30)$	445 608 557.6(6.3) MHz	1.4×10^{-8}	CERN-06	4.2
$C19$	$\nu_{\bar{p}^4He^+}(37,35:38,34)$	412 885 132.2(3.9) MHz	9.4×10^{-9}	CERN-06	4.2
$C20$	$\nu_{\bar{p}^3He^+}(32,31:31,30)$	1 043 128 608(13) MHz	1.3×10^{-8}	CERN-06	4.2
$C21$	$\nu_{\bar{p}^3He^+}(34,32:33,31)$	822 809 190(12) MHz	1.5×10^{-8}	CERN-06	4.2
$C22$	$\nu_{\bar{p}^3He^+}(36,33:34,32)$	646 180 434(12) MHz	1.9×10^{-8}	CERN-06	4.2
$C23$	$\nu_{\bar{p}^3He^+}(38,34:37,33)$	505 222 295.7(8.2) MHz	1.6×10^{-8}	CERN-06	4.2
$C24$	$\nu_{\bar{p}^3He^+}(36,34:37,33)$	414 147 507.8(4.0) MHz	9.7×10^{-9}	CERN-06	4.2

[a]The values in brackets are relative to the corresponding transition frequency.

The consistency of measurements of various quantities of different types is shown mainly by comparing the values of the fine-structure constant α or the Planck constant h inferred from the measured values of the quantities. Such inferred values of α and h are given throughout the data review sections, and the results are summarized and discussed further here.

The consistency of a significant fraction of the data of Tables 28 and 30 is indicated in Table 34 and Figs. 3, 4, and 5, which give and graphically compare the values of α inferred from that data. Figures 3 and 4 compare the data that yield values of α with $u_r < 10^{-7}$ and $u_r < 10^{-8}$, respectively; Fig. 5 also compares the data that yield values of α with $u_r < 10^{-7}$, but does so through combined values of α obtained from similar experiments. Most of the values of α are in reasonable agreement, implying that most of the data from which they are obtained are reasonably consistent. There are, however, two important exceptions.

The value of α inferred from the PTB measurement of $h/m_n d_{220}(\text{W04})$, item $B55$, is based on the mean value $\overline{d_{220}}$ of $d_{220}(\text{W04})$ implied by the four direct {220} XROI

TABLE 33 Non-negligible correlation coefficients $r(x_i, x_j)$ of the input data in Table 32. For simplicity, the two items of data to which a particular correlation coefficient corresponds are identified by their item numbers in Table 32.

$r(C1, C2) = 0.929$	$r(C9, C10) = 0.925$	$r(C14, C23) = 0.132$	$r(C17, C24) = 0.287$
$r(C1, C3) = 0.912$	$r(C9, C11) = 0.949$	$r(C14, C24) = 0.271$	$r(C18, C19) = 0.235$
$r(C1, C4) = 0.936$	$r(C9, C12) = 0.978$	$r(C15, C16) = 0.223$	$r(C18, C20) = 0.107$
$r(C1, C5) = 0.883$	$r(C10, C11) = 0.907$	$r(C15, C17) = 0.198$	$r(C18, C21) = 0.118$
$r(C1, C6) = 0.758$	$r(C10, C12) = 0.934$	$r(C15, C18) = 0.140$	$r(C18, C22) = 0.122$
$r(C1, C7) = 0.957$	$r(C11, C12) = 0.959$	$r(C15, C19) = 0.223$	$r(C18, C23) = 0.112$
$r(C2, C3) = 0.900$	$r(C13, C14) = 0.210$	$r(C15, C20) = 0.128$	$r(C18, C24) = 0.229$
$r(C2, C4) = 0.924$	$r(C13, C15) = 0.167$	$r(C15, C21) = 0.142$	$r(C19, C20) = 0.170$
$r(C2, C5) = 0.872$	$r(C13, C16) = 0.224$	$r(C15, C22) = 0.141$	$r(C19, C21) = 0.188$
$r(C2, C6) = 0.748$	$r(C13, C17) = 0.197$	$r(C15, C23) = 0.106$	$r(C19, C22) = 0.191$
$r(C2, C7) = 0.945$	$r(C13, C18) = 0.138$	$r(C15, C24) = 0.217$	$r(C19, C23) = 0.158$
$r(C3, C4) = 0.907$	$r(C13, C19) = 0.222$	$r(C16, C17) = 0.268$	$r(C19, C24) = 0.324$
$r(C3, C5) = 0.856$	$r(C13, C20) = 0.129$	$r(C16, C18) = 0.193$	$r(C20, C21) = 0.109$
$r(C3, C6) = 0.734$	$r(C13, C21) = 0.142$	$r(C16, C19) = 0.302$	$r(C20, C22) = 0.108$
$r(C3, C7) = 0.927$	$r(C13, C22) = 0.141$	$r(C16, C20) = 0.172$	$r(C20, C23) = 0.081$
$r(C4, C5) = 0.878$	$r(C13, C23) = 0.106$	$r(C16, C21) = 0.190$	$r(C20, C24) = 0.166$
$r(C4, C6) = 0.753$	$r(C13, C24) = 0.216$	$r(C16, C22) = 0.189$	$r(C21, C22) = 0.120$
$r(C4, C7) = 0.952$	$r(C14, C15) = 0.209$	$r(C16, C23) = 0.144$	$r(C21, C23) = 0.090$
$r(C5, C6) = 0.711$	$r(C14, C16) = 0.280$	$r(C16, C24) = 0.294$	$r(C21, C24) = 0.184$
$r(C5, C7) = 0.898$	$r(C14, C17) = 0.247$	$r(C17, C18) = 0.210$	$r(C22, C23) = 0.091$
$r(C6, C7) = 0.770$	$r(C14, C18) = 0.174$	$r(C17, C19) = 0.295$	$r(C22, C24) = 0.186$
$r(C8, C9) = 0.978$	$r(C14, C19) = 0.278$	$r(C17, C20) = 0.152$	$r(C23, C24) = 0.154$
$r(C8, C10) = 0.934$	$r(C14, C20) = 0.161$	$r(C17, C21) = 0.167$	
$r(C8, C11) = 0.959$	$r(C14, C21) = 0.178$	$r(C17, C22) = 0.169$	
$r(C8, C12) = 0.988$	$r(C14, C22) = 0.177$	$r(C17, C23) = 0.141$	

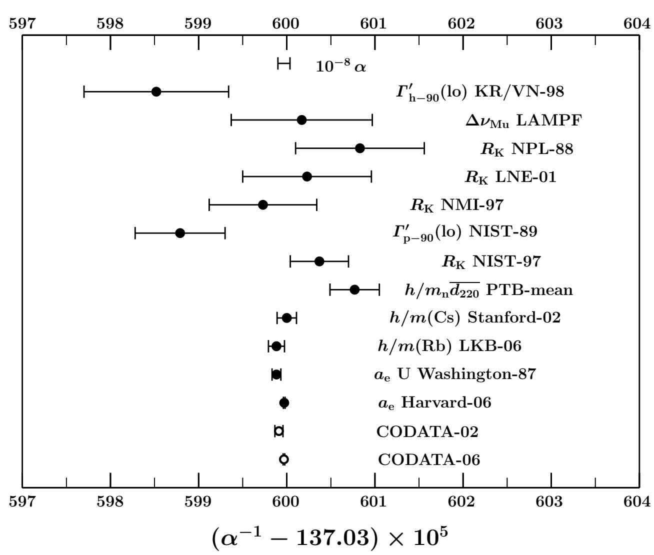

FIG. 3 Values of the fine-structure constant α with $u_{\mathrm{r}} < 10^{-7}$ implied by the input data in Table 30, in order of decreasing uncertainty from top to bottom, and the 2002 and 2006 CODATA recommended values of α. (See Table 34.) Here "mean" indicates the PTB-99 result for $h/m_{\mathrm{n}}d_{220}(\mathrm{W04})$ using the value of $d_{220}(\mathrm{W04})$ implied by the four XROI lattice-spacing measurements.

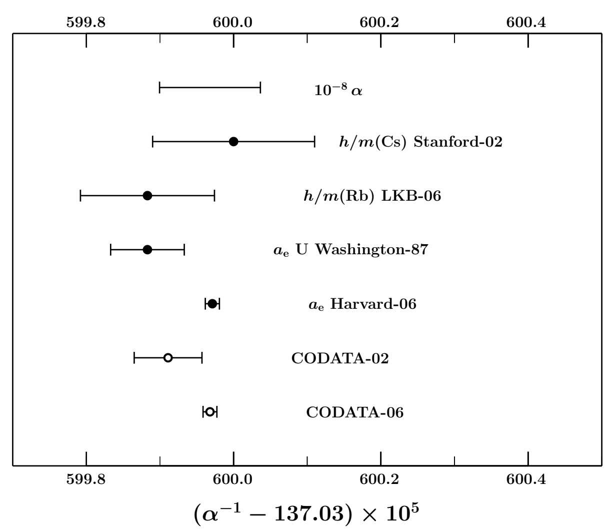

FIG. 4 Values of the fine-structure constant α with $u_{\mathrm{r}} < 10^{-8}$ implied by the input data in Table 30, in order of decreasing uncertainty from top to bottom. (See Table 34.)

lattice spacing measurements, items $B38.1$-$B40$. It disagrees by about $2.8u_{\mathrm{diff}}$ with the value of α with the smallest uncertainty, that inferred from the Harvard University measurement of a_{e}. Also, the value of α inferred

TABLE 34 Comparison of the input data in Table 30 through inferred values of the fine-structure constant α in order of increasing standard uncertainty.

Primary source	Item number	Identification	Sec. and Eq.	α^{-1}	Relative standard uncertainty u_r
a_e	$B11.2$	HarvU-06	5.1.3 (105)	137.035 999 711(96)	7.0×10^{-10}
a_e	$B11.1$	UWash-87	5.1.3 (104)	137.035 998 83(50)	3.7×10^{-9}
$h/m(\mathrm{Rb})$	$B57$	LKB-06	8.4.3 (334)	137.035 998 83(91)	6.7×10^{-9}
$h/m(\mathrm{Cs})$	$B56$	StanfU-02	8.4.2 (331)	137.036 0000(11)	7.7×10^{-9}
$h/m_n d_{220}(\mathrm{W04})$	$B55$	PTB-99			
$\overline{d_{220}}$	$B38.1$-$B40$	Mean	8.4.1 (324)	137.036 0077(28)	2.1×10^{-8}
R_K	$B34.1$	NIST-97	7.2.1 (266)	137.036 0037(33)	2.4×10^{-8}
$\Gamma'_{p-90}(\mathrm{lo})$	$B31.1$	NIST-89	7.1.1.1 (254)	137.035 9879(51)	3.7×10^{-8}
R_K	$B34.2$	NMI-97	7.2.2 (268)	137.035 9973(61)	4.4×10^{-8}
R_K	$B34.5$	LNE-01	7.2.5 (274)	137.036 0023(73)	5.3×10^{-8}
R_K	$B34.3$	NPL-88	7.2.3 (270)	137.036 0083(73)	5.4×10^{-8}
$\Delta\nu_{\mathrm{Mu}}$	$B28.1$,$B28.2$	LAMPF	6.2.2.3 (244)	137.036 0017(80)	5.8×10^{-8}
$\Gamma'_{h-90}(\mathrm{lo})$	$B32$	KR/VN-98	7.1.1.3 (258)	137.035 9852(82)	6.0×10^{-8}
R_K	$B34.4$	NIM-95	7.2.4 (272)	137.036 004(18)	1.3×10^{-7}
$\Gamma'_{p-90}(\mathrm{lo})$	$B31.2$	NIM-95	7.1.1.2 (256)	137.036 006(30)	2.2×10^{-7}
ν_H, ν_D	$A26$-$A47$	Various	4.1.1.13 (65)	137.036 002(48)	3.5×10^{-7}
$\overline{R}$	$B13$	BNL-02	5.2.2.1 (132)	137.035 67(26)	1.9×10^{-6}

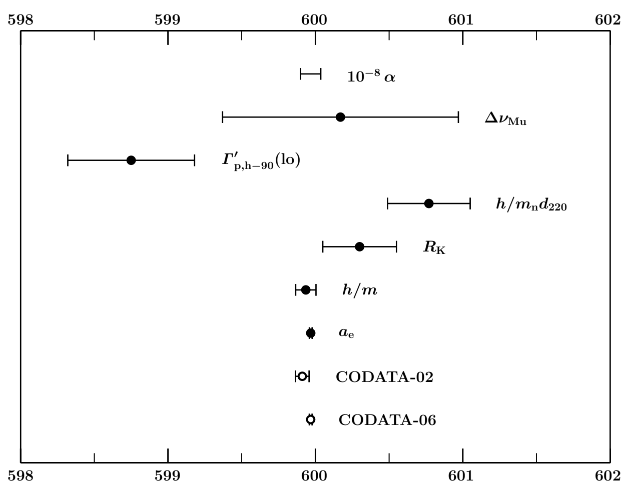

FIG. 5 Values of the fine-structure constant α with $u_r < 10^{-7}$ implied by the input data in Table 30, taken as a weighted mean when more than one measurement of a given type is considered [see Eqs. (361) to (366)], in order of decreasing uncertainty from top to bottom.

from the NIST measurement of $\Gamma'_{p-90}(\mathrm{lo})$ disagrees with the latter by about $2.3u_{\mathrm{diff}}$. But it is also worth noting that the value $\alpha^{-1} = 137.036\,0000(38)$ [2.8×10^{-8}] implied by $h/m_n d_{220}(\mathrm{W04})$ together with item $B39$ alone, the NMIJ XROI measurement of $d_{220}(\mathrm{NR3})$, agrees well with the Harvard a_e value of α. If instead one uses the three other direct XROI lattice spacing measurements, items $B38.1$, $B38.2$, and $B40$, which agree among themselves, one finds $\alpha^{-1} = 137.036\,0092(28)$ [2.1×10^{-8}]. This value disagrees with α from the Harvard a_e by $3.3u_{\mathrm{diff}}$.

The values of α compared in Fig. 5 follow from Table 34 and are, again in order of increasing uncertainty,

$$\alpha^{-1}[a_e] = 137.035\,999\,683(94) \quad [6.9 \times 10^{-10}] \qquad (361)$$

$$\alpha^{-1}[h/m] = 137.035\,999\,35(69) \quad [5.0 \times 10^{-9}] \qquad (362)$$

$$\alpha^{-1}[R_K] = 137.036\,0030(25) \quad [1.8 \times 10^{-8}] \qquad (363)$$

$$\alpha^{-1}[h/m_n d_{220}] = 137.036\,0077(28) \quad [2.1 \times 10^{-8}] \qquad (364)$$

$$\alpha^{-1}[\Gamma'_{p,h-90}(\mathrm{lo})] = 137.035\,9875(43) \quad [3.1 \times 10^{-8}] \qquad (365)$$

$$\alpha^{-1}[\Delta\nu_{\mathrm{Mu}}] = 137.036\,0017(80) \quad [5.8 \times 10^{-8}]\,. \qquad (366)$$

Here $\alpha^{-1}[a_e]$ is the weighted mean of the two a_e values of α; $\alpha^{-1}[h/m]$ is the weighted mean of the $h/m(^{87}\mathrm{Rb})$ and $h/m(^{133}\mathrm{Cs})$ values; $\alpha^{-1}[R_K]$ is the weighted mean of the five quantum Hall effect-calculable capacitor values; $\alpha^{-1}[h/m_n d_{220}]$ is the value as given in Table 34 and is based on the measurement of $h/m_n d_{220}(\mathrm{W04})$ and

the value of $d_{220}(\text{W04})$ inferred from the four XROI determinations of the {220} lattice spacing of three different silicon crystals; $\alpha^{-1}[\Gamma'_{\text{p,h}-90}(\text{lo})]$ is the weighted mean of the two values of $\alpha^{-1}[\Gamma'_{\text{p}-90}(\text{lo})]$ and one value of $\alpha^{-1}[\Gamma'_{\text{h}-90}(\text{lo})]$; and $\alpha^{-1}[\Delta\nu_{\text{Mu}}]$ is the value as given in Table 34 and is based on the 1982 and 1999 measurements at LAMPF on muonium.

Figures 3, 4, and 5 show that even if all of the data of Table 30 were retained, the 2006 recommended value of α would be determined to a great extent by a_e, and in particular, the Harvard University determination of a_e.

The consistency of a significant fraction of the data of Table 30 is indicated in Table 35 and Figs. 6 and 7, which give and graphically compare the values of h inferred from those data. Figure 6 compares the data by showing each inferred value of h in the table, while Fig. 7 compares the data through combined values of h from similar experiments. The values of h are in good agreement, implying that the data from which they are obtained are consistent, with one important exception. The value of h inferred from $V_\text{m}(\text{Si})$, item $B53$, disagrees by $2.9u_\text{diff}$ with the value of h from the weighted mean of the three watt-balance values of $K_\text{J}^2 R_\text{K}$ [uncertainty $u_\text{r} = 3.4 \times 10^{-8}$—see Eq. (360)].

In this regard, it is worth noting that a value of d_{220} of an ideal silicon crystal is required to obtain a value of h from $V_\text{m}(\text{Si})$ [see Eq. (316)], and the value used to obtain the inferred value of h given in Eq. (318) and Table 35 is based on all four XROI lattice spacing measurements, items $B38.1$-$B40$, plus the indirect value from $h/m_\text{n}d_{220}(\text{W04})$ (see Table 24 and Fig. 1). However, the NMIJ measurement of $d_{220}(\text{NR3})$, item $B39$, and the indirect value of d_{220} from $h/m_\text{n}d_{220}(\text{W04})$, yield values of h from $V_\text{m}(\text{Si})$ that are less consistent with the watt-balance mean value than the three other direct XROI lattice spacing measurements, items $B38.1$, $B38.2$, and $B40$, which agree among themselves (a disagreement of about $3.8u_\text{diff}$ compared to $2.5u_\text{diff}$). In contrast, the NMIJ measurement of $d_{220}(\text{NR3})$ yields a value of α from $h/m_\text{n}d_{220}(\text{W04})$ that is in excellent agreement with the Harvard University value from a_e, while the three other lattice spacing measurements yield a value of α in poor agreement with alpha from a_e ($3.3u_\text{diff}$).

The values of h compared in Fig. 7 follow from Table 35 and are, again in order of increasing uncertainty,

$$h[K_\text{J}^2 R_\text{K}] = 6.626\,068\,89(23) \times 10^{-34} \quad [3.4 \times 10^{-8}] \quad (367)$$

$$h[V_\text{m}(\text{Si})] = 6.626\,0745(19) \times 10^{-34} \quad [2.9 \times 10^{-7}] \quad (368)$$

$$h[K_\text{J}] = 6.626\,0678(27) \times 10^{-34} \quad [4.1 \times 10^{-7}] \quad (369)$$

$$h[\Gamma'_{\text{p}-90}(\text{hi})] = 6.626\,0724(57) \times 10^{-34} \quad [8.6 \times 10^{-7}] \quad (370)$$

$$h[\mathcal{F}_{90}] = 6.626\,0657(88) \times 10^{-34} \quad [1.3 \times 10^{-6}]\ . \quad (371)$$

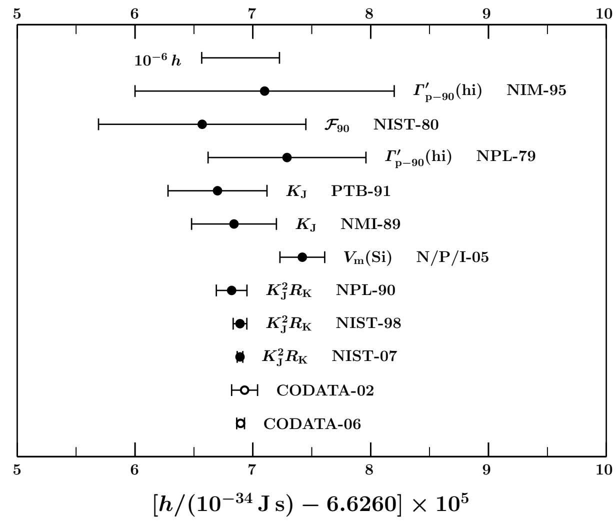

FIG. 6 Values of the Planck constant h implied by the input data in Table 30, in order of decreasing uncertainty from top to bottom. (See Table 35.)

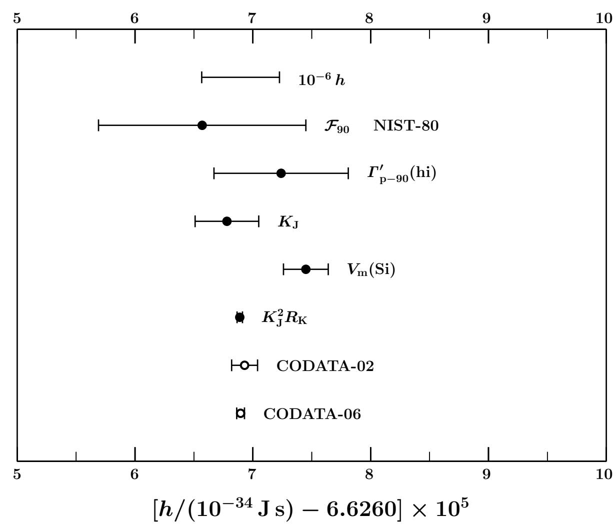

FIG. 7 Values of the Planck constant h implied by the input data in Table 30, taken as a weighted mean when more than one measurement of a given type is considered [see Eqs. (367) to (371)], in order of decreasing uncertainty from top to bottom.

Here $h[K_\text{J}^2 R_\text{K}]$ is the weighted mean of the three values of h from the three watt-balance measurements of $K_\text{J}^2 R_\text{K}$; $h[V_\text{m}(\text{Si})]$ is the value as given in Table 35 and is based on all four XROI d_{220} lattice spacing measurements plus the indirect lattice spacing value from $h/m_\text{n}d_{220}(\text{W04})$; $h[K_\text{J}]$ is the weighted mean of the two direct Josephson

TABLE 35 Comparison of the input data in Table 30 through inferred values of the Planck constant h in order of increasing standard uncertainty.

Primary source	Item number	Identification	Sec. and Eq.	$h/(\mathrm{J\ s})$	Relative standard uncertainty u_r
$K_\mathrm{J}^2 R_\mathrm{K}$	$B36.3$	NIST-07	7.4.2.2 (288)	$6.626\,068\,91(24) \times 10^{-34}$	3.6×10^{-8}
$K_\mathrm{J}^2 R_\mathrm{K}$	$B36.2$	NIST-98	7.4.2.1 (284)	$6.626\,068\,91(58) \times 10^{-34}$	8.7×10^{-8}
$K_\mathrm{J}^2 R_\mathrm{K}$	$B36.1$	NPL-90	7.4.1 (282)	$6.626\,0682(13) \times 10^{-34}$	2.0×10^{-7}
$V_\mathrm{m}(\mathrm{Si})$	$B53$	N/P/I-05	8.2 (318)	$6.626\,0745(19) \times 10^{-34}$	2.9×10^{-7}
K_J	$B35.1$	NMI-89	7.3.1 (277)	$6.626\,0684(36) \times 10^{-34}$	5.4×10^{-7}
K_J	$B35.2$	PTB-91	7.3.2 (279)	$6.626\,0670(42) \times 10^{-34}$	6.3×10^{-7}
$\Gamma'_{\mathrm{p}-90}(\mathrm{hi})$	$B33.2$	NPL-79	7.1.2.2 (263)	$6.626\,0729(67) \times 10^{-34}$	1.0×10^{-6}
$\mathcal{F}_{90}$	$B37$	NIST-80	7.5.1 (296)	$6.626\,0657(88) \times 10^{-34}$	1.3×10^{-6}
$\Gamma'_{\mathrm{p}-90}(\mathrm{hi})$	$B33.1$	NIM-95	7.1.2.1 (261)	$6.626\,071(11) \times 10^{-34}$	1.6×10^{-6}

effect measurements of K_J; $h[\Gamma_{\mathrm{p}-90}(\mathrm{hi})]$ is the weighted mean of the two values of h from the two measurements of $\Gamma_{\mathrm{p}-90}(\mathrm{hi})$; and $h[\mathcal{F}_{90}]$ is the value as given in Table 35 and comes from the silver coulometer measurement of $\mathcal{F}_{90}$. Figures 6 and 7 show that even if all of the data of Table 30 were retained, the 2006 recommended value of h would be determined to a large extent by $K_\mathrm{J}^2 R_\mathrm{K}$, and in particular, the NIST 2007 determination of this quantity.

We conclude our data comparisons by listing in Table 36 the four available values of $A_\mathrm{r}(\mathrm{e})$. The reasonable agreement of these values shows that the corresponding input data are consistent. The most important of these data are the University of Washington value of $A_\mathrm{r}(\mathrm{e})$, δ_C, δ_O, $f_\mathrm{s}(^{12}\mathrm{C}^{5+})/f_\mathrm{c}(^{12}C^{5+})$, $f_\mathrm{s}(^{12}\mathrm{O}^{7+})/f_\mathrm{c}(^{12}\mathrm{O}^{7+})$, and the antiprotonic helium data, items $B9$, $B14$-$B17$, and $C1$-$C24$.

In summary, the data comparisons of this section of the paper have identified the following potential problems: (i) the measurement of $V_\mathrm{m}(\mathrm{Si})$, item $B53$, is inconsistent with the watt-balance measurements of $K_\mathrm{J}^2 R_\mathrm{K}$, items $B36.1$-$B36.3$, and somewhat inconsistent with the mercury-electrometer and voltage-balance measurements of K_J; (ii) the three XROI {220} lattice spacing values $d_{220}(\textsc{w4.2a})$, $d_{220}(\textsc{w4.2a})$, and $d_{220}(\textsc{mo}^*)$, items $B38.1$, $B38.2$, and $B40$, are inconsistent with the value of $d_{220}(\textsc{nr3})$, item $B39$, and the measurement of $h/m_\mathrm{n}d_{220}(\textsc{w04})$, item $B55$; (iii) the NIST-89 measurement of $\Gamma'_{\mathrm{p}-90}(\mathrm{lo})$, item $B33.1$, is inconsistent with the most accurate data that also determine the value of the fine-structure constant; (iv) although not a problem in the sense of (i)-(iii), there are a number of input data with uncertainties so large that they are unlikely to make a contribution to the determination of the 2006 CODATA recommended values.

Furthermore, we note that some of the inferred values of α in Table 34 and most of the inferred values of h in Table 35 depend on either one or both of the relations $K_\mathrm{J} = 2e/h$ and $R_\mathrm{K} = h/e^2$. The question of whether relaxing the assumption that these relations are exact would reduce or possibly even eliminate some of the observed inconsistencies, considered in Appendix F of CODATA-02, is addressed in the section following the next section. This study indeed confirms the Josephson and quantum Hall effect relations.

2. Multivariate analysis of data

The multivariate analysis of the data is based on the fact that measured quantities can be expressed as theoretical functions of fundamental constants. These expressions, or observational equations, are written in terms of a particular independent subset of the constants whose members are here called *adjusted constants*. The goal of the analysis is to find the values of the adjusted constants that predict values for the measured data that best agree with the data themselves in the least-squares sense (see Appendix E of CODATA-98).

The symbol $\doteq$ is used to indicate that an observed value of an input datum of the particular type shown on the left-hand side is ideally given by the function of the adjusted constants on the right-hand side; however, the two sides are not necessarily equal, because the equation is one of an overdetermined set relating the data to the adjusted constants. The best estimate of a quantity is given by its observational equation evaluated with the least-squares estimated values of the adjusted constants on which it depends.

In essence, we follow the least-squares approach of Aitken (1934) [see also Sheppard (1912)], who treated the case where the input data are correlated. The 150 input data of Tables 28, 30, and 32 are of 135 distinct types and are expressed as functions of the 79 adjusted constants listed in Tables 37, 39, and 41. The observational equations that relate the input data to the adjusted constants are given in Tables 38, 40, and 42.

Note that the various binding energies $E_\mathrm{b}(X)/m_\mathrm{u}c^2$ in Table 40, such as in the equation for item $B1$, are treated as fixed quantities with negligible uncertainties. Similarly, the bound-state g-factor ratios in this table, such as in the equation for item $B18$, are treated in the same way. Further, the frequency f_p is not an adjusted constant but is included in the equation for items $B29$

and $B30$ to indicate that they are functions of f_p. Finally, the observational equation for items $B29$ and $B30$, based on Eqs. (215), (216), and (217) of Sec. 6.2, includes the functions $a_\mathrm{e}(\alpha, \delta_\mathrm{e})$ and $a_\mu(\alpha, \delta_\mu)$ as well as the theoretical expression for input data of type $B28$, $\Delta\nu_\mathrm{Mu}$. The latter expression is discussed in Sec. 6.2.1 and is a function of $R_\infty, \alpha, m_\mathrm{e}/m_\mu, a_\mu(\alpha, \delta_\mu)$, and δ_Mu.

1. Summary of adjustments

A number of adjustments were carried out to gauge the compatibility of the input data in Tables 28, 30, and 32 (together with their covariances in Tables 29, 31, and 33) and to assess their influence on the values of the adjusted constants. The results of 11 of these are given in Tables 43 to 45 and are discussed in the following paragraphs. Because the adjusted value of the Rydberg constant R_∞ is essentially the same for all six adjustments summarized in Table 43 and equal to that of adjustment 4 of Table 45, the value of R_∞ is not listed in Table 43. It should also be noted that adjustment 4 of all three tables is the same adjustment.

Adjustment 1. This initial adjustment includes all of the input data, four of which have normalized residuals r_i with absolute magnitudes significantly greater than 2; the values of r_i for these four data resulting from adjustments 1-6 are given in Table 44. Consistent with the previous discussion, the four most inconsistent items are the molar volume of silicon $V_\mathrm{m}(\mathrm{Si})$, the quotient $h/m_\mathrm{n}d_{220}(\textsc{w04})$, the XROI measurement of the $\{220\}$ lattice spacing $d_{220}(\textsc{nr3})$, and the NIST-89 value of $\Gamma'_{\mathrm{p}-90}(\mathrm{lo})$. All other input data have values of r_i considerably less than 2, except those for $\nu_{\bar{p}\,{}^3\mathrm{He}}(32, 31 : 31, 30)$ and $\nu_{\bar{p}\,{}^3\mathrm{He}}(36, 33 : 34, 32)$, items $C20$ and $C22$, for which $r_{20} = 2.09$ and $r_{22} = 2.06$. However, the self sensitivity coefficients S_c for these input data are considerably less than 0.01; hence, because their contribution to the adjustment is small, their marginally large normalized residuals are of little concern. In this regard, we see from Table 44 that three of the four inconsistent data have values of S_c considerably larger than 0.01; the exception is $\Gamma'_{\mathrm{p}-90}(\mathrm{lo})$ with $S_\mathrm{c} = 0.0099$, which is rounded to 0.010 in the table.

Adjustment 2. Since the four direct lattice spacing measurements, items $B38.1$-$B40$, are credible, as is the measurement of $h/m_\mathrm{n}d_{220}(\textsc{w04})$, item $B55$, after due consideration the CODATA Task Group on Fundamental Constants decided that all five of these input data should be considered for retention, but that each of their *a priori* assigned uncertainties should be weighted by the multiplicative factor 1.5 to reduce $|r_\mathrm{i}|$ of $h/m_\mathrm{n}d_{220}(\textsc{w04})$ and of $d_{220}(\textsc{nr3})$ to a more acceptable level, that is, to about 2, while maintaining their relative weights. This has been done in adjustment 2. As can be seen from Table 43, this increase of uncertainties has an inconsequential impact on the value of α, and no impact on the value of h. It does reduce R_B, as would be expected.

Adjustment 3. Again, since the measurement of $V_\mathrm{m}(\mathrm{Si})$, item $B53$, as well as the three measurements of $K_\mathrm{J}^2\,R_\mathrm{K}$, items $B36.1$-$B36.3$, and the two measurements of K_J, items $B35.1$ and $B35.2$, are credible, the Task Group decided that all six should be considered for retention, but that each of their *a priori* assigned uncertainties should be weighted by the multiplicative factor 1.5 to reduce $|r_i|$ of $V_\mathrm{m}(\mathrm{Si})$ to about 2, while maintaining their relative weights. This has been done in adjustment 3. Note that this also reduces $|r_i|$ of $h/m_\mathrm{n}d_{220}(\textsc{w04})$ from 2.03 in adjustment 2 to 1.89 in adjustment 3. We see from Table 43 that this increase in uncertainty has negligible consequences for the value of α, but it does increase the uncertainty of h by about the same factor, as would be expected. Also as would be expected, R_B is further reduced.

It may be recalled that faced with a similar situation in the 2002 adjustment, the Task Group decided to use a multiplicative weighting factor of 2.325 in order to reduce $|r_i|$ of $V_\mathrm{m}(\mathrm{Si})$ to 1.50. The reduced weighting factor of 1.5 in the 2006 adjustment recognizes the new value of $K_\mathrm{J}^2\,R_\mathrm{K}$ now available and the excellent agreement with the two earlier values.

Adjustment 4. In adjustment 3, a number of input data, as measured by their self-sensitivity coefficients S_c, do not contribute in a significant way to the determination of the adjusted constants. We therefore omit in adjustment 4 those input data with $S_\mathrm{c} < 0.01$ in adjustment 3 unless they are a subset of the data of an experiment that provides other input data with $S_\mathrm{c} > 0.01$. The 14 input data deleted in adjustment 4 for this reason are $B31.1$-$B35.2$, $B37$, and $B56$, which are the five low- and high-field proton and helion gyromagnetic ratio results; the five calculable capacitor values of R_K; both values of K_J as obtained using a Hg electrometer and a voltage balance; the Ag coulometer result for the Faraday constant; and the recoil/atom interferometry result for the quotient of the Planck constant and mass of the cesium-133 atom. The respective values of S_c for these data in adjustment 3 are in the range 0.0000 to 0.0099. Deleting such marginal data is in keeping with the practice followed in the 1998 and 2002 adjustments; see Sec. I.D of CODATA-98.

Because $h/m(^{133}\mathrm{Cs})$, item $B56$, has been deleted as an input datum due to its low weight, $A_\mathrm{r}(^{133}\mathrm{Cs})$, item $B8$, which is not coupled to any other input datum, has also been omitted as an input datum and as an adjusted constant from adjustment 4. This brings the total number of omitted items to 15. Table 43 shows that deleting these 15 data has virtually no impact on the values of α and h.

Adjustment 4 is the adjustment on which the 2006 CODATA recommended values are based, and as such it is referred to as the "final adjustment."

Adjustments 5 and 6. These adjustments are intended to check the robustness of adjustment 4, the final adjustment, while adjustments 7-11, which are summarized in Table 45, probe various aspects of the R_∞ data in Ta-

ble 28.

Adjustment 5 only differs from adjustment 3 in that it does not include the input data that lead to the four most accurate values of α: the two measurements of $a_{\rm e}$, items $B11.1$ and $B11.2$, the measurement of $h/m(^{133}{\rm Cs})$, item $B56$, and the measurement of $h/m(^{87}{\rm Rb})$, item $B57$. The $u_{\rm r}$ of the inferred values of α from these data are 7.0×10^{-10}, 3.7×10^{-9}, 7.7×10^{-9}, and 6.7×10^{-9}. We see from Table 43 that the value of α from adjustment 5 is consistent with the 2006 recommended value from adjustment 4 (the difference is $0.8u_{\rm diff}$), but its uncertainty is about 20 times larger. Moreover, the resulting value of h is the same as the recommended value.

Adjustment 6 only differs from adjustment 3 in that it does not include the input data that yield the three most accurate values of h, namely, the watt-balance measurements of $K_{\rm J}^2R_{\rm K}$, items $B36.1$-$B36.3$. The $u_{\rm r}$ of the inferred values of h from these data, as they are used in adjustment 3 (that is, after their uncertainties are multiplied by the weighting factor 1.5), are 5.4×10^{-8}, 1.3×10^{-7}, and 3.0×10^{-7}. From Table 43, we see that the value of h from adjustment 6 is consistent with the 2006 recommended value from adjustment 4 (the difference is $1.4u_{\rm diff}$), but its uncertainty is well over 6 times larger. Furthermore, the resulting value of α is the same as the recommended value. Therefore adjustments 5 and 6 suggest that the less accurate input data are consistent with the more accurate data, thereby providing a consistency check on the 2006 recommended values of the constants.

Adjustments 7-11. These adjustments differ from adjustment 4, the final adjustment, in the following ways. In adjustment 7, the scattering-data input values for both $R_{\rm p}$ and $R_{\rm d}$, items $A48$ and $A49$, are omitted; in adjustment 8, only $R_{\rm p}$ is omitted, and in adjustment 9, only $R_{\rm d}$ is omitted; adjustment 10 includes only the hydrogen data, and adjustment 11 includes only the deuterium data, but for both, the H-D isotope shift, item $A47$, is omitted. Although a somewhat improved value of the $1{\rm S}_{1/2}$-$2{\rm S}_{1/2}$ hydrogen transition frequency and improvements in the theory of H and D energy levels have become available since the completion of the 2002 adjustment, the value of R_∞, which is determined almost entirely by these data, has changed very little. The values of $R_{\rm p}$ and $R_{\rm d}$, which are also determined mainly by these data, have changed by less than one third of their uncertainties. The experimental and theoretical H and D data remain highly consistent.

2. Test of the Josephson and quantum Hall effect relations

Investigation of the exactness of the relations $K_{\rm J}=2e/h$ and $R_{\rm K}=h/e^2$ is carried out, as in CODATA-02, by writing

$$K_{\rm J} = \frac{2e}{h}(1+\varepsilon_{\rm J}) = \left(\frac{8\alpha}{\mu_0 ch}\right)^{1/2}(1+\varepsilon_{\rm J}) \quad (372)$$

$$R_{\rm K} = \frac{h}{e^2}(1+\varepsilon_{\rm K}) = \frac{\mu_0 c}{2\alpha}(1+\varepsilon_{\rm K}) \ , \quad (373)$$

where $\varepsilon_{\rm J}$ and $\varepsilon_{\rm K}$ are unknown correction factors taken to be additional adjusted constants determined by least-squares calculations. Replacing the relations $K_{\rm J}=2e/h$ and $R_{\rm K}=h/e^2$ with the generalizations in Eqs. (372) and (373) in the analysis leading to the observational equations in Table 40 leads to the modified observational equations given in Table 46.

The results of seven different adjustments are presented in Table 47. In addition to the adjusted values of α, h, $\varepsilon_{\rm J}$, and $\varepsilon_{\rm K}$, we also give the normalized residuals r_i of the four input data with the largest values of $|r_i|$: $V_{\rm m}({\rm Si})$, item $B53$, $h/m_{\rm n}d_{220}({\rm W04})$, item $B55$, $d_{220}({\rm NR3})$, item $B39$, and the NIST-89 value for $\Gamma'_{\rm p-90}({\rm lo})$, item $B31.1$. The residuals are included as additional indicators of whether relaxing the assumption $K_{\rm J}=2e/h$ and $R_{\rm K}=h/e^2$ reduces the disagreements among the data.

The adjusted value of R_∞ is not included in Table 47, because it remains essentially unchanged from one adjustment to the next and equal to the 2006 recommended value. An entry of 0 in the $\varepsilon_{\rm K}$ column means that it is assumed that $R_{\rm K}=h/e^2$ in the corresponding adjustment; similarly, an entry of 0 in the $\varepsilon_{\rm J}$ column means that it is assumed that $K_{\rm J}=2e/h$ in the corresponding adjustment. The following comments apply to the adjustments of Table 47.

Adjustment (i) is identical to adjustment 1 of Tables 43 and 44 in the previous section and is included here simply for reference; all of the input data are included and multiplicative weighting factors have not been applied to any uncertainties. For this adjustment, $N=150$, $M=79$, $\nu=71$, and $\chi^2=92.1$.

The next three adjustments differ from adjustment (i) in that in adjustment (ii) the relation $K_{\rm J}=2e/h$ is relaxed, in adjustment (iii) the relation $R_{\rm K}=h/e^2$ is relaxed, and in adjustment (iv) both of the relations are relaxed. For these three adjustments, $N=150$, $M=80$, $\nu=70$, and $\chi^2=91.5$; $N=150$, $M=80$, $\nu=70$, and $\chi^2=91.3$; and $N=150$, $M=81$, $\nu=69$, and $\chi^2=90.4$, respectively.

It is clear from Table 47 that there is no evidence for the inexactness of either of the relations $K_{\rm J}=2e/h$ or $R_{\rm K}=h/e^2$. This conclusion is also true if instead of taking adjustment 1 of Table 43 as our starting point, we had taken adjustment 2 in which the uncertainties of the five x-ray related data are multiplied by the factor 1.5. That is, none of the numbers in Table 47 would change significantly, except $R_{\rm B}$ would be reduced from 1.14 to about 1.08. The reason adjustments (iii)-(vii) summarized in Table 47 give values of $\epsilon_{\rm K}$ consistent with zero within about 2 parts in 10^8 is mainly because the value of alpha inferred from the mean of the five measured

values of $R_{\rm K}$ under the assumption $R_{\rm K} = h/e^2$, which has $u_{\rm r} = 1.8 \times 10^{-8}$, agrees with the value of α with $u_{\rm r} = 7.0 \times 10^{-10}$ inferred from the Harvard University measured value of $a_{\rm e}$.

Table 46 and the uncertainties of the 2006 input data indicate that the values of $\epsilon_{\rm J}$ from adjustments (ii) and (iv) are determined mainly by the input data for the quantities $\Gamma'_{\rm p-90}({\rm lo})$ and $\Gamma'_{\rm h-90}({\rm lo})$ with observational equations that depend on $\epsilon_{\rm J}$ but not on h; and by the input data for the quantities $\Gamma'_{\rm p-90}({\rm hi})$, $K_{\rm J}$, $K_{\rm J}^2\,R_{\rm K}$, and $\mathcal{F}_{90}$, with observational equations that depend on both $\epsilon_{\rm J}$ and h. Because the value of h in these least-squares calculations arises primarily from the measured value of the molar volume of silicon, $V_{\rm m}({\rm Si})$, the values of $\epsilon_{\rm J}$ in adjustments (ii) and (iv) arise mainly from a combination of individual values of $\epsilon_{\rm J}$ that either depend on $V_{\rm m}({\rm Si})$ or on $\Gamma'_{\rm p-90}({\rm lo})$ and $\Gamma'_{\rm h-90}({\rm lo})$. It is therefore of interest to repeat adjustment (iv), first with $V_{\rm m}({\rm Si})$ deleted but with the $\Gamma'_{\rm p-90}({\rm lo})$ and $\Gamma'_{\rm h-90}({\rm lo})$ data included, and then with the latter deleted but with $V_{\rm m}({\rm Si})$ included. These are, in fact, adjustments (v) and (vi) of Table 47.

In each of these adjustments, the absolute values of $\epsilon_{\rm J}$ are comparable and significantly larger than the uncertainties, which are also comparable, but the values have different signs. Consequently, when $V_{\rm m}({\rm Si})$ and the $\Gamma'_{\rm p-90}({\rm lo})$ and $\Gamma'_{\rm h-90}({\rm lo})$ data are included at the same time as in adjustment (iv), the result for $\epsilon_{\rm J}$ is consistent with zero.

The values of $\epsilon_{\rm J}$ from adjustments (v) and (vi) reflect some of the inconsistencies among the data: the disagreement of the values of h implied by $V_{\rm m}({\rm Si})$ and $K_{\rm J}^2\,R_{\rm K}$ when it is assumed that the relations $K_{\rm J} = 2e/h$ and $R_{\rm K} = h/e^2$ are exact; and the disagreement of the values of α implied by the electron magnetic moment anomaly $a_{\rm e}$ and by $\Gamma'_{\rm p-90}({\rm lo})$ and $\Gamma'_{\rm h-90}({\rm lo})$ under the same assumption.

In adjustment (vii), the problematic input data for $V_{\rm m}({\rm Si})$, $\Gamma'_{\rm p-90}({\rm lo})$, and $\Gamma'_{\rm h-90}({\rm lo})$ are simultaneously deleted from the calculation. Then the value of $\epsilon_{\rm J}$ arises mainly from the input data for $\Gamma'_{\rm p-90}({\rm hi})$, $K_{\rm J}$, $K_{\rm J}^2 R_{\rm K}$, and $\mathcal{F}_{90}$. Like adjustment (iv), adjustment (vii) shows that $\epsilon_{\rm J}$ is consistent with zero, although not within 8 parts in 10^8 but within 7 parts in 10^7. However, adjustment (vii) has the advantage of being based on consistent data.

The comparatively narrow range of values of alpha in Table 47 is due to the fact that the input data that mainly determine alpha do not depend on the Josephson or quantum Hall effects. This is not the case for the input data that primarily determine h, hence the values of h vary over a wide range.

TABLE 36 Values of $A_r(e)$ implied by the input data in Table 30 in order of increasing standard uncertainty.

Primary source	Item number	Identification	Sec. and Eq.	$A_r(e)$	Relative standard uncertainty u_r
$f_s(C)/f_c(C)$	$B16$	GSI-02	5.3.2.1 (177)	0.000 548 579 909 32(29)	5.2×10^{-10}
$f_s(O)/f_c(O)$	$B17$	GSI-02	5.3.2.2 (181)	0.000 548 579 909 58(42)	7.6×10^{-10}
$\Delta\nu_{\bar{p}\,He^+}$	$C1 - C24$	JINR/CERN-06	4.2.3 (74)	0.000 548 579 908 81(91)	1.7×10^{-9}
$A_r(e)$	$B9$	UWash-95	3.3 (5)	0.000 548 579 9111(12)	2.1×10^{-9}

TABLE 37 The 28 adjusted constants (variables) used in the least-squares multivariate analysis of the Rydberg-constant data given in Table 28. These adjusted constants appear as arguments of the functions on the right-hand side of the observational equations of Table 38. The notation for hydrogenic energy levels $E_X(n\mathrm{L}_j)$ and for additive corrections $\delta_X(n\mathrm{L}_j)$ in this table have the same meaning as the notations $E^X_{n\mathrm{L}j}$ and $\delta^X_{n\mathrm{L}j}$ in Sec. 4.1.1.12.

Adjusted constant	Symbol
Rydberg constant	R_∞
bound-state proton rms charge radius	R_p
bound-state deuteron rms charge radius	R_d
additive correction to $E_\mathrm{H}(1\mathrm{S}_{1/2})/h$	$\delta_\mathrm{H}(1\mathrm{S}_{1/2})$
additive correction to $E_\mathrm{H}(2\mathrm{S}_{1/2})/h$	$\delta_\mathrm{H}(2\mathrm{S}_{1/2})$
additive correction to $E_\mathrm{H}(3\mathrm{S}_{1/2})/h$	$\delta_\mathrm{H}(3\mathrm{S}_{1/2})$
additive correction to $E_\mathrm{H}(4\mathrm{S}_{1/2})/h$	$\delta_\mathrm{H}(4\mathrm{S}_{1/2})$
additive correction to $E_\mathrm{H}(6\mathrm{S}_{1/2})/h$	$\delta_\mathrm{H}(6\mathrm{S}_{1/2})$
additive correction to $E_\mathrm{H}(8\mathrm{S}_{1/2})/h$	$\delta_\mathrm{H}(8\mathrm{S}_{1/2})$
additive correction to $E_\mathrm{H}(2\mathrm{P}_{1/2})/h$	$\delta_\mathrm{H}(2\mathrm{P}_{1/2})$
additive correction to $E_\mathrm{H}(4\mathrm{P}_{1/2})/h$	$\delta_\mathrm{H}(4\mathrm{P}_{1/2})$
additive correction to $E_\mathrm{H}(2\mathrm{P}_{3/2})/h$	$\delta_\mathrm{H}(2\mathrm{P}_{3/2})$
additive correction to $E_\mathrm{H}(4\mathrm{P}_{3/2})/h$	$\delta_\mathrm{H}(4\mathrm{P}_{3/2})$
additive correction to $E_\mathrm{H}(8\mathrm{D}_{3/2})/h$	$\delta_\mathrm{H}(8\mathrm{D}_{3/2})$
additive correction to $E_\mathrm{H}(12\mathrm{D}_{3/2})/h$	$\delta_\mathrm{H}(12\mathrm{D}_{3/2})$
additive correction to $E_\mathrm{H}(4\mathrm{D}_{5/2})/h$	$\delta_\mathrm{H}(4\mathrm{D}_{5/2})$
additive correction to $E_\mathrm{H}(6\mathrm{D}_{5/2})/h$	$\delta_\mathrm{H}(6\mathrm{D}_{5/2})$
additive correction to $E_\mathrm{H}(8\mathrm{D}_{5/2})/h$	$\delta_\mathrm{H}(8\mathrm{D}_{5/2})$
additive correction to $E_\mathrm{H}(12\mathrm{D}_{5/2})/h$	$\delta_\mathrm{H}(12\mathrm{D}_{5/2})$
additive correction to $E_\mathrm{D}(1\mathrm{S}_{1/2})/h$	$\delta_\mathrm{D}(1\mathrm{S}_{1/2})$
additive correction to $E_\mathrm{D}(2\mathrm{S}_{1/2})/h$	$\delta_\mathrm{D}(2\mathrm{S}_{1/2})$
additive correction to $E_\mathrm{D}(4\mathrm{S}_{1/2})/h$	$\delta_\mathrm{D}(4\mathrm{S}_{1/2})$
additive correction to $E_\mathrm{D}(8\mathrm{S}_{1/2})/h$	$\delta_\mathrm{D}(8\mathrm{S}_{1/2})$
additive correction to $E_\mathrm{D}(8\mathrm{D}_{3/2})/h$	$\delta_\mathrm{D}(8\mathrm{D}_{3/2})$
additive correction to $E_\mathrm{D}(12\mathrm{D}_{3/2})/h$	$\delta_\mathrm{D}(12\mathrm{D}_{3/2})$
additive correction to $E_\mathrm{D}(4\mathrm{D}_{5/2})/h$	$\delta_\mathrm{D}(4\mathrm{D}_{5/2})$
additive correction to $E_\mathrm{D}(8\mathrm{D}_{5/2})/h$	$\delta_\mathrm{D}(8\mathrm{D}_{5/2})$
additive correction to $E_\mathrm{D}(12\mathrm{D}_{5/2})/h$	$\delta_\mathrm{D}(12\mathrm{D}_{5/2})$

TABLE 38 Observational equations that express the input data related to R_∞ in Table 28 as functions of the adjusted constants in Table 37. The numbers in the first column correspond to the numbers in the first column of Table 28. The expressions for the energy levels of hydrogenic atoms are discussed in Sec. 4.1.1. As pointed out in Sec. 4.1.1.12, $E_X(n\mathrm{L}_j)/h$ is in fact proportional to cR_∞ and independent of h, hence h is not an adjusted constant in these equations. The notation for hydrogenic energy levels $E_X(n\mathrm{L}_j)$ and for additive corrections $\delta_X(n\mathrm{L}_j)$ in this table have the same meaning as the notations $E^X_{n\mathrm{L}j}$ and $\delta^X_{n\mathrm{L}j}$ in Sec. 4.1.1.12. See Sec. 12.2 for an explanation of the symbol $\doteq$.

Type of input datum	Observational equation
$A1$–$A16$	$\delta_{\mathrm{H}}(n\mathrm{L}_j) \doteq \delta_{\mathrm{H}}(n\mathrm{L}_j)$
$A17$–$A25$	$\delta_{\mathrm{D}}(n\mathrm{L}_j) \doteq \delta_{\mathrm{D}}(n\mathrm{L}_j)$
$A26$–$A31$ $A38$, $A39$	$\nu_{\mathrm{H}}(n_1\mathrm{L}_{1j_1} - n_2\mathrm{L}_{2j_2}) \doteq \big[E_{\mathrm{H}}\big(n_2\mathrm{L}_{2j_2}; R_\infty, \alpha, A_{\mathrm{r}}(\mathrm{e}), A_{\mathrm{r}}(\mathrm{p}), R_{\mathrm{p}}, \delta_{\mathrm{H}}(n_2\mathrm{L}_{2j_2})\big) - E_{\mathrm{H}}\big(n_1\mathrm{L}_{1j_1}; R_\infty, \alpha, A_{\mathrm{r}}(\mathrm{e}), A_{\mathrm{r}}(\mathrm{p}), R_{\mathrm{p}}, \delta_{\mathrm{H}}(n_1\mathrm{L}_{1j_1})\big)\big]/h$
$A32$–$A37$	$\nu_{\mathrm{H}}(n_1\mathrm{L}_{1j_1} - n_2\mathrm{L}_{2j_2}) - \frac{1}{4}\nu_{\mathrm{H}}(n_3\mathrm{L}_{3j_3} - n_4\mathrm{L}_{4j_4}) \doteq \Big\{E_{\mathrm{H}}\big(n_2\mathrm{L}_{2j_2}; R_\infty, \alpha, A_{\mathrm{r}}(\mathrm{e}), A_{\mathrm{r}}(\mathrm{p}), R_{\mathrm{p}}, \delta_{\mathrm{H}}(n_2\mathrm{L}_{2j_2})\big) - E_{\mathrm{H}}\big(n_1\mathrm{L}_{1j_1}; R_\infty, \alpha, A_{\mathrm{r}}(\mathrm{e}), A_{\mathrm{r}}(\mathrm{p}), R_{\mathrm{p}}, \delta_{\mathrm{H}}(n_1\mathrm{L}_{1j_1})\big) - \frac{1}{4}\big[E_{\mathrm{H}}\big(n_4\mathrm{L}_{4j_4}; R_\infty, \alpha, A_{\mathrm{r}}(\mathrm{e}), A_{\mathrm{r}}(\mathrm{p}), R_{\mathrm{p}}, \delta_{\mathrm{H}}(n_4\mathrm{L}_{4j_4})\big) - E_{\mathrm{H}}\big(n_3\mathrm{L}_{3j_3}; R_\infty, \alpha, A_{\mathrm{r}}(\mathrm{e}), A_{\mathrm{r}}(\mathrm{p}), R_{\mathrm{p}}, \delta_{\mathrm{H}}(n_3\mathrm{L}_{3j_3})\big)\big]\Big\}/h$
$A40$–$A44$	$\nu_{\mathrm{D}}(n_1\mathrm{L}_{1j_1} - n_2\mathrm{L}_{2j_2}) \doteq \big[E_{\mathrm{D}}\big(n_2\mathrm{L}_{2j_2}; R_\infty, \alpha, A_{\mathrm{r}}(\mathrm{e}), A_{\mathrm{r}}(\mathrm{d}), R_{\mathrm{d}}, \delta_{\mathrm{D}}(n_2\mathrm{L}_{2j_2})\big) - E_{\mathrm{D}}\big(n_1\mathrm{L}_{1j_1}; R_\infty, \alpha, A_{\mathrm{r}}(\mathrm{e}), A_{\mathrm{r}}(\mathrm{d}), R_{\mathrm{d}}, \delta_{\mathrm{D}}(n_1\mathrm{L}_{1j_1})\big)\big]/h$
$A45$–$A46$	$\nu_{\mathrm{D}}(n_1\mathrm{L}_{1j_1} - n_2\mathrm{L}_{2j_2}) - \frac{1}{4}\nu_{\mathrm{D}}(n_3\mathrm{L}_{3j_3} - n_4\mathrm{L}_{4j_4}) \doteq \Big\{E_{\mathrm{D}}\big(n_2\mathrm{L}_{2j_2}; R_\infty, \alpha, A_{\mathrm{r}}(\mathrm{e}), A_{\mathrm{r}}(\mathrm{d}), R_{\mathrm{d}}, \delta_{\mathrm{D}}(n_2\mathrm{L}_{2j_2})\big) - E_{\mathrm{D}}\big(n_1\mathrm{L}_{1j_1}; R_\infty, \alpha, A_{\mathrm{r}}(\mathrm{e}), A_{\mathrm{r}}(\mathrm{d}), R_{\mathrm{d}}, \delta_{\mathrm{D}}(n_1\mathrm{L}_{1j_1})\big) - \frac{1}{4}\big[E_{\mathrm{D}}\big(n_4\mathrm{L}_{4j_4}; R_\infty, \alpha, A_{\mathrm{r}}(\mathrm{e}), A_{\mathrm{r}}(\mathrm{d}), R_{\mathrm{d}}, \delta_{\mathrm{D}}(n_4\mathrm{L}_{4j_4})\big) - E_{\mathrm{D}}\big(n_3\mathrm{L}_{3j_3}; R_\infty, \alpha, A_{\mathrm{r}}(\mathrm{e}), A_{\mathrm{r}}(\mathrm{d}), R_{\mathrm{d}}, \delta_{\mathrm{D}}(n_3\mathrm{L}_{3j_3})\big)\big]\Big\}/h$
$A47$	$\nu_{\mathrm{D}}(1\mathrm{S}_{1/2} - 2\mathrm{S}_{1/2}) - \nu_{\mathrm{H}}(1\mathrm{S}_{1/2} - 2\mathrm{S}_{1/2}) \doteq \Big\{E_{\mathrm{D}}\big(2\mathrm{S}_{1/2}; R_\infty, \alpha, A_{\mathrm{r}}(\mathrm{e}), A_{\mathrm{r}}(\mathrm{d}), R_{\mathrm{d}}, \delta_{\mathrm{D}}(2\mathrm{S}_{1/2})\big) - E_{\mathrm{D}}\big(1\mathrm{S}_{1/2}; R_\infty, \alpha, A_{\mathrm{r}}(\mathrm{e}), A_{\mathrm{r}}(\mathrm{d}), R_{\mathrm{d}}, \delta_{\mathrm{D}}(1\mathrm{S}_{1/2})\big) - \big[E_{\mathrm{H}}\big(2\mathrm{S}_{1/2}; R_\infty, \alpha, A_{\mathrm{r}}(\mathrm{e}), A_{\mathrm{r}}(\mathrm{p}), R_{\mathrm{p}}, \delta_{\mathrm{H}}(2\mathrm{S}_{1/2})\big) - E_{\mathrm{H}}\big(1\mathrm{S}_{1/2}; R_\infty, \alpha, A_{\mathrm{r}}(\mathrm{e}), A_{\mathrm{r}}(\mathrm{p}), R_{\mathrm{p}}, \delta_{\mathrm{H}}(1\mathrm{S}_{1/2})\big)\big]\Big\}/h$
$A48$	$R_{\mathrm{p}} \doteq R_{\mathrm{p}}$
$A49$	$R_{\mathrm{d}} \doteq R_{\mathrm{d}}$

TABLE 39 The 39 adjusted constants (variables) used in the least-squares multivariate analysis of the input data in Table 30. These adjusted constants appear as arguments of the functions on the right-hand side of the observational equations of Table 40.

Adjusted constant	Symbol
electron relative atomic mass	$A_r(e)$
proton relative atomic mass	$A_r(p)$
neutron relative atomic mass	$A_r(n)$
deuteron relative atomic mass	$A_r(d)$
triton relative atomic mass	$A_r(t)$
helion relative atomic mass	$A_r(h)$
alpha particle relative atomic mass	$A_r(\alpha)$
$^{16}O^{7+}$ relative atomic mass	$A_r(^{16}O^{7+})$
^{87}Rb relative atomic mass	$A_r(^{87}Rb)$
^{133}Cs relative atomic mass	$A_r(^{133}Cs)$
fine-structure constant	α
additive correction to a_e(th)	δ_e
additive correction to a_μ(th)	δ_μ
additive correction to g_C(th)	δ_C
additive correction to g_O(th)	δ_O
electron-proton magnetic moment ratio	μ_{e^-}/μ_p
deuteron-electron magnetic moment ratio	μ_d/μ_{e^-}
triton-proton magnetic moment ratio	μ_t/μ_p
shielding difference of d and p in HD	σ_{dp}
shielding difference of t and p in HT	σ_{tp}
electron to shielded proton magnetic moment ratio	μ_{e^-}/μ'_p
shielded helion to shielded proton magnetic moment ratio	μ'_h/μ'_p
neutron to shielded proton magnetic moment ratio	μ_n/μ'_p
electron-muon mass ratio	m_e/m_μ
additive correction to $\Delta\nu_{Mu}$(th)	δ_{Mu}
Planck constant	h
molar gas constant	R
copper Kα_1 x unit	xu(CuKα_1)
molybdenum Kα_1 x unit	xu(MoKα_1)
ångstrom star	Å*
d_{220} of Si crystal ILL	d_{220}(ILL)
d_{220} of Si crystal N	d_{220}(N)
d_{220} of Si crystal WASO 17	d_{220}(W17)
d_{220} of Si crystal WASO 04	d_{220}(W04)
d_{220} of Si crystal WASO 4.2a	d_{220}(W4.2a)
d_{220} of Si crystal MO*	d_{220}(MO*)
d_{220} of Si crystal NR3	d_{220}(NR3)
d_{220} of Si crystal NR4	d_{220}(NR4)
d_{220} of an ideal Si crystal	d_{220}

TABLE 40 Observational equations that express the input data in Table 30 as functions of the adjusted constants in Table 39. The numbers in the first column correspond to the numbers in the first column of Table 30. For simplicity, the lengthier functions are not explicitly given. See Sec. 12.2 for an explanation of the symbol $\doteq$.

Type of input datum	Observational equation	Sec.
$B1$	$A_{\rm r}(^1{\rm H}) \doteq A_{\rm r}({\rm p}) + A_{\rm r}({\rm e}) - E_{\rm b}(^1{\rm H})/m_{\rm u}c^2$	3.2
$B2$	$A_{\rm r}(^2{\rm H}) \doteq A_{\rm r}({\rm d}) + A_{\rm r}({\rm e}) - E_{\rm b}(^2{\rm H})/m_{\rm u}c^2$	3.2
$B3$	$A_{\rm r}(^3{\rm H}) \doteq A_{\rm r}({\rm t}) + A_{\rm r}({\rm e}) - E_{\rm b}(^3{\rm H})/m_{\rm u}c^2$	3.2
$B4$	$A_{\rm r}(^3{\rm He}) \doteq A_{\rm r}({\rm h}) + 2A_{\rm r}({\rm e}) - E_{\rm b}(^3{\rm He})/m_{\rm u}c^2$	3.2
$B5$	$A_{\rm r}(^4{\rm He}) \doteq A_{\rm r}(\alpha) + 2A_{\rm r}({\rm e}) - E_{\rm b}(^4{\rm He})/m_{\rm u}c^2$	3.2
$B6$	$A_{\rm r}(^{16}{\rm O}) \doteq A_{\rm r}(^{16}{\rm O}^{7+}) + 7A_{\rm r}({\rm e}) - \left[E_{\rm b}(^{16}{\rm O}) - E_{\rm b}(^{16}{\rm O}^{7+})\right]/m_{\rm u}c^2$	5.3.2.2
$B7$	$A_{\rm r}(^{87}{\rm Rb}) \doteq A_{\rm r}(^{87}{\rm Rb})$	
$B8$	$A_{\rm r}(^{133}{\rm Cs}) \doteq A_{\rm r}(^{133}{\rm Cs})$	
$B9$	$A_{\rm r}({\rm e}) \doteq A_{\rm r}({\rm e})$	
$B10$	$\delta_{\rm e} \doteq \delta_{\rm e}$	
$B11$	$a_{\rm e} \doteq a_{\rm e}(\alpha, \delta_e)$	5.1.1
$B12$	$\delta_\mu \doteq \delta_\mu$	
$B13$	$\overline{R} \doteq -\dfrac{a_\mu(\alpha,\delta_\mu)}{1+a_{\rm e}(\alpha,\delta_{\rm e})}\dfrac{m_{\rm e}}{m_\mu}\dfrac{\mu_{\rm e^-}}{\mu_{\rm p}}$	5.2.2
$B14$	$\delta_{\rm C} \doteq \delta_{\rm C}$	
$B15$	$\delta_{\rm O} \doteq \delta_{\rm O}$	
$B16$	$\dfrac{f_{\rm s}\left(^{12}{\rm C}^{5+}\right)}{f_{\rm c}\left(^{12}{\rm C}^{5+}\right)} \doteq -\dfrac{g_{\rm C}(\alpha,\delta_{\rm C})}{10A_{\rm r}({\rm e})}\left[12 - 5A_{\rm r}({\rm e}) + \dfrac{E_{\rm b}\left(^{12}{\rm C}\right) - E_{\rm b}\left(^{12}{\rm C}^{5+}\right)}{m_{\rm u}c^2}\right]$	5.3.2.1
$B17$	$\dfrac{f_{\rm s}\left(^{16}{\rm O}^{7+}\right)}{f_{\rm c}\left(^{16}{\rm O}^{7+}\right)} \doteq -\dfrac{g_{\rm O}(\alpha,\delta_{\rm O})}{14A_{\rm r}({\rm e})}A_{\rm r}(^{16}{\rm O}^{7+})$	5.3.2.2
$B18$	$\dfrac{\mu_{\rm e^-}({\rm H})}{\mu_{\rm p}({\rm H})} \doteq \dfrac{g_{\rm e^-}({\rm H})}{g_{\rm e^-}}\left(\dfrac{g_{\rm p}({\rm H})}{g_{\rm p}}\right)^{-1}\dfrac{\mu_{\rm e^-}}{\mu_{\rm p}}$	6.1.2.1
$B19$	$\dfrac{\mu_{\rm d}({\rm D})}{\mu_{\rm e^-}({\rm D})} \doteq \dfrac{g_{\rm d}({\rm D})}{g_{\rm d}}\left(\dfrac{g_{\rm e^-}({\rm D})}{g_{\rm e^-}}\right)^{-1}\dfrac{\mu_{\rm d}}{\mu_{\rm e^-}}$	6.1.2.2
$B20$	$\dfrac{\mu_{\rm p}({\rm HD})}{\mu_{\rm d}({\rm HD})} \doteq [1+\sigma_{\rm dp}]\dfrac{\mu_{\rm p}}{\mu_{\rm e^-}}\dfrac{\mu_{\rm e^-}}{\mu_{\rm d}}$	6.1.2.3
$B21$	$\sigma_{\rm dp} \doteq \sigma_{\rm dp}$	
$B22$	$\dfrac{\mu_{\rm t}({\rm HT})}{\mu_{\rm p}({\rm HT})} \doteq [1-\sigma_{\rm tp}]\dfrac{\mu_{\rm t}}{\mu_{\rm p}}$	6.1.2.3
$B23$	$\sigma_{\rm tp} \doteq \sigma_{\rm tp}$	
$B24$	$\dfrac{\mu_{\rm e^-}({\rm H})}{\mu'_{\rm p}} \doteq \dfrac{g_{\rm e^-}({\rm H})}{g_{\rm e^-}}\dfrac{\mu_{\rm e^-}}{\mu'_{\rm p}}$	6.1.2.4
$B25$	$\dfrac{\mu'_{\rm h}}{\mu'_{\rm p}} \doteq \dfrac{\mu'_{\rm h}}{\mu'_{\rm p}}$	
$B26$	$\dfrac{\mu_{\rm n}}{\mu'_{\rm p}} \doteq \dfrac{\mu_{\rm n}}{\mu'_{\rm p}}$	

TABLE 40 *(Continued).* Observational equations that express the input data in Table 30 as functions of the adjusted constants in Table 39. The numbers in the first column correspond to the numbers in the first column of Table 30. For simplicity, the lengthier functions are not explicitly given. See Sec. 12.2 for an explanation of the symbol $\doteq$.

Type of input datum	Observational equation	Sec.
$B27$	$\delta_{\mathrm{Mu}} \doteq \delta_{\mathrm{Mu}}$	
$B28$	$\Delta\nu_{\mathrm{Mu}} \doteq \Delta\nu_{\mathrm{Mu}}\left(R_\infty, \alpha, \dfrac{m_{\mathrm{e}}}{m_{\mu}}, \delta_{\mu}, \delta_{\mathrm{Mu}}\right)$	6.2.1
$B29, B30$	$\nu(f_{\mathrm{p}}) \doteq \nu\left(f_{\mathrm{p}}; R_\infty, \alpha, \dfrac{m_{\mathrm{e}}}{m_{\mu}}, \dfrac{\mu_{\mathrm{e}^-}}{\mu_{\mathrm{p}}}, \delta_{\mathrm{e}}, \delta_{\mu}, \delta_{\mathrm{Mu}}\right)$	6.2
$B31$	$\Gamma'_{\mathrm{p}-90}(\mathrm{lo}) \doteq -\dfrac{K_{\mathrm{J}-90}R_{\mathrm{K}-90}[1+a_{\mathrm{e}}(\alpha,\delta_{\mathrm{e}})]\alpha^3}{2\mu_0 R_\infty}\left(\dfrac{\mu_{\mathrm{e}^-}}{\mu'_{\mathrm{p}}}\right)^{-1}$	7.1.1
$B32$	$\Gamma'_{\mathrm{h}-90}(\mathrm{lo}) \doteq \dfrac{K_{\mathrm{J}-90}R_{\mathrm{K}-90}[1+a_{\mathrm{e}}(\alpha,\delta_{\mathrm{e}})]\alpha^3}{2\mu_0 R_\infty}\left(\dfrac{\mu_{\mathrm{e}^-}}{\mu'_{\mathrm{p}}}\right)^{-1}\dfrac{\mu'_{\mathrm{h}}}{\mu'_{\mathrm{p}}}$	7.1.1
$B33$	$\Gamma'_{\mathrm{p}-90}(\mathrm{hi}) \doteq -\dfrac{c[1+a_{\mathrm{e}}(\alpha,\delta_{\mathrm{e}})]\alpha^2}{K_{\mathrm{J}-90}R_{\mathrm{K}-90}R_\infty h}\left(\dfrac{\mu_{\mathrm{e}^-}}{\mu'_{\mathrm{p}}}\right)^{-1}$	7.1.2
$B34$	$R_{\mathrm{K}} \doteq \dfrac{\mu_0 c}{2\alpha}$	7.2
$B35$	$K_{\mathrm{J}} \doteq \left(\dfrac{8\alpha}{\mu_0 c h}\right)^{1/2}$	7.3
$B36$	$K_{\mathrm{J}}^2 R_{\mathrm{K}} \doteq \dfrac{4}{h}$	7.4
$B37$	$\mathcal{F}_{90} \doteq \dfrac{cM_{\mathrm{u}}A_{\mathrm{r}}(\mathrm{e})\alpha^2}{K_{\mathrm{J}-90}R_{\mathrm{K}-90}R_\infty h}$	7.5
$B38$-$B40$	$d_{220}(\mathit{X}) \doteq d_{220}(\mathit{X})$	
$B41$-$B52$	$\dfrac{d_{220}(\mathit{X})}{d_{220}(\mathit{Y})} - 1 \doteq \dfrac{d_{220}(\mathit{X})}{d_{220}(\mathit{Y})} - 1$	
$B53$	$V_{\mathrm{m}}(\mathrm{Si}) \doteq \dfrac{\sqrt{2}\,cM_{\mathrm{u}}A_{\mathrm{r}}(\mathrm{e})\alpha^2 d_{220}^3}{R_\infty h}$	8.2
$B54$	$\dfrac{\lambda_{\mathrm{meas}}}{d_{220}(\textsc{ILL})} \doteq \dfrac{\alpha^2 A_{\mathrm{r}}(\mathrm{e})}{R_\infty d_{220}(\textsc{ILL})}\dfrac{A_{\mathrm{r}}(\mathrm{n})+A_{\mathrm{r}}(\mathrm{p})}{[A_{\mathrm{r}}(\mathrm{n})+A_{\mathrm{r}}(\mathrm{p})]^2 - A_{\mathrm{r}}^2(\mathrm{d})}$	8.3
$B55$	$\dfrac{h}{m_{\mathrm{n}}d_{220}(\textsc{W04})} \doteq \dfrac{A_{\mathrm{r}}(\mathrm{e})}{A_{\mathrm{r}}(\mathrm{n})}\dfrac{c\alpha^2}{2R_\infty d_{220}(\textsc{W04})}$	8.4.1
$B56, B57$	$\dfrac{h}{m(X)} \doteq \dfrac{A_{\mathrm{r}}(\mathrm{e})}{A_{\mathrm{r}}(X)}\dfrac{c\alpha^2}{2R_\infty}$	8.4
$B58$	$R \doteq R$	
$B59, B62$	$\dfrac{\lambda(\mathrm{CuK}\alpha_1)}{d_{220}(\mathit{X})} \doteq \dfrac{1\,537.400\ \mathrm{xu}(\mathrm{CuK}\alpha_1)}{d_{220}(\mathit{X})}$	11.1
$B60$	$\dfrac{\lambda(\mathrm{WK}\alpha_1)}{d_{220}(\textsc{N})} \doteq \dfrac{0.209\,010\,0\ \text{Å}^*}{d_{220}(\textsc{N})}$	11.1
$B61$	$\dfrac{\lambda(\mathrm{MoK}\alpha_1)}{d_{220}(\textsc{N})} \doteq \dfrac{707.831\ \mathrm{xu}(\mathrm{MoK}\alpha_1)}{d_{220}(\textsc{N})}$	11.1

TABLE 41 The 12 adjusted constants (variables) relevant to the antiprotonic helium data given in Table 32. These adjusted constants appear as arguments of the theoretical expressions on the right-hand side of the observational equations of Table 42.

Transition	Adjusted constant
$\bar{\mathrm{p}}^4\mathrm{He}^+$: $(32,31) \rightarrow (31,30)$	$\delta_{\bar{\mathrm{p}}^4\mathrm{He}^+}(32,31{:}31,30)$
$\bar{\mathrm{p}}^4\mathrm{He}^+$: $(35,33) \rightarrow (34,32)$	$\delta_{\bar{\mathrm{p}}^4\mathrm{He}^+}(35,33{:}34,32)$
$\bar{\mathrm{p}}^4\mathrm{He}^+$: $(36,34) \rightarrow (35,33)$	$\delta_{\bar{\mathrm{p}}^4\mathrm{He}^+}(36,34{:}35,33)$
$\bar{\mathrm{p}}^4\mathrm{He}^+$: $(37,34) \rightarrow (36,33)$	$\delta_{\bar{\mathrm{p}}^4\mathrm{He}^+}(37,34{:}36,33)$
$\bar{\mathrm{p}}^4\mathrm{He}^+$: $(39,35) \rightarrow (38,34)$	$\delta_{\bar{\mathrm{p}}^4\mathrm{He}^+}(39,35{:}38,34)$
$\bar{\mathrm{p}}^4\mathrm{He}^+$: $(40,35) \rightarrow (39,34)$	$\delta_{\bar{\mathrm{p}}^4\mathrm{He}^+}(40,35{:}39,34)$
$\bar{\mathrm{p}}^4\mathrm{He}^+$: $(37,35) \rightarrow (38,34)$	$\delta_{\bar{\mathrm{p}}^4\mathrm{He}^+}(37,35{:}38,34)$
$\bar{\mathrm{p}}^3\mathrm{He}^+$: $(32,31) \rightarrow (31,30)$	$\delta_{\bar{\mathrm{p}}^3\mathrm{He}^+}(32,31{:}31,30)$
$\bar{\mathrm{p}}^3\mathrm{He}^+$: $(34,32) \rightarrow (33,31)$	$\delta_{\bar{\mathrm{p}}^3\mathrm{He}^+}(34,32{:}33,31)$
$\bar{\mathrm{p}}^3\mathrm{He}^+$: $(36,33) \rightarrow (35,32)$	$\delta_{\bar{\mathrm{p}}^3\mathrm{He}^+}(36,33{:}35,32)$
$\bar{\mathrm{p}}^3\mathrm{He}^+$: $(38,34) \rightarrow (37,33)$	$\delta_{\bar{\mathrm{p}}^3\mathrm{He}^+}(38,34{:}37,33)$
$\bar{\mathrm{p}}^3\mathrm{He}^+$: $(36,34) \rightarrow (37,33)$	$\delta_{\bar{\mathrm{p}}^3\mathrm{He}^+}(36,34{:}37,33)$

TABLE 42 Observational equations that express the input data related to antiprotonic helium in Table 32 as functions of adjusted constants in Tables 39 and 41. The numbers in the first column correspond to the numbers in the first column of Table 32. Definitions of the symbols and values of the parameters in these equations are given in Sec. 4.2. See Sec. 12.2 for an explanation of the symbol $\doteq$.

Type of input datum	Observational equation
$C1$–$C7$	$\delta_{\bar{p}^4\mathrm{He}^+}(n,l:n',l') \doteq \delta_{\bar{p}^4\mathrm{He}^+}(n,l:n',l')$
$C8$–$C12$	$\delta_{\bar{p}^3\mathrm{He}^+}(n,l:n',l') \doteq \delta_{\bar{p}^3\mathrm{He}^+}(n,l:n',l')$
$C13$–$C19$	$\nu_{\bar{p}^4\mathrm{He}^+}(n,l:n',l') \doteq \nu^{(0)}_{\bar{p}^4\mathrm{He}^+}(n,l:n',l') + a_{\bar{p}^4\mathrm{He}^+}(n,l:n',l')\left[\left(\frac{A_\mathrm{r}(\mathrm{e})}{A_\mathrm{r}(\mathrm{p})}\right)^{(0)}\left(\frac{A_\mathrm{r}(\mathrm{p})}{A_\mathrm{r}(\mathrm{e})}\right)-1\right] + b_{\bar{p}^4\mathrm{He}^+}(n,l:n',l')\left[\left(\frac{A_\mathrm{r}(\mathrm{e})}{A_\mathrm{r}(\alpha)}\right)^{(0)}\left(\frac{A_\mathrm{r}(\alpha)}{A_\mathrm{r}(\mathrm{e})}\right)-1\right] + \delta_{\bar{p}^4\mathrm{He}^+}(n,l:n',l')$
$C20$–$C24$	$\nu_{\bar{p}^3\mathrm{He}^+}(n,l:n',l') \doteq \nu^{(0)}_{\bar{p}^3\mathrm{He}^+}(n,l:n',l') + a_{\bar{p}^3\mathrm{He}^+}(n,l:n',l')\left[\left(\frac{A_\mathrm{r}(\mathrm{e})}{A_\mathrm{r}(\mathrm{p})}\right)^{(0)}\left(\frac{A_\mathrm{r}(\mathrm{p})}{A_\mathrm{r}(\mathrm{e})}\right)-1\right] + b_{\bar{p}^3\mathrm{He}^+}(n,l:n',l')\left[\left(\frac{A_\mathrm{r}(\mathrm{e})}{A_\mathrm{r}(\mathrm{h})}\right)^{(0)}\left(\frac{A_\mathrm{r}(\mathrm{h})}{A_\mathrm{r}(\mathrm{e})}\right)-1\right] + \delta_{\bar{p}^3\mathrm{He}^+}(n,l:n',l')$

TABLE 43 Summary of the results of some of the least-squares adjustments used to analyze all of the input data given in Tables 28, 29, 30, and 31. The values of α and h are those obtained in the adjustment, N is the number of input data, M is the number of adjusted constants, $\nu = N - M$ is the degrees of freedom, and $R_\mathrm{B} = \sqrt{\chi^2/\nu}$ is the Birge ratio. See the text for an explanation and discussion of each adjustment, but in brief, 1 is all the data; 2 is 1 with the uncertainties of the key x-ray/silicon data multiplied by 1.5; 3 is 2 with the uncertainties of the key electrical data also multiplied by 1.5; 4 is the final adjustment from which the 2006 recommended values are obtained and is 3 with the input data with low weights deleted; 5 is 3 with the four data that provide the most accurate values of α deleted; and 6 is 3 with the three data that provide the most accurate values of h deleted.

Adj.	N	M	ν	χ^2	R_B	α^{-1}	$u_\mathrm{r}(\alpha^{-1})$	$h/(\mathrm{J\ s})$	$u_\mathrm{r}(h)$
1	150	79	71	92.1	1.14	137.035 999 687(93)	6.8×10^{-10}	$6.626\,068\,96(22)\times10^{-34}$	3.4×10^{-8}
2	150	79	71	82.0	1.07	137.035 999 682(93)	6.8×10^{-10}	$6.626\,068\,96(22)\times10^{-34}$	3.4×10^{-8}
3	150	79	71	77.5	1.04	137.035 999 681(93)	6.8×10^{-10}	$6.626\,068\,96(33)\times10^{-34}$	5.0×10^{-8}
4	135	78	57	65.0	1.07	137.035 999 679(94)	6.8×10^{-10}	$6.626\,068\,96(33)\times10^{-34}$	5.0×10^{-8}
5	144	77	67	72.9	1.04	137.036 0012(19)	1.4×10^{-8}	$6.626\,068\,96(33)\times10^{-34}$	5.0×10^{-8}
6	147	79	68	75.4	1.05	137.035 999 680(93)	6.8×10^{-10}	$6.626\,0719(21)\times10^{-34}$	3.2×10^{-7}

TABLE 44 Normalized residuals r_i and self-sensitivity coefficients S_c that result from the six least-squares adjustments summarized in Table 43 for the four input data whose absolute values of r_i in Adj. 1 exceed 1.50. S_c is a measure of how the least-squares estimated value of a given type of input datum depends on a particular measured or calculated value of that type of datum; see Appendix E of CODATA-98. See the text for an explanation and discussion of each adjustment; brief explanations are given at the end of the caption to the previous table.

Item number	Input quantity	Identification	Adj. 1		Adj. 2		Adj. 3		Adj. 4		Adj. 5		Adj. 6	
			r_i	S_c	r_i	S_c	r_i	S_c	r_i	S_c	r_i	S_c	r_i	S_c
$B53$	$V_\mathrm{m}(\mathrm{Si})$	N/P/I-05	−2.82	0.065	−2.68	0.085	−1.86	0.046	−1.86	0.047	−1.79	0.053	−0.86	0.556
$B55$	$h/m_\mathrm{n}d_{220}(\mathrm{W04})$	PTB-99	−2.71	0.155	−2.03	0.118	−1.89	0.121	−1.89	0.121	−1.57	0.288	−1.82	0.123
$B39$	$d_{220}(\mathrm{NR3})$	NMIJ-04	2.37	0.199	1.86	0.145	1.74	0.148	1.74	0.148	1.78	0.151	−1.00	0.353
$B31.1$	$\Gamma'_{\mathrm{p}-90}(\mathrm{lo})$	NIST-89	2.31	0.010	2.30	0.010	2.30	0.010	deleted		2.60	0.143	2.30	0.010

TABLE 45 Summary of the results of some of the least-squares adjustments used to analyze the input data related to R_∞. The values of R_∞, R_p, and R_d are those obtained in the indicated adjustment, N is the number of input data, M is the number of adjusted constants, $\nu = N - M$ is the degrees of freedom, and $R_\mathrm{B} = \sqrt{\chi^2/\nu}$ is the Birge ratio. See the text for an explanation and discussion of each adjustment, but in brief, 4 is the final adjustment; 7 is 4 with the input data for R_p and R_d deleted; 8 is 4 with just the R_p datum deleted; 9 is 4 with just the R_d datum deleted; 10 is 4 but with only the hydrogen data included; and 11 is 4 but with only the deuterium data included.

Adj.	N	M	ν	χ^2	R_B	R_∞/m^{-1}	$u_\mathrm{r}(R_\infty)$	R_p/fm	R_d/fm
4	135	78	57	65.0	1.07	10 973 731.568 527(73)	6.6×10^{-12}	0.8768(69)	2.1402(28)
7	133	78	55	63.0	1.07	10 973 731.568 518(82)	7.5×10^{-12}	0.8760(78)	2.1398(32)
8	134	78	56	63.8	1.07	10 973 731.568 495(78)	7.1×10^{-12}	0.8737(75)	2.1389(30)
9	134	78	56	63.9	1.07	10 973 731.568 549(76)	6.9×10^{-12}	0.8790(71)	2.1411(29)
10	117	68	49	60.8	1.11	10 973 731.568 562(85)	7.8×10^{-12}	0.8802(80)	
11	102	61	41	54.7	1.16	10 973 731.568 39(13)	1.1×10^{-11}		2.1286(93)

TABLE 46 Generalized observational equations that express input data $B31$-$B37$ in Table 30 as functions of the adjusted constants in Tables 39 and 37 with the additional adjusted constants ε_J and ε_K as given in Eqs. (372) and (373). The numbers in the first column correspond to the numbers in the first column of Table 30. For simplicity, the lengthier functions are not explicitly given. See Sec. 12.2 for an explanation of the symbol $\doteq$.

Type of input datum	Generalized observational equation
$B31^*$	$\Gamma'_{\mathrm{p}-90}(\mathrm{lo}) \doteq -\dfrac{K_{\mathrm{J}-90}R_{\mathrm{K}-90}[1+a_\mathrm{e}(\alpha,\delta_\mathrm{e})]\alpha^3}{2\mu_0 R_\infty(1+\varepsilon_\mathrm{J})(1+\varepsilon_\mathrm{K})}\left(\dfrac{\mu_{\mathrm{e}^-}}{\mu'_\mathrm{p}}\right)^{-1}$
$B32^*$	$\Gamma'_{\mathrm{h}-90}(\mathrm{lo}) \doteq \dfrac{K_{\mathrm{J}-90}R_{\mathrm{K}-90}[1+a_\mathrm{e}(\alpha,\delta_\mathrm{e})]\alpha^3}{2\mu_0 R_\infty(1+\varepsilon_\mathrm{J})(1+\varepsilon_\mathrm{K})}\left(\dfrac{\mu_{\mathrm{e}^-}}{\mu'_\mathrm{p}}\right)^{-1}\dfrac{\mu'_\mathrm{h}}{\mu'_\mathrm{p}}$
$B33^*$	$\Gamma'_{\mathrm{p}-90}(\mathrm{hi}) \doteq -\dfrac{c[1+a_\mathrm{e}(\alpha,\delta_\mathrm{e})]\alpha^2}{K_{\mathrm{J}-90}R_{\mathrm{K}-90}R_\infty h}(1+\varepsilon_\mathrm{J})(1+\varepsilon_\mathrm{K})\left(\dfrac{\mu_{\mathrm{e}^-}}{\mu'_\mathrm{p}}\right)^{-1}$
$B34^*$	$R_\mathrm{K} \doteq \dfrac{\mu_0 c}{2\alpha}(1+\varepsilon_\mathrm{K})$
$B35^*$	$K_\mathrm{J} \doteq \left(\dfrac{8\alpha}{\mu_0 c h}\right)^{1/2}(1+\varepsilon_\mathrm{J})$
$B36^*$	$K_\mathrm{J}^2 R_\mathrm{K} \doteq \dfrac{4}{h}(1+\varepsilon_\mathrm{J})^2(1+\varepsilon_\mathrm{K})$
$B37^*$	$\mathcal{F}_{90} \doteq \dfrac{cM_\mathrm{u}A_\mathrm{r}(\mathrm{e})\alpha^2}{K_{\mathrm{J}-90}R_{\mathrm{K}-90}R_\infty h}(1+\varepsilon_\mathrm{J})(1+\varepsilon_\mathrm{K})$
$B62^*$	$\varepsilon_\mathrm{J} \doteq \varepsilon_\mathrm{J}$
$B63^*$	$\varepsilon_\mathrm{K} \doteq \varepsilon_\mathrm{K}$

TABLE 47 Summary of the results of several least-squares adjustments carried out to investigate the effect of assuming the relations for $K_{\rm J}$ and $R_{\rm K}$ given in Eqs. (372) and (373). The values of α, h, $\varepsilon_{\rm K}$, and $\varepsilon_{\rm J}$ are those obtained in the indicated adjustments. The quantity $R_{\rm B} = \sqrt{\chi^2/\nu}$ is the Birge ratio and r_i is the normalized residual of the indicated input datum (see Table 30). These four data have the largest $|r_i|$ of all the input data and are the only data in Adj. (i) with $|r_i| > 1.50$. See the text for an explanation and discussion of each adjustment, but in brief, (i) assumes $K_{\rm J} = 2e/h$ and $R_{\rm K} = h/e^2$ and uses all the data; (ii) is (i) with the relation $K_{\rm J} = 2e/h$ relaxed; (iii) is (i) with the relation $R_{\rm K} = h/e^2$ relaxed; (iv) is (i) with both relations relaxed; (v) is (iv) with the $V_{\rm m}({\rm Si})$ datum deleted; (vi) is (iv) with the $\Gamma'_{\rm p-90}({\rm lo})$ and $\Gamma'_{\rm h-90}({\rm lo})$ data deleted; and (vii) is (iv) with the $V_{\rm m}({\rm Si})$, $\Gamma'_{\rm p-90}({\rm lo})$, and $\Gamma'_{\rm h-90}({\rm lo})$ data deleted.

Adj.	$R_{\rm B}$	α^{-1}	$h/({\rm J\ s})$	$\varepsilon_{\rm K}$	$\varepsilon_{\rm J}$	r_{B53}	r_{B55}	r_{B39}	$r_{B31.1}$
(i)	1.14	137.035 999 687(93)	$6.626\,068\,96(22) \times 10^{-34}$	0	0	−2.82	−2.71	2.37	2.31
(ii)	1.14	137.035 999 688(93)	$6.626\,0682(10) \times 10^{-34}$	0	$-61(79) \times 10^{-9}$	−3.22	−2.75	2.39	1.77
(iii)	1.14	137.035 999 683(93)	$6.626\,069\,06(25) \times 10^{-34}$	$16(18) \times 10^{-9}$	0	−2.77	−2.71	2.36	2.45
(iv)	1.14	137.035 999 685(93)	$6.626\,0681(11) \times 10^{-34}$	$20(18) \times 10^{-9}$	$-77(80) \times 10^{-9}$	−3.27	−2.75	2.39	1.79
(v)	1.05	137.035 999 686(93)	$6.626\,0653(13) \times 10^{-34}$	$23(18) \times 10^{-9}$	$-281(95) \times 10^{-9}$	deleted	−2.45	2.19	0.01
(vi)	1.05	137.035 999 686(93)	$6.626\,0744(19) \times 10^{-34}$	$24(18) \times 10^{-9}$	$407(143) \times 10^{-9}$	−0.05	−2.45	2.19	deleted
(vii)	1.06	137.035 999 686(93)	$6.626\,0722(95) \times 10^{-34}$	$24(18) \times 10^{-9}$	$238(720) \times 10^{-9}$	deleted	−2.45	2.19	deleted

13. THE 2006 CODATA RECOMMENDED VALUES

1. Calculational details

As indicated in Sec. 12.2, the 2006 recommended values of the constants are based on adjustment 4 of Tables 43 to 45. This adjustment is obtained by (i) deleting 15 items from the originally considered 150 items of input data of Tables 28, 30, and 32, namely, items $B8$, $B31.1$-$B35.2$, $B37$, and $B56$, because of their low weight (self sensitivity coefficient $S_{\rm c} < 0.01$); and (ii) weighting the uncertainties of the nine input data $B36.1$-$B36.3$, $B38.1$-$B40$, $B53$, and $B55$ by the multiplicative factor 1.5 in order to reduce the absolute values of their normalized residuals $|r_i|$ to less than 2. The correlation coefficients of the data, as given in Tables 29, 31, and 33, are also taken into account. The 135 final input data are expressed in terms of the 78 adjusted constants of Tables 37, 39, and 41, corresponding to $N - M = \nu = 57$ degrees of freedom. Because $h/m(^{133}{\rm Cs})$, item $B56$, has been deleted as an input datum due to its low weight, $A_{\rm r}(^{133}{\rm Cs})$, item $B8$, has also been deleted as an input datum and as an adjusted constant.

For the final adjustment, $\chi^2 = 65.0$, $\sqrt{\chi^2/\nu} = R_{\rm B} = 1.04$, and $Q(65.0|57) = 0.22$, where $Q(\chi^2|\nu)$ is the probability that the observed value of χ^2 for degrees of freedom ν would have exceeded that observed value (see Appendix E of CODATA-98). Each input datum in the final adjustment has $S_{\rm c} > 0.01$, or is a subset of the data of an experiment that provides an input datum or input data with $S_{\rm c} > 0.01$. Not counting such input data with $S_{\rm c} < 0.01$, the six input data with the largest $|r_i|$ are $B55$, $B53$, $B39$, $C18$, $B11.1$, and $B9$; their values of r_i are -1.89, -1.86, 1.74, -1.73, 1.69, and 1.45, respectively. The next largest r_i are 1.22 and 1.11.

The output of the final adjustment is the set of best estimated values, in the least-squares sense, of the 78 adjusted constants and their variances and covariances. Together with (i) those constants that have exact values such as μ_0 and c; (ii) the value of G obtained in Sec. 10; and (iii) the values of m_τ, $G_{\rm F}$, and $\sin^2\theta_{\rm W}$ given in Sec. 11.2, all of the 2006 recommended values, including their uncertainties, are obtained from the 78 adjusted constants. How this is done can be found in Sec. V.B of CODATA-98.

2. Tables of values

The 2006 CODATA recommended values of the basic constants and conversion factors of physics and chemistry and related quantities are given in Tables 49 to 56. These tables are very similar in form to their 2002 counterparts; the principal difference is that a number of new recommended values have been included in the 2006 list, in particular, in Table 50. These are $m_{\rm P}c^2$ in GeV, where $m_{\rm P}$ is the Planck mass; the g-factor of the deuteron $g_{\rm d}$; $b' = \nu_{\rm max}/T$, the Wien displacement law-constant for frequency; and for the first time, 14 recommended values of a number of constants that characterize the triton, including its mass $m_{\rm t}$, magnetic moment $\mu_{\rm t}$, g-factor $g_{\rm t}$, and the magnetic moment ratios $\mu_{\rm t}/\mu_{\rm e}$ and $\mu_{\rm t}/\mu_{\rm p}$. The addition of the triton-related constants is a direct consequence of the improved measurement of $A_{\rm r}(^3{\rm H})$ (item $B3$ in Table 30) and the new NMR measurements on, and re-examined shielding correction differences for, the HT molecule (items $B22$ and $B23$ in Table 30).

Table 49 is a highly abbreviated list containing the values of the constants and conversion factors most commonly used. Table 50 is a much more extensive list of values categorized as follows: UNIVERSAL; ELECTROMAGNETIC; ATOMIC AND NUCLEAR; and PHYSICOCHEMICAL. The ATOMIC AND NUCLEAR category is subdivided into 11 subcategories: General; Electroweak; Electron, e$^-$; Muon, μ^-; Tau, τ^-; Proton, p; Neutron, n; Deuteron, d; Triton, t; Helion, h; and Alpha particle, α. Table 51 gives the variances, covariances, and correlation coefficients of a selected group of constants. (Application of the covariance matrix is discussed in Appendix E of CODATA-98.) Table 52 gives the internationally adopted values of various quantities; Table 53 lists the values of a number of x-ray related quantities; Table 54 lists the values of various non-SI units; and Tables 55 and 56 give the values of various energy equivalents.

All of the values given in Tables 49 to 56 are available on the Web pages of the Fundamental Constants Data Center of the NIST Physics Laboratory at physics.nist.gov/constants. This electronic version of the 2006 CODATA recommended values of the constants also includes a much more extensive correlation coefficient matrix. Indeed, the correlation coefficient of any two constants listed in the tables is accessible on the Web site, as well as the automatic conversion of the value of an energy-related quantity expressed in one unit to the corresponding value expressed in another unit (in essence, an automated version of Tables 55 and 56).

As discussed in Sec. 5, well after the 31 December 2006 closing date of the 2006 adjustment and the 29 March 2007 distribution date of the 2006 recommended values on the Web, Aoyama *et al.* (2007) reported their discovery of an error in the coefficient $A_1^{(8)}$ in the theoretical expression for the electron magnetic moment anomaly $a_{\rm e}$. Use of the new coefficient would lead to an increase in the 2006 recommended value of α by 6.8 times its uncertainty, and an increase of its uncertainty by a factor of 1.02. The recommended values and uncertainties of constants that depend solely on α, or on α in combination with other constants with $u_{\rm r}$ no larger than a few parts in 10^{10}, would change in the same way. However, the changes in the recommended values of the vast majority of the constants listed in the tables would lie in the range 0 to 0.5 times their 2006 uncertainties, and their uncertainties would remain essentially unchanged.

14. SUMMARY AND CONCLUSION

We conclude this report by (i) comparing the 2006 and 2002 CODATA recommended values of the constants and identifying those new results that have contributed most to the changes from the 2002 values; (ii) presenting some of the conclusions that can be drawn from the 2006 recommended values and analysis of the 2006 input data; and (iii) looking to the future and identifying experimental and theoretical work that can advance our knowledge of the values of the constants.

1. Comparison of 2006 and 2002 CODATA recommended values

The 2006 and 2002 recommended values of a representative group of constants are compared in Table 48. Regularities in the numbers in columns 2-4 arise because many constants are obtained from expressions proportional to α, h, or R raised to various powers. Thus, the first six quantities in the table are calculated from expressions proportional to α^a, where $|a| = 1$, 2, 3, or 6. The next 15 quantities, h through μ_p, are calculated from expressions containing the factor h^a, where $|a| = 1$ or $\frac{1}{2}$. And the five quantities R through σ are proportional to R^a, where $|a| = 1$ or 4.

Further comments on the entries in Table 48 are as follows.

(i) The uncertainty of the 2002 recommended value of α has been reduced by nearly a factor of five by the measurement of a_e at Harvard University and the improved theoretical expression for a_e(th). The difference between the Harvard result and the earlier University of Washington result, which played a major role in the determination of α in the 2002 adjustment, accounts for most of the change in the recommended value of α from 2002 to 2006.

(ii) The uncertainty of the 2002 recommended value of h has been reduced by over a factor of three due to the new NIST watt-balance result for $K_J^2 R_K$ and because the factor used to increase the uncertainties of the data related to h (applied to reduce the inconsistencies among the data), was reduced from 2.325 in the 2002 adjustment to 1.5 in the 2006 adjustment. That the change in value from 2002 to 2006 is small is due to the excellent agreement between the new value of $K_J^2 R_K$ and the earlier NIST and NPL values, which played a major role in the determination of h in the 2002 adjustment.

(iii) The updating of two measurements that contributed to the determination of the 2002 recommended value of G reduced the spread in the values and reinforced the most accurate result, that from the University of Washington. On this basis, the Task Group reduced the assigned uncertainty from $u_r = 1.5 \times 10^{-5}$ in 2002 to $u_r = 1.0 \times 10^{-5}$ in 2006. This uncertainty reflects the historical difficulty of measuring G. Although the recommended value is the weighted mean of the eight available

TABLE 48 Comparison of the 2006 and 2002 CODATA adjustments of the values of the constants by the comparison of the corresponding recommended values of a representative group of constants. Here D_r is the 2006 value minus the 2002 value divided by the standard uncertainty u of the 2002 value (*i.e.,* D_r is the change in the value of the constant from 2002 to 2006 relative to its 2002 standard uncertainty).

Quantity	2006 rel. std. uncert. u_r	Ratio 2002 u_r to 2006 u_r	D_r
α	6.8×10^{-10}	4.9	−1.3
R_K	6.8×10^{-10}	4.9	1.3
a_0	6.8×10^{-10}	4.9	−1.3
λ_C	1.4×10^{-9}	4.9	−1.3
r_e	2.1×10^{-9}	4.9	−1.3
σ_e	4.1×10^{-9}	4.9	−1.3
h	5.0×10^{-8}	3.4	−0.3
m_e	5.0×10^{-8}	3.4	−0.3
m_h	5.0×10^{-8}	3.4	−0.3
m_α	5.0×10^{-8}	3.4	−0.3
N_A	5.0×10^{-8}	3.4	0.3
E_h	5.0×10^{-8}	3.4	−0.3
c_1	5.0×10^{-8}	3.4	−0.3
e	2.5×10^{-8}	3.4	−0.3
K_J	2.5×10^{-8}	3.4	0.3
F	2.5×10^{-8}	3.4	0.2
γ_p'	2.7×10^{-8}	3.2	0.2
μ_B	2.5×10^{-8}	3.4	−0.4
μ_N	2.5×10^{-8}	3.4	−0.4
μ_e	2.5×10^{-8}	3.4	0.4
μ_p	2.6×10^{-8}	3.3	−0.4
R	1.7×10^{-6}	1.0	0.0
k	1.7×10^{-6}	1.0	0.0
V_m	1.7×10^{-6}	1.0	0.0
c_2	1.7×10^{-6}	1.0	0.0
σ	7.0×10^{-6}	1.0	0.0
G	1.0×10^{-4}	1.5	0.1
R_∞	6.6×10^{-12}	1.0	0.0
m_e/m_p	4.3×10^{-10}	1.1	0.2
m_e/m_μ	2.5×10^{-8}	1.0	0.3
$A_r(e)$	4.2×10^{-10}	1.0	−0.1
$A_r(p)$	1.0×10^{-10}	1.3	−0.9
$A_r(n)$	4.3×10^{-10}	1.3	0.7
$A_r(d)$	3.9×10^{-11}	4.5	0.1
$A_r(h)$	8.6×10^{-10}	2.3	0.7
$A_r(\alpha)$	1.5×10^{-11}	0.9	−0.4
d_{220}	2.6×10^{-8}	1.4	−2.9
g_e	7.4×10^{-13}	5.0	1.3
g_μ	6.0×10^{-10}	1.0	−1.4
μ_p/μ_B	8.1×10^{-9}	1.2	0.2
μ_p/μ_N	8.2×10^{-9}	1.2	0.2
μ_n/μ_N	2.4×10^{-7}	1.0	0.0
μ_d/μ_N	8.4×10^{-9}	1.3	−0.2
μ_e/μ_p	8.1×10^{-9}	1.2	0.2
μ_n/μ_p	2.4×10^{-7}	1.0	0.0
μ_d/μ_p	7.7×10^{-9}	1.9	−0.3

values, the assigned uncertainty is still over four times the uncertainty of the mean multiplied by the corresponding Birge ratio R_B.

(iv) The large shift in the recommended value of d_{220} from 2002 to 2006 is due to the fact that in the 2002 adjustment only the NMIJ result for d_{220}(NR3) was included, while in the 2006 adjustment this result (but updated by more recent NMIJ measurements) was included together with the PTB result for d_{220}(W4.2a) and the new INRIM results for d_{220}(W4.2a) and d_{220}(MO*). Moreover, the NMIJ value of d_{220} inferred from d_{220}(NR3) strongly disagrees with the values of d_{220} inferred from the other three results.

(v) The marginally significant shift in the recommended value of g_μ from 2002 to 2006 is mainly due to the following: In the 2002 adjustment, the principal hadronic contribution to the theoretical expression for a_μ was based on both a calculation that included only e^+e^- annihilation data and a calculation that used data from hadronic decays of the τ in place of some of the e^+e^- annihilation data. In the 2006 adjustment, the principal hadronic contribution was based on a calculation that used only annihilation data because of various concerns that subsequently arose about the reliability of incorporating the τ data in the calculation; the calculation based on both e^+e^- annihilation data and τ decay data was only used to estimate the uncertainty of the hadronic contribution. Because the results from the two calculations are in significant disagreement, the uncertainty of a_μ(th) is comparatively large: $u_r = 1.8 \times 10^{-6}$.

(vi) The reduction of the uncertainties of the magnetic moment ratios μ_p/μ_B, μ_p/μ_N, μ_d/μ_N, μ_e/μ_p, and μ_d/μ_p are due to the new NMR measurement of $\mu_p(\mathrm{HD})/\mu_d(\mathrm{HD})$ and careful re-examination of the calculation of the D-H shielding correction difference σ_{dp}. Because the value of the product $(\mu_p/\mu_e)(\mu_e/\mu_d)$ implied by the new measurement is highly consistent with the same product implied by the individual measurements of $\mu_e(\mathrm{H})/\mu_p(\mathrm{H})$ and $\mu_d(\mathrm{D})/\mu_e(\mathrm{D})$, the changes in the values of the ratios are small.

In summary, the most important differences between the 2006 and 2002 adjustments are that the 2006 adjustment had available new experimental and theoretical results for a_e, which provided a dramatically improved value of α, and a new result for $K_J^2 R_K$, which provided a significantly improved value of h. These two advances from 2002 to 2006 have resulted in major reductions in the uncertainties of many of the 2006 recommended values compared with their 2002 counterparts.

2. Some implications of the 2006 CODATA recommended values and adjustment for physics and metrology

A number of conclusions that can be drawn from the 2006 adjustment concerning metrology and the basic theories and experimental methods of physics are presented here, where the focus is on those conclusions that are new or are different from those drawn from the 2002 and 1998 adjustments.

Conventional electric units. One can interpret the adoption of the conventional values $K_{J-90} = 483\,597.9$ GHz/V and $R_{K-90} = 25\,812.807\ \Omega$ for the Josephson and von Klitzing constants as establishing conventional, practical units of voltage and resistance, V_{90} and Ω_{90}, given by $V_{90} = (K_{J-90}/K_J)$ V and $\Omega_{90} = (R_K/R_{K-90})\ \Omega$. Other conventional electric units follow from V_{90} and Ω_{90}, for example, $A_{90} = V_{90}/\Omega_{90}$, $C_{90} = A_{90}$ s, $W_{90} = A_{90}V_{90}$, $F_{90} = C_{90}/V_{90}$, and $H_{90} = \Omega_{90}$ s, which are the conventional, practical units of current, charge, power, capacitance, and inductance, respectively (Taylor and Mohr, 2001). For the relations between K_J and K_{J-90}, and R_K and R_{K-90}, the 2006 adjustment gives

$$K_J = K_{J-90}[1 - 1.9(2.5) \times 10^{-8}] \tag{374}$$
$$R_K = R_{K-90}[1 + 2.159(68) \times 10^{-8}]\ , \tag{375}$$

which lead to

$$V_{90} = [1 + 1.9(2.5) \times 10^{-8}]\ \mathrm{V} \tag{376}$$
$$\Omega_{90} = [1 + 2.159(68) \times 10^{-8}]\ \Omega \tag{377}$$
$$A_{90} = [1 - 0.3(2.5) \times 10^{-8}]\ \mathrm{A} \tag{378}$$
$$C_{90} = [1 - 0.3(2.5) \times 10^{-8}]\ \mathrm{C} \tag{379}$$
$$W_{90} = [1 + 1.6(5.0) \times 10^{-8}]\ \mathrm{W} \tag{380}$$
$$F_{90} = [1 - 2.159(68) \times 10^{-8}]\ \mathrm{F} \tag{381}$$
$$H_{90} = [1 + 2.159(68) \times 10^{-8}]\ \mathrm{H}\ . \tag{382}$$

Equations (376) and (377) show that V_{90} exceeds V and Ω_{90} exceeds Ω by $1.9(2.5) \times 10^{-8}$ and $2.159(68) \times 10^{-8}$, respectively. This means that measured voltages and resistances traceable to the Josephson effect and K_{J-90} and the quantum Hall effect and R_{K-90}, respectively, are too small relative to the SI by these same fractional amounts. However, these differences are well within the 40×10^{-8} uncertainty assigned to V_{90}/V and the 10×10^{-8} uncertainty assigned to Ω_{90}/Ω by the Consultative Committee for Electricity and Magnetism (CCEM) of the CIPM (Quinn, 1989, 2001).

Josephson and quantum Hall effects. The study in Sec. 12.2.2 provides no statistically significant evidence that the fundamental Josephson and quantum Hall effect relations $K_J = 2e/h$ and $R_K = h/e^2$ are not exact. The theories of two of the most important phenomena of condensed-matter physics are thereby further supported.

Antiprotonic helium. The good agreement between the value of $A_r(\mathrm{e})$ obtained from the measured values and theoretical expressions for a number of transition frequencies in antiprotonic ^{4}He and ^{3}He with three other values obtained by entirely different methods indicates that these rather complex atoms are reasonably well understood both experimentally and theoretically.

Newtonian constant of gravitation. Although the inconsistencies among the values of G have been reduced somewhat as a result of modifications to two of the eight

results available in 2002, the situation remains problematic; there is no evidence that the historic difficulty of measuring G has been overcome.

Tests of QED. The good agreement of the highly accurate values of α inferred from $h/m(^{133}\mathrm{Cs})$ and $h/m(^{87}\mathrm{Rb})$, which are only weakly dependent on QED theory, with the values of α inferred from a_e, muonium transition frequencies, and H and D transition frequencies, provide support for the QED theory of a_e as well as the bound-state QED theory of muonium and H and D. In particular, the weighted mean of the two values of α inferred from $h/m(^{133}\mathrm{Cs})$ and $h/m(^{87}\mathrm{Rb})$, $\alpha^{-1} = 137.035\,999\,34(69)$ $[5.0 \times 10^{-9}]$, and the weighted mean of the two values of α inferred from the two experimental values of a_e, $\alpha^{-1} = 137.035\,999\,680(94)$ $[6.9 \times 10^{-10}]$, differ by only $0.5u_\mathrm{diff}$, with $u_\mathrm{diff} = 5.1 \times 10^{-9}$. This is a truly impressive confirmation of QED theory.

Physics beyond the Standard Model. If the principal hadronic contribution to $a_\mu(\mathrm{th})$ obtained from the $\mathrm{e}^+\mathrm{e}^-$ annihilation-data plus τ hadronic-decay-data calculation (see previous section) is completely ignored, and the value based on the annihilation-data-only calculation with its uncertainty of 45×10^{-11} is used in $a_\mu(\mathrm{th})$, then the value of α inferred from the BNL experimentally determined value of $a_\mu(\mathrm{exp})$, $\alpha^{-1} = 137.035\,670(91)$ $[6.6 \times 10^{-7}]$, differs from the $h/m(^{133}\mathrm{Cs})$-$h/m(^{87}\mathrm{Rb})$ mean value of α by $3.6u_\mathrm{diff}$. Although such a large discrepancy may suggest "New Physics," the consensus is that such a view is premature (Davier, 2006).

Electrical and silicon crystal-related measurements. The previously discussed inconsistencies involving the watt-balance determinations of $K_\mathrm{J}^2 R_\mathrm{K}$, the mercury electrometer and voltage balance measurements of K_J, the XROI determinations of the $\{220\}$ lattice spacing of various silicon crystals, the measurement of $h/m_\mathrm{n}d_{220}(\mathrm{W04})$, and the measurement of $V_\mathrm{m}(\mathrm{Si})$ hint at possible problems with one or more of these these rather complex experiments. This suggests that some of the many different measurement techniques required for their execution may not be as well understood as is currently believed.

Redefinition of the kilogram. There has been considerable discussion of late about the possibility of the 24th General Conference on Weights and Measures (CGPM), which convenes in 2011, redefining the kilogram, ampere, kelvin, and mole by linking these SI base units to fixed values of h, e, k, and N_A, respectively (Mills *et al.*, 2006; Stock and Witt, 2006), in much the same way that the current definition of the meter is linked to a fixed value of c (BIPM, 2006). Before such a definition of the kilogram can be accepted, h should be known with a u_r of a few parts in 10^{-8}. It is therefore noteworthy that the 2006 CODATA recommended value of h has $u_\mathrm{r} = 5.0 \times 10^{-8}$ and the most accurate measured value of h (the 2007 NIST watt-balance result) has $u_\mathrm{r} = 3.6 \times 10^{-8}$.

3. Outlook and suggestions for future work

Because there is little redundancy among some of the key input data, the 2006 CODATA set of recommended values, like its 2002 and 1998 predecessors, does not rest on as solid a foundation as one might wish. The constants α, h, and R play a critical role in determining many other constants, yet the recommended value of each is determined by a severely limited number of input data. Moreover, some input data for the same quantity have uncertainties of considerably different magnitudes and hence these data contribute to the final adjustment with considerably different weights.

The input datum that primarily determines α is the 2006 experimental result for a_e from Harvard University with $u_\mathrm{r} = 6.5 \times 10^{-10}$; the uncertainty $u_\mathrm{r} = 37 \times 10^{-10}$ of the next most accurate experimental result for a_e, that reported by the University of Washington in 1987, is 5.7 times larger. Furthermore, there is only a single value of the eighth-order coefficient $A_1^{(8)}$, that due to Kinoshita and Nio; it plays a critical role in the theoretical expression for a_e from which α is obtained and requires lengthy QED calculations.

The 2007 NIST watt-balance result for $K_\mathrm{J}^2 R_\mathrm{K}$ with $u_\mathrm{r} = 3.6 \times 10^{-8}$ is the primary input datum that determines h, since the uncertainty of the next most accurate value of $K_\mathrm{J}^2 R_\mathrm{K}$, the NIST 1998 result, is 2.4 times larger. Further, the 2005 consensus value of $V_\mathrm{m}(\mathrm{Si})$ disagrees with all three high accuracy measurements of $K_\mathrm{J}^2 R_\mathrm{K}$ currently available.

For R, the key input datum is the 1998 NIST value based on speed-of-sound measurements in argon using a spherical acoustic resonator with $u_\mathrm{r} = 1.7 \times 10^{-6}$. The uncertainty of the next most accurate value, the 1979 NPL result, also obtained from speed of sound measurements in argon but using an acoustic interferometer, is 4.7 times larger.

Lack of redundancy is, of course, not the only difficulty with the 2006 adjustment. An equally important but not fully independent issue is the several inconsistencies involving some of the electrical and silicon crystal-related input data as already discussed, including the recently reported preliminary result for $K_\mathrm{J}^2 R_\mathrm{K}$ from the NPL watt balance given in Sec 7.4.1. There is also the issue of the recently corrected (but still tentative) value for the coefficient $A_1^{(8)}$ in the theoretical expression for a_e given in Sec 5, which would directly effect the recommended value of α.

With these problems in mind, some of which impact the possible redefinition of the kilogram, ampere, kelvin, and mole in terms of exact values of h, e, k, and N_A in 2011, we offer the following "wish list" for new work. If these needs, some of which appeared in our similar 2002 list, are successfully met, the key issues facing the precision measurement-fundamental constants and fundamental metrology fields should be resolved. As a consequence, our knowledge of the values of the constants, together with the International System of Units (SI), would

be significantly advanced.

(i) A watt-balance determination of $K_J^2 R_K$ from a laboratory other than NIST or NPL with a u_r fully competitive with $u_r = 3.6 \times 10^{-8}$, the uncertainty of the most accurate value currently available from NIST.

(ii) A timely completion of the current international effort to determine N_A with a u_r of a few parts in 10^8 using highly enriched silicon crystals with $x(^{28}\mathrm{Si}) > 0.999\,85$ (Becker *et al.*, 2006). This will require major advances in determining the $\{220\}$ lattice spacing, density, and molar mass of silicon.

(iii) A determination of R (or Boltzmann constant $k = R/N_A$) with a u_r fully competitive with $u_r = 1.7 \times 10^{-6}$, the uncertainty of the most accurate value of R currently available, preferably using a method other than measuring the velocity of sound in argon.

(iv) An independent calculation of the eighth order coefficient $A_1^{(8)}$ in the QED theoretical expression for a_e.

(v) A determination of α that is only weakly dependent on QED theory with a value of u_r fully competitive with $u_r = 7.0 \times 10^{-10}$, the uncertainty of the most accurate value currently available as obtained from $a_e(\mathrm{exp})$ and $a_e(\mathrm{th})$.

(vi) A determination of the Newtonian constant of gravitation G with a u_r fully competitive with $u_r = 1.4 \times 10^{-5}$, the uncertainty of the most accurate value of G currently available.

(vii) A measurement of a transition frequency in hydrogen or deuterium, other than the already well-known hydrogen $1\mathrm{S}_{1/2} - 2\mathrm{S}_{1/2}$ frequency, with an uncertainty within an order of magnitude of the current uncertainty of that frequency, $u_r = 1.4 \times 10^{-14}$, thereby providing an improved value of the Rydberg constant R_∞.

(viii) Improved theory of the principal hadronic contribution to the theoretical expression for the muon magnetic moment anomaly $a_\mu(\mathrm{th})$ and improvements in the experimental data underlying the calculation of this contribution so that the origin of the current disagreement between $a_\mu(\mathrm{th})$ and $a_\mu(\mathrm{exp})$ can be better understood.

(ix) Although there is no experimental or theoretical evidence that the relations $K_J = 2e/h$ and $R_K = h/e^2$ are not exact, improved calculable-capacitor measurements of R_K and low-field measurements of the gyromagnetic ratios of the shielded proton and shielded helion, which could provide further tests of the exactness of these relations, would not be unwelcome, nor would high accuracy results ($u_r \approx 10^{-8}$) from experiments to close the "quantum electrical triangle" (Drake and Grigorescu, 2005; Piquemal *et al.*, 2007).

It will be most interesting to see what portion, if any, of this very ambitious program of work is completed by the 31 December 2010 closing date of the next CODATA adjustment of the values of the constants. Indeed, the progress made, especially in meeting needs (i)-(iii), may very likely determine whether the 24th CGPM, which convenes in October 2011, will approve new definitions of the kilogram, ampere, kelvin, and mole as discussed in the previous section. If such new definitions are adopted, h, e, k, and N_A as well as a number of other fundamental constants, for example, K_J, R_K (assuming $K_J = 2e/h$ and $R_K = h/e^2$), R, and σ, would be exactly known, and many others would have significantly reduced uncertainties. The result would be a significant advance in our knowledge of the values of the constants.

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TABLE 49 An abbreviated list of the CODATA recommended values of the fundamental constants of physics and chemistry based on the 2006 adjustment.

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. u_r
speed of light in vacuum	c, c_0	299 792 458	m s^{-1}	(exact)
magnetic constant	μ_0	$4\pi \times 10^{-7}$	N A^{-2}	
		$= 12.566\,370\,614... \times 10^{-7}$	N A^{-2}	(exact)
electric constant $1/\mu_0 c^2$	ϵ_0	$8.854\,187\,817... \times 10^{-12}$	F m^{-1}	(exact)
Newtonian constant of gravitation	G	$6.674\,28(67) \times 10^{-11}$	m^3 kg^{-1} s^{-2}	1.0×10^{-4}
Planck constant	h	$6.626\,068\,96(33) \times 10^{-34}$	J s	5.0×10^{-8}
$h/2\pi$	$\hbar$	$1.054\,571\,628(53) \times 10^{-34}$	J s	5.0×10^{-8}
elementary charge	e	$1.602\,176\,487(40) \times 10^{-19}$	C	2.5×10^{-8}
magnetic flux quantum $h/2e$	Φ_0	$2.067\,833\,667(52) \times 10^{-15}$	Wb	2.5×10^{-8}
conductance quantum $2e^2/h$	G_0	$7.748\,091\,7004(53) \times 10^{-5}$	S	6.8×10^{-10}
electron mass	m_e	$9.109\,382\,15(45) \times 10^{-31}$	kg	5.0×10^{-8}
proton mass	m_p	$1.672\,621\,637(83) \times 10^{-27}$	kg	5.0×10^{-8}
proton-electron mass ratio	m_p/m_e	1836.152 672 47(80)		4.3×10^{-10}
fine-structure constant $e^2/4\pi\epsilon_0\hbar c$	α	$7.297\,352\,5376(50) \times 10^{-3}$		6.8×10^{-10}
inverse fine-structure constant	α^{-1}	137.035 999 679(94)		6.8×10^{-10}
Rydberg constant $\alpha^2 m_e c/2h$	R_∞	10 973 731.568 527(73)	m^{-1}	6.6×10^{-12}
Avogadro constant	N_A, L	$6.022\,141\,79(30) \times 10^{23}$	mol^{-1}	5.0×10^{-8}
Faraday constant $N_A e$	F	96 485.3399(24)	C mol^{-1}	2.5×10^{-8}
molar gas constant	R	8.314 472(15)	J mol^{-1} K^{-1}	1.7×10^{-6}
Boltzmann constant R/N_A	k	$1.380\,6504(24) \times 10^{-23}$	J K^{-1}	1.7×10^{-6}
Stefan-Boltzmann constant $(\pi^2/60)k^4/\hbar^3 c^2$	σ	$5.670\,400(40) \times 10^{-8}$	W m^{-2} K^{-4}	7.0×10^{-6}
Non-SI units accepted for use with the SI				
electron volt: (e/C) J	eV	$1.602\,176\,487(40) \times 10^{-19}$	J	2.5×10^{-8}
(unified) atomic mass unit $1\ \mathrm{u} = m_u = \frac{1}{12}m(^{12}\mathrm{C}) = 10^{-3}$ kg mol$^{-1}/N_A$	u	$1.660\,538\,782(83) \times 10^{-27}$	kg	5.0×10^{-8}

TABLE 50: The CODATA recommended values of the fundamental constants of physics and chemistry based on the 2006 adjustment.

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. u_r
		UNIVERSAL		
speed of light in vacuum	c, c_0	$299\,792\,458$	m s^{-1}	(exact)
magnetic constant	μ_0	$4\pi \times 10^{-7}$	N A^{-2}	
		$= 12.566\,370\,614... \times 10^{-7}$	N A^{-2}	(exact)
electric constant $1/\mu_0 c^2$	ϵ_0	$8.854\,187\,817... \times 10^{-12}$	F m^{-1}	(exact)
characteristic impedance of vacuum $\sqrt{\mu_0/\epsilon_0} = \mu_0 c$	Z_0	$376.730\,313\,461...$	Ω	(exact)
Newtonian constant of gravitation	G	$6.674\,28(67) \times 10^{-11}$	m^3 kg^{-1} s^{-2}	1.0×10^{-4}
	$G/\hbar c$	$6.708\,81(67) \times 10^{-39}$	$(\mathrm{GeV}/c^2)^{-2}$	1.0×10^{-4}
Planck constant	h	$6.626\,068\,96(33) \times 10^{-34}$	J s	5.0×10^{-8}
in eV s		$4.135\,667\,33(10) \times 10^{-15}$	eV s	2.5×10^{-8}
$h/2\pi$	$\hbar$	$1.054\,571\,628(53) \times 10^{-34}$	J s	5.0×10^{-8}
in eV s		$6.582\,118\,99(16) \times 10^{-16}$	eV s	2.5×10^{-8}
$\hbar c$ in MeV fm		$197.326\,9631(49)$	MeV fm	2.5×10^{-8}
Planck mass $(\hbar c/G)^{1/2}$	m_{P}	$2.176\,44(11) \times 10^{-8}$	kg	5.0×10^{-5}
energy equivalent in GeV	$m_{\mathrm{P}}c^2$	$1.220\,892(61) \times 10^{19}$	GeV	5.0×10^{-5}
Planck temperature $(\hbar c^5/G)^{1/2}/k$	T_{P}	$1.416\,785(71) \times 10^{32}$	K	5.0×10^{-5}
Planck length $\hbar/m_{\mathrm{P}}c = (\hbar G/c^3)^{1/2}$	l_{P}	$1.616\,252(81) \times 10^{-35}$	m	5.0×10^{-5}
Planck time $l_{\mathrm{P}}/c = (\hbar G/c^5)^{1/2}$	t_{P}	$5.391\,24(27) \times 10^{-44}$	s	5.0×10^{-5}
		ELECTROMAGNETIC		
elementary charge	e	$1.602\,176\,487(40) \times 10^{-19}$	C	2.5×10^{-8}
	e/h	$2.417\,989\,454(60) \times 10^{14}$	A J^{-1}	2.5×10^{-8}
magnetic flux quantum $h/2e$	Φ_0	$2.067\,833\,667(52) \times 10^{-15}$	Wb	2.5×10^{-8}
conductance quantum $2e^2/h$	G_0	$7.748\,091\,7004(53) \times 10^{-5}$	S	6.8×10^{-10}
inverse of conductance quantum	G_0^{-1}	$12\,906.403\,7787(88)$	Ω	6.8×10^{-10}
Josephson constant[a] $2e/h$	K_{J}	$483\,597.891(12) \times 10^9$	Hz V^{-1}	2.5×10^{-8}
von Klitzing constant[b] $h/e^2 = \mu_0 c/2\alpha$	R_{K}	$25\,812.807\,557(18)$	Ω	6.8×10^{-10}
Bohr magneton $e\hbar/2m_{\mathrm{e}}$	μ_{B}	$927.400\,915(23) \times 10^{-26}$	J T^{-1}	2.5×10^{-8}
in eV T^{-1}		$5.788\,381\,7555(79) \times 10^{-5}$	eV T^{-1}	1.4×10^{-9}
	μ_{B}/h	$13.996\,246\,04(35) \times 10^9$	Hz T^{-1}	2.5×10^{-8}
	μ_{B}/hc	$46.686\,4515(12)$	m^{-1} T^{-1}	2.5×10^{-8}
	μ_{B}/k	$0.671\,7131(12)$	K T^{-1}	1.7×10^{-6}
nuclear magneton $e\hbar/2m_{\mathrm{p}}$	μ_{N}	$5.050\,783\,24(13) \times 10^{-27}$	J T^{-1}	2.5×10^{-8}
in eV T^{-1}		$3.152\,451\,2326(45) \times 10^{-8}$	eV T^{-1}	1.4×10^{-9}
	μ_{N}/h	$7.622\,593\,84(19)$	MHz T^{-1}	2.5×10^{-8}
	μ_{N}/hc	$2.542\,623\,616(64) \times 10^{-2}$	m^{-1} T^{-1}	2.5×10^{-8}
	μ_{N}/k	$3.658\,2637(64) \times 10^{-4}$	K T^{-1}	1.7×10^{-6}
		ATOMIC AND NUCLEAR		
		General		
fine-structure constant $e^2/4\pi\epsilon_0\hbar c$	α	$7.297\,352\,5376(50) \times 10^{-3}$		6.8×10^{-10}
inverse fine-structure constant	α^{-1}	$137.035\,999\,679(94)$		6.8×10^{-10}
Rydberg constant $\alpha^2 m_{\mathrm{e}}c/2h$	R_∞	$10\,973\,731.568\,527(73)$	m^{-1}	6.6×10^{-12}

[a] See Table 52 for the conventional value adopted internationally for realizing representations of the volt using the Josephson effect.
[b] See Table 52 for the conventional value adopted internationally for realizing representations of the ohm using the quantum Hall effect.

TABLE 50: *(Continued).*

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. u_r
	$R_\infty c$	$3.289\,841\,960\,361(22) \times 10^{15}$	Hz	6.6×10^{-12}
	$R_\infty hc$	$2.179\,871\,97(11) \times 10^{-18}$	J	5.0×10^{-8}
$R_\infty hc$ in eV		$13.605\,691\,93(34)$	eV	2.5×10^{-8}
Bohr radius $\alpha/4\pi R_\infty = 4\pi\epsilon_0\hbar^2/m_e e^2$	a_0	$0.529\,177\,208\,59(36) \times 10^{-10}$	m	6.8×10^{-10}
Hartree energy $e^2/4\pi\epsilon_0 a_0 = 2R_\infty hc$ $= \alpha^2 m_e c^2$	E_h	$4.359\,743\,94(22) \times 10^{-18}$	J	5.0×10^{-8}
in eV		$27.211\,383\,86(68)$	eV	2.5×10^{-8}
quantum of circulation	$h/2m_e$	$3.636\,947\,5199(50) \times 10^{-4}$	$m^2\ s^{-1}$	1.4×10^{-9}
	h/m_e	$7.273\,895\,040(10) \times 10^{-4}$	$m^2\ s^{-1}$	1.4×10^{-9}
	Electroweak			
Fermi coupling constant[c]	$G_F/(\hbar c)^3$	$1.166\,37(1) \times 10^{-5}$	GeV^{-2}	8.6×10^{-6}
weak mixing angle[d] θ_W (on-shell scheme) $\sin^2\theta_W = s_W^2 \equiv 1 - (m_W/m_Z)^2$	$\sin^2\theta_W$	$0.222\,55(56)$		2.5×10^{-3}
	Electron, e^-			
electron mass	m_e	$9.109\,382\,15(45) \times 10^{-31}$	kg	5.0×10^{-8}
in u, $m_e = A_r(e)$ u (electron relative atomic mass times u)		$5.485\,799\,0943(23) \times 10^{-4}$	u	4.2×10^{-10}
energy equivalent	$m_e c^2$	$8.187\,104\,38(41) \times 10^{-14}$	J	5.0×10^{-8}
in MeV		$0.510\,998\,910(13)$	MeV	2.5×10^{-8}
electron-muon mass ratio	m_e/m_μ	$4.836\,331\,71(12) \times 10^{-3}$		2.5×10^{-8}
electron-tau mass ratio	m_e/m_τ	$2.875\,64(47) \times 10^{-4}$		1.6×10^{-4}
electron-proton mass ratio	m_e/m_p	$5.446\,170\,2177(24) \times 10^{-4}$		4.3×10^{-10}
electron-neutron mass ratio	m_e/m_n	$5.438\,673\,4459(33) \times 10^{-4}$		6.0×10^{-10}
electron-deuteron mass ratio	m_e/m_d	$2.724\,437\,1093(12) \times 10^{-4}$		4.3×10^{-10}
electron to alpha particle mass ratio	m_e/m_α	$1.370\,933\,555\,70(58) \times 10^{-4}$		4.2×10^{-10}
electron charge to mass quotient	$-e/m_e$	$-1.758\,820\,150(44) \times 10^{11}$	$C\ kg^{-1}$	2.5×10^{-8}
electron molar mass $N_A m_e$	$M(e), M_e$	$5.485\,799\,0943(23) \times 10^{-7}$	$kg\ mol^{-1}$	4.2×10^{-10}
Compton wavelength $h/m_e c$	λ_C	$2.426\,310\,2175(33) \times 10^{-12}$	m	1.4×10^{-9}
$\lambda_C/2\pi = \alpha a_0 = \alpha^2/4\pi R_\infty$	$\bar{\lambda}_C$	$386.159\,264\,59(53) \times 10^{-15}$	m	1.4×10^{-9}
classical electron radius $\alpha^2 a_0$	r_e	$2.817\,940\,2894(58) \times 10^{-15}$	m	2.1×10^{-9}
Thomson cross section $(8\pi/3)r_e^2$	σ_e	$0.665\,245\,8558(27) \times 10^{-28}$	m^2	4.1×10^{-9}
electron magnetic moment	μ_e	$-928.476\,377(23) \times 10^{-26}$	$J\ T^{-1}$	2.5×10^{-8}
to Bohr magneton ratio	μ_e/μ_B	$-1.001\,159\,652\,181\,11(74)$		7.4×10^{-13}
to nuclear magneton ratio	μ_e/μ_N	$-1838.281\,970\,92(80)$		4.3×10^{-10}
electron magnetic moment anomaly $\lvert\mu_e\rvert/\mu_B - 1$	a_e	$1.159\,652\,181\,11(74) \times 10^{-3}$		6.4×10^{-10}
electron g-factor $-2(1 + a_e)$	g_e	$-2.002\,319\,304\,3622(15)$		7.4×10^{-13}
electron-muon magnetic moment ratio	μ_e/μ_μ	$206.766\,9877(52)$		2.5×10^{-8}
electron-proton magnetic moment ratio	μ_e/μ_p	$-658.210\,6848(54)$		8.1×10^{-9}
electron to shielded proton magnetic moment ratio (H_2O, sphere, 25 °C)	μ_e/μ'_p	$-658.227\,5971(72)$		1.1×10^{-8}

[c] Value recommended by the Particle Data Group (Yao *et al.*, 2006).
[d] Based on the ratio of the masses of the W and Z bosons m_W/m_Z recommended by the Particle Data Group (Yao *et al.*, 2006). The value for $\sin^2\theta_W$ they recommend, which is based on a particular variant of the modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme, is $\sin^2\hat{\theta}_W(M_Z) = 0.231\,22(15)$.

TABLE 50: *(Continued).*

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. u_r
electron-neutron magnetic moment ratio	μ_e/μ_n	960.920 50(23)		2.4×10^{-7}
electron-deuteron magnetic moment ratio	μ_e/μ_d	−2143.923 498(18)		8.4×10^{-9}
electron to shielded helion magnetic moment ratio (gas, sphere, 25 °C)	μ_e/μ'_h	864.058 257(10)		1.2×10^{-8}
electron gyromagnetic ratio $2\|\mu_e\|/\hbar$	γ_e	$1.760\,859\,770(44) \times 10^{11}$	$s^{-1}\ T^{-1}$	2.5×10^{-8}
	$\gamma_e/2\pi$	28 024.953 64(70)	MHz T^{-1}	2.5×10^{-8}
		Muon, μ^-		
muon mass	m_μ	$1.883\,531\,30(11) \times 10^{-28}$	kg	5.6×10^{-8}
in u, $m_\mu = A_r(\mu)$ u (muon relative atomic mass times u)		0.113 428 9256(29)	u	2.5×10^{-8}
energy equivalent	$m_\mu c^2$	$1.692\,833\,510(95) \times 10^{-11}$	J	5.6×10^{-8}
in MeV		105.658 3668(38)	MeV	3.6×10^{-8}
muon-electron mass ratio	m_μ/m_e	206.768 2823(52)		2.5×10^{-8}
muon-tau mass ratio	m_μ/m_τ	$5.945\,92(97) \times 10^{-2}$		1.6×10^{-4}
muon-proton mass ratio	m_μ/m_p	0.112 609 5261(29)		2.5×10^{-8}
muon-neutron mass ratio	m_μ/m_n	0.112 454 5167(29)		2.5×10^{-8}
muon molar mass $N_A m_\mu$	$M(\mu), M_\mu$	$0.113\,428\,9256(29) \times 10^{-3}$	kg mol^{-1}	2.5×10^{-8}
muon Compton wavelength $h/m_\mu c$	$\lambda_{C,\mu}$	$11.734\,441\,04(30) \times 10^{-15}$	m	2.5×10^{-8}
$\lambda_{C,\mu}/2\pi$	$\bar{\lambda}_{C,\mu}$	$1.867\,594\,295(47) \times 10^{-15}$	m	2.5×10^{-8}
muon magnetic moment	μ_μ	$-4.490\,447\,86(16) \times 10^{-26}$	J T^{-1}	3.6×10^{-8}
to Bohr magneton ratio	μ_μ/μ_B	$-4.841\,970\,49(12) \times 10^{-3}$		2.5×10^{-8}
to nuclear magneton ratio	μ_μ/μ_N	−8.890 597 05(23)		2.5×10^{-8}
muon magnetic moment anomaly $\|\mu_\mu\|/(e\hbar/2m_\mu) - 1$	a_μ	$1.165\,920\,69(60) \times 10^{-3}$		5.2×10^{-7}
muon g-factor $-2(1 + a_\mu)$	g_μ	−2.002 331 8414(12)		6.0×10^{-10}
muon-proton magnetic moment ratio	μ_μ/μ_p	−3.183 345 137(85)		2.7×10^{-8}
		Tau, τ^-		
tau mass[e]	m_τ	$3.167\,77(52) \times 10^{-27}$	kg	1.6×10^{-4}
in u, $m_\tau = A_r(\tau)$ u (tau relative atomic mass times u)		1.907 68(31)	u	1.6×10^{-4}
energy equivalent	$m_\tau c^2$	$2.847\,05(46) \times 10^{-10}$	J	1.6×10^{-4}
in MeV		1776.99(29)	MeV	1.6×10^{-4}
tau-electron mass ratio	m_τ/m_e	3477.48(57)		1.6×10^{-4}
tau-muon mass ratio	m_τ/m_μ	16.8183(27)		1.6×10^{-4}
tau-proton mass ratio	m_τ/m_p	1.893 90(31)		1.6×10^{-4}
tau-neutron mass ratio	m_τ/m_n	1.891 29(31)		1.6×10^{-4}
tau molar mass $N_A m_\tau$	$M(\tau), M_\tau$	$1.907\,68(31) \times 10^{-3}$	kg mol^{-1}	1.6×10^{-4}
tau Compton wavelength $h/m_\tau c$	$\lambda_{C,\tau}$	$0.697\,72(11) \times 10^{-15}$	m	1.6×10^{-4}
$\lambda_{C,\tau}/2\pi$	$\bar{\lambda}_{C,\tau}$	$0.111\,046(18) \times 10^{-15}$	m	1.6×10^{-4}
		Proton, p		
proton mass	m_p	$1.672\,621\,637(83) \times 10^{-27}$	kg	5.0×10^{-8}
in u, $m_p = A_r(p)$ u (proton				

[e] This and all other values involving m_τ are based on the value of $m_\tau c^2$ in MeV recommended by the Particle Data Group (Yao *et al.*, 2006), but with a standard uncertainty of 0.29 MeV rather than the quoted uncertainty of −0.26 MeV, +0.29 MeV.

TABLE 50: *(Continued).*

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. u_r
relative atomic mass times u)		1.007 276 466 77(10)	u	1.0×10^{-10}
energy equivalent	$m_p c^2$	$1.503\,277\,359(75) \times 10^{-10}$	J	5.0×10^{-8}
in MeV		938.272 013(23)	MeV	2.5×10^{-8}
proton-electron mass ratio	m_p/m_e	1836.152 672 47(80)		4.3×10^{-10}
proton-muon mass ratio	m_p/m_μ	8.880 243 39(23)		2.5×10^{-8}
proton-tau mass ratio	m_p/m_τ	0.528 012(86)		1.6×10^{-4}
proton-neutron mass ratio	m_p/m_n	0.998 623 478 24(46)		4.6×10^{-10}
proton charge to mass quotient	e/m_p	$9.578\,833\,92(24) \times 10^{7}$	C kg^{-1}	2.5×10^{-8}
proton molar mass $N_A m_p$	$M(\text{p})$, M_p	$1.007\,276\,466\,77(10) \times 10^{-3}$	kg mol^{-1}	1.0×10^{-10}
proton Compton wavelength $h/m_p c$	$\lambda_{C,p}$	$1.321\,409\,8446(19) \times 10^{-15}$	m	1.4×10^{-9}
$\lambda_{C,p}/2\pi$	$\bar{\lambda}_{C,p}$	$0.210\,308\,908\,61(30) \times 10^{-15}$	m	1.4×10^{-9}
proton rms charge radius	R_p	$0.8768(69) \times 10^{-15}$	m	7.8×10^{-3}
proton magnetic moment	μ_p	$1.410\,606\,662(37) \times 10^{-26}$	J T^{-1}	2.6×10^{-8}
to Bohr magneton ratio	μ_p/μ_B	$1.521\,032\,209(12) \times 10^{-3}$		8.1×10^{-9}
to nuclear magneton ratio	μ_p/μ_N	2.792 847 356(23)		8.2×10^{-9}
proton g-factor $2\mu_p/\mu_N$	g_p	5.585 694 713(46)		8.2×10^{-9}
proton-neutron magnetic moment ratio	μ_p/μ_n	$-1.459\,898\,06(34)$		2.4×10^{-7}
shielded proton magnetic moment (H_2O, sphere, 25 °C)	μ'_p	$1.410\,570\,419(38) \times 10^{-26}$	J T^{-1}	2.7×10^{-8}
to Bohr magneton ratio	μ'_p/μ_B	$1.520\,993\,128(17) \times 10^{-3}$		1.1×10^{-8}
to nuclear magneton ratio	μ'_p/μ_N	2.792 775 598(30)		1.1×10^{-8}
proton magnetic shielding correction $1 - \mu'_p/\mu_p$ (H_2O, sphere, 25 °C)	σ'_p	$25.694(14) \times 10^{-6}$		5.3×10^{-4}
proton gyromagnetic ratio $2\mu_p/\hbar$	γ_p	$2.675\,222\,099(70) \times 10^{8}$	s^{-1} T^{-1}	2.6×10^{-8}
	$\gamma_p/2\pi$	42.577 4821(11)	MHz T^{-1}	2.6×10^{-8}
shielded proton gyromagnetic ratio $2\mu'_p/\hbar$ (H_2O, sphere, 25 °C)	γ'_p	$2.675\,153\,362(73) \times 10^{8}$	s^{-1} T^{-1}	2.7×10^{-8}
	$\gamma'_p/2\pi$	42.576 3881(12)	MHz T^{-1}	2.7×10^{-8}
		Neutron, n		
neutron mass	m_n	$1.674\,927\,211(84) \times 10^{-27}$	kg	5.0×10^{-8}
in u, $m_n = A_r(\text{n})$ u (neutron relative atomic mass times u)		1.008 664 915 97(43)	u	4.3×10^{-10}
energy equivalent	$m_n c^2$	$1.505\,349\,505(75) \times 10^{-10}$	J	5.0×10^{-8}
in MeV		939.565 346(23)	MeV	2.5×10^{-8}
neutron-electron mass ratio	m_n/m_e	1838.683 6605(11)		6.0×10^{-10}
neutron-muon mass ratio	m_n/m_μ	8.892 484 09(23)		2.5×10^{-8}
neutron-tau mass ratio	m_n/m_τ	0.528 740(86)		1.6×10^{-4}
neutron-proton mass ratio	m_n/m_p	1.001 378 419 18(46)		4.6×10^{-10}
neutron molar mass $N_A m_n$	$M(\text{n})$, M_n	$1.008\,664\,915\,97(43) \times 10^{-3}$	kg mol^{-1}	4.3×10^{-10}
neutron Compton wavelength $h/m_n c$	$\lambda_{C,n}$	$1.319\,590\,8951(20) \times 10^{-15}$	m	1.5×10^{-9}
$\lambda_{C,n}/2\pi$	$\bar{\lambda}_{C,n}$	$0.210\,019\,413\,82(31) \times 10^{-15}$	m	1.5×10^{-9}
neutron magnetic moment	μ_n	$-0.966\,236\,41(23) \times 10^{-26}$	J T^{-1}	2.4×10^{-7}
to Bohr magneton ratio	μ_n/μ_B	$-1.041\,875\,63(25) \times 10^{-3}$		2.4×10^{-7}
to nuclear magneton ratio	μ_n/μ_N	$-1.913\,042\,73(45)$		2.4×10^{-7}
neutron g-factor $2\mu_n/\mu_N$	g_n	$-3.826\,085\,45(90)$		2.4×10^{-7}
neutron-electron magnetic moment ratio	μ_n/μ_e	$1.040\,668\,82(25) \times 10^{-3}$		2.4×10^{-7}

TABLE 50: *(Continued).*

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. u_r
neutron-proton magnetic moment ratio	μ_n/μ_p	$-0.684\,979\,34(16)$		2.4×10^{-7}
neutron to shielded proton magnetic moment ratio (H_2O, sphere, 25 °C)	μ_n/μ'_p	$-0.684\,996\,94(16)$		2.4×10^{-7}
neutron gyromagnetic ratio $2\lvert\mu_n\rvert/\hbar$	γ_n	$1.832\,471\,85(43)\times10^{8}$	$s^{-1}\ T^{-1}$	2.4×10^{-7}
	$\gamma_n/2\pi$	$29.164\,6954(69)$	MHz T^{-1}	2.4×10^{-7}
		Deuteron, d		
deuteron mass	m_d	$3.343\,583\,20(17)\times10^{-27}$	kg	5.0×10^{-8}
in u, $m_d = A_r(d)$ u (deuteron relative atomic mass times u)		$2.013\,553\,212\,724(78)$	u	3.9×10^{-11}
energy equivalent	m_dc^2	$3.005\,062\,72(15)\times10^{-10}$	J	5.0×10^{-8}
in MeV		$1875.612\,793(47)$	MeV	2.5×10^{-8}
deuteron-electron mass ratio	m_d/m_e	$3670.482\,9654(16)$		4.3×10^{-10}
deuteron-proton mass ratio	m_d/m_p	$1.999\,007\,501\,08(22)$		1.1×10^{-10}
deuteron molar mass N_Am_d	$M(d), M_d$	$2.013\,553\,212\,724(78)\times10^{-3}$	kg mol^{-1}	3.9×10^{-11}
deuteron rms charge radius	R_d	$2.1402(28)\times10^{-15}$	m	1.3×10^{-3}
deuteron magnetic moment	μ_d	$0.433\,073\,465(11)\times10^{-26}$	J T^{-1}	2.6×10^{-8}
to Bohr magneton ratio	μ_d/μ_B	$0.466\,975\,4556(39)\times10^{-3}$		8.4×10^{-9}
to nuclear magneton ratio	μ_d/μ_N	$0.857\,438\,2308(72)$		8.4×10^{-9}
deuteron g-factor μ_d/μ_N	g_d	$0.857\,438\,2308(72)$		8.4×10^{-9}
deuteron-electron magnetic moment ratio	μ_d/μ_e	$-4.664\,345\,537(39)\times10^{-4}$		8.4×10^{-9}
deuteron-proton magnetic moment ratio	μ_d/μ_p	$0.307\,012\,2070(24)$		7.7×10^{-9}
deuteron-neutron magnetic moment ratio	μ_d/μ_n	$-0.448\,206\,52(11)$		2.4×10^{-7}
		Triton, t		
triton mass	m_t	$5.007\,355\,88(25)\times10^{-27}$	kg	5.0×10^{-8}
in u, $m_t = A_r(t)$ u (triton relative atomic mass times u)		$3.015\,500\,7134(25)$	u	8.3×10^{-10}
energy equivalent	m_tc^2	$4.500\,387\,03(22)\times10^{-10}$	J	5.0×10^{-8}
in MeV		$2808.920\,906(70)$	MeV	2.5×10^{-8}
triton-electron mass ratio	m_t/m_e	$5496.921\,5269(51)$		9.3×10^{-10}
triton-proton mass ratio	m_t/m_p	$2.993\,717\,0309(25)$		8.4×10^{-10}
triton molar mass N_Am_t	$M(t), M_t$	$3.015\,500\,7134(25)\times10^{-3}$	kg mol^{-1}	8.3×10^{-10}
triton magnetic moment	μ_t	$1.504\,609\,361(42)\times10^{-26}$	J T^{-1}	2.8×10^{-8}
to Bohr magneton ratio	μ_t/μ_B	$1.622\,393\,657(21)\times10^{-3}$		1.3×10^{-8}
to nuclear magneton ratio	μ_t/μ_N	$2.978\,962\,448(38)$		1.3×10^{-8}
triton g-factor $2\mu_t/\mu_N$	g_t	$5.957\,924\,896(76)$		1.3×10^{-8}
triton-electron magnetic moment ratio	μ_t/μ_e	$-1.620\,514\,423(21)\times10^{-3}$		1.3×10^{-8}
triton-proton magnetic moment ratio	μ_t/μ_p	$1.066\,639\,908(10)$		9.8×10^{-9}
triton-neutron magnetic moment ratio	μ_t/μ_n	$-1.557\,185\,53(37)$		2.4×10^{-7}
		Helion, h		
helion mass[e]	m_h	$5.006\,411\,92(25)\times10^{-27}$	kg	5.0×10^{-8}
in u, $m_h = A_r(h)$ u (helion relative atomic mass times u)		$3.014\,932\,2473(26)$	u	8.6×10^{-10}

TABLE 50: *(Continued)*.

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. u_r
energy equivalent	$m_h c^2$	$4.499\,538\,64(22) \times 10^{-10}$	J	5.0×10^{-8}
in MeV		2808.391 383(70)	MeV	2.5×10^{-8}
helion-electron mass ratio	m_h/m_e	5495.885 2765(52)		9.5×10^{-10}
helion-proton mass ratio	m_h/m_p	2.993 152 6713(26)		8.7×10^{-10}
helion molar mass $N_A m_h$	$M(h), M_h$	$3.014\,932\,2473(26) \times 10^{-3}$	kg mol^{-1}	8.6×10^{-10}
shielded helion magnetic moment (gas, sphere, 25 °C)	μ'_h	$-1.074\,552\,982(30) \times 10^{-26}$	J T^{-1}	2.8×10^{-8}
to Bohr magneton ratio	μ'_h/μ_B	$-1.158\,671\,471(14) \times 10^{-3}$		1.2×10^{-8}
to nuclear magneton ratio	μ'_h/μ_N	$-2.127\,497\,718(25)$		1.2×10^{-8}
shielded helion to proton magnetic moment ratio (gas, sphere, 25 °C)	μ'_h/μ_p	$-0.761\,766\,558(11)$		1.4×10^{-8}
shielded helion to shielded proton magnetic moment ratio (gas/H_2O, spheres, 25 °C)	μ'_h/μ'_p	$-0.761\,786\,1313(33)$		4.3×10^{-9}
shielded helion gyromagnetic ratio $2\lvert\mu'_h\rvert/\hbar$ (gas, sphere, 25 °C)	γ'_h	$2.037\,894\,730(56) \times 10^{8}$	s^{-1} T^{-1}	2.8×10^{-8}
	$\gamma'_h/2\pi$	32.434 101 98(90)	MHz T^{-1}	2.8×10^{-8}
	Alpha particle, α			
alpha particle mass	m_α	$6.644\,656\,20(33) \times 10^{-27}$	kg	5.0×10^{-8}
in u, $m_\alpha = A_r(\alpha)$ u (alpha particle relative atomic mass times u)		4.001 506 179 127(62)	u	1.5×10^{-11}
energy equivalent	$m_\alpha c^2$	$5.971\,919\,17(30) \times 10^{-10}$	J	5.0×10^{-8}
in MeV		3727.379 109(93)	MeV	2.5×10^{-8}
alpha particle to electron mass ratio	m_α/m_e	7294.299 5365(31)		4.2×10^{-10}
alpha particle to proton mass ratio	m_α/m_p	3.972 599 689 51(41)		1.0×10^{-10}
alpha particle molar mass $N_A m_\alpha$	$M(\alpha), M_\alpha$	$4.001\,506\,179\,127(62) \times 10^{-3}$	kg mol^{-1}	1.5×10^{-11}
	PHYSICOCHEMICAL			
Avogadro constant	N_A, L	$6.022\,141\,79(30) \times 10^{23}$	mol^{-1}	5.0×10^{-8}
atomic mass constant $m_u = \frac{1}{12}m(^{12}C) = 1$ u $= 10^{-3}$ kg mol$^{-1}/N_A$	m_u	$1.660\,538\,782(83) \times 10^{-27}$	kg	5.0×10^{-8}
energy equivalent	$m_u c^2$	$1.492\,417\,830(74) \times 10^{-10}$	J	5.0×10^{-8}
in MeV		931.494 028(23)	MeV	2.5×10^{-8}
Faraday constant[f] $N_A e$	F	96 485.3399(24)	C mol^{-1}	2.5×10^{-8}
molar Planck constant	$N_A h$	$3.990\,312\,6821(57) \times 10^{-10}$	J s mol^{-1}	1.4×10^{-9}
	$N_A hc$	0.119 626 564 72(17)	J m mol^{-1}	1.4×10^{-9}
molar gas constant	R	8.314 472(15)	J mol^{-1} K^{-1}	1.7×10^{-6}
Boltzmann constant R/N_A	k	$1.380\,6504(24) \times 10^{-23}$	J K^{-1}	1.7×10^{-6}
in eV K^{-1}		$8.617\,343(15) \times 10^{-5}$	eV K^{-1}	1.7×10^{-6}
	k/h	$2.083\,6644(36) \times 10^{10}$	Hz K^{-1}	1.7×10^{-6}
	k/hc	69.503 56(12)	m^{-1} K^{-1}	1.7×10^{-6}
molar volume of ideal gas RT/p $T = 273.15$ K, $p = 101.325$ kPa	V_m	$22.413\,996(39) \times 10^{-3}$	m^3 mol^{-1}	1.7×10^{-6}

[f] The numerical value of F to be used in coulometric chemical measurements is 96 485.3401(48) [5.0×10^{-8}] when the relevant current is measured in terms of representations of the volt and ohm based on the Josephson and quantum Hall effects and the internationally adopted conventional values of the Josephson and von Klitzing constants K_{J-90} and R_{K-90} given in Table 52.

TABLE 50: *(Continued).*

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. $u_{\rm r}$
Loschmidt constant $N_{\rm A}/V_{\rm m}$	n_0	$2.686\,7774(47) \times 10^{25}$	$\rm m^{-3}$	1.7×10^{-6}
$T = 273.15$ K, $p = 100$ kPa	$V_{\rm m}$	$22.710\,981(40) \times 10^{-3}$	$\rm m^3\ mol^{-1}$	1.7×10^{-6}
Sackur-Tetrode constant				
(absolute entropy constant)[g]				
$\frac{5}{2} + \ln[(2\pi m_{\rm u} kT_1/h^2)^{3/2} kT_1/p_0]$				
$T_1 = 1$ K, $p_0 = 100$ kPa	S_0/R	$-1.151\,7047(44)$		3.8×10^{-6}
$T_1 = 1$ K, $p_0 = 101.325$ kPa		$-1.164\,8677(44)$		3.8×10^{-6}
Stefan-Boltzmann constant				
$(\pi^2/60)k^4/\hbar^3 c^2$	σ	$5.670\,400(40) \times 10^{-8}$	$\rm W\ m^{-2}\ K^{-4}$	7.0×10^{-6}
first radiation constant $2\pi hc^2$	c_1	$3.741\,771\,18(19) \times 10^{-16}$	$\rm W\ m^2$	5.0×10^{-8}
first radiation constant for spectral radiance $2hc^2$	$c_{\rm 1L}$	$1.191\,042\,759(59) \times 10^{-16}$	$\rm W\ m^2\ sr^{-1}$	5.0×10^{-8}
second radiation constant hc/k	c_2	$1.438\,7752(25) \times 10^{-2}$	m K	1.7×10^{-6}
Wien displacement law constants				
$b = \lambda_{\rm max} T = c_2/4.965\,114\,231...$	b	$2.897\,7685(51) \times 10^{-3}$	m K	1.7×10^{-6}
$b' = \nu_{\rm max}/T = 2.821\,439\,372...\,c/c_2$	b'	$5.878\,933(10) \times 10^{10}$	$\rm Hz\ K^{-1}$	1.7×10^{-6}

[g] The entropy of an ideal monoatomic gas of relative atomic mass $A_{\rm r}$ is given by $S = S_0 + \frac{3}{2}R \ln A_{\rm r} - R \ln(p/p_0) + \frac{5}{2}R \ln(T/{\rm K})$.

TABLE 51 The variances, covariances, and correlation coefficients of the values of a selected group of constants based on the 2006 CODATA adjustment. The numbers in bold above the main diagonal are 10^{16} times the numerical values of the relative covariances; the numbers in bold on the main diagonal are 10^{16} times the numerical values of the relative variances; and the numbers in italics below the main diagonal are the correlation coefficients.[a]

	α	h	e	m_e	N_A	m_e/m_μ	F
α	**0.0047**	**0.0002**	**0.0024**	**−0.0092**	**0.0092**	**−0.0092**	**0.0116**
h	*0.0005*	**24.8614**	**12.4308**	**24.8611**	**−24.8610**	**−0.0003**	**−12.4302**
e	*0.0142*	*0.9999*	**6.2166**	**12.4259**	**−12.4259**	**−0.0048**	**−6.2093**
m_e	*−0.0269*	*0.9996*	*0.9992*	**24.8795**	**−24.8794**	**0.0180**	**−12.4535**
N_A	*0.0269*	*−0.9996*	*−0.9991*	*−1.0000*	**24.8811**	**−0.0180**	**12.4552**
m_e/m_μ	*−0.0528*	*0.0000*	*−0.0008*	*0.0014*	*−0.0014*	**6.4296**	**−0.0227**
F	*0.0679*	*−0.9975*	*−0.9965*	*−0.9990*	*0.9991*	*−0.0036*	**6.2459**

[a]The relative covariance is $u_r(x_i, x_j) = u(x_i, x_j)/(x_i x_j)$, where $u(x_i, x_j)$ is the covariance of x_i and x_j; the relative variance is $u_r^2(x_i) = u_r(x_i, x_i)$: and the correlation coefficient is $r(x_i, x_j) = u(x_i, x_j)/[u(x_i)u(x_j)]$.

TABLE 52 Internationally adopted values of various quantities.

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. u_r
relative atomic mass[a] of ^{12}C	$A_r(^{12}\mathrm{C})$	12		(exact)
molar mass constant	M_u	1×10^{-3}	kg mol^{-1}	(exact)
molar mass of ^{12}C	$M(^{12}\mathrm{C})$	12×10^{-3}	kg mol^{-1}	(exact)
conventional value of Josephson constant[b]	K_{J-90}	483 597.9	GHz V^{-1}	(exact)
conventional value of von Klitzing constant[c]	R_{K-90}	25 812.807	Ω	(exact)
standard atmosphere		101 325	Pa	(exact)

[a]The relative atomic mass $A_r(X)$ of particle X with mass $m(X)$ is defined by $A_r(X) = m(X)/m_u$, where $m_u = m(^{12}\mathrm{C})/12 = M_u/N_A =$ 1 u is the atomic mass constant, M_u is the molar mass constant, N_A is the Avogadro constant, and u is the unified atomic mass unit. Thus the mass of particle X is $m(X) = A_r(X)$ u and the molar mass of X is $M(X) = A_r(X)M_u$.
[b]This is the value adopted internationally for realizing representations of the volt using the Josephson effect.
[c]This is the value adopted internationally for realizing representations of the ohm using the quantum Hall effect.

TABLE 53 Values of some x-ray-related quantities based on the 2006 CODATA adjustment of the values of the constants.

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. u_r
Cu x unit: $\lambda(\mathrm{CuK}\alpha_1)/1\,537.400$	xu(CuKα_1)	$1.002\,076\,99(28) \times 10^{-13}$	m	2.8×10^{-7}
Mo x unit: $\lambda(\mathrm{MoK}\alpha_1)/707.831$	xu(MoKα_1)	$1.002\,099\,55(53) \times 10^{-13}$	m	5.3×10^{-7}
ångstrom star: $\lambda(\mathrm{WK}\alpha_1)/0.209\,010\,0$	Å*	$1.000\,014\,98(90) \times 10^{-10}$	m	9.0×10^{-7}
lattice parameter[a] of Si (in vacuum, 22.5 °C)	a	$543.102\,064(14) \times 10^{-12}$	m	2.6×10^{-8}
{220} lattice spacing of Si $a/\sqrt{8}$ (in vacuum, 22.5 °C)	d_{220}	$192.015\,5762(50) \times 10^{-12}$	m	2.6×10^{-8}
molar volume of Si $M(\mathrm{Si})/\rho(\mathrm{Si}) = N_A a^3/8$ (in vacuum, 22.5 °C)	$V_m(\mathrm{Si})$	$12.058\,8349(11) \times 10^{-6}$	m^3 mol^{-1}	9.1×10^{-8}

[a]This is the lattice parameter (unit cell edge length) of an ideal single crystal of naturally occurring Si free of impurities and imperfections, and is deduced from measurements on extremely pure and nearly perfect single crystals of Si by correcting for the effects of impurities.

TABLE 54 The values in SI units of some non-SI units based on the 2006 CODATA adjustment of the values of the constants.

Quantity	Symbol	Numerical value	Unit	Relative std. uncert. u_r
		Non-SI units accepted for use with the SI		
electron volt: (e/C) J	eV	$1.602\,176\,487(40) \times 10^{-19}$	J	2.5×10^{-8}
(unified) atomic mass unit:				
1 u $= m_u = \frac{1}{12} m(^{12}\mathrm{C})$ $= 10^{-3}$ kg mol$^{-1}/N_A$	u	$1.660\,538\,782(83) \times 10^{-27}$	kg	5.0×10^{-8}
		Natural units (n.u.)		
n.u. of velocity:				
speed of light in vacuum	c, c_0	299 792 458	m s^{-1}	(exact)
n.u. of action:				
reduced Planck constant ($h/2\pi$)	$\hbar$	$1.054\,571\,628(53) \times 10^{-34}$	J s	5.0×10^{-8}
in eV s		$6.582\,118\,99(16) \times 10^{-16}$	eV s	2.5×10^{-8}
in MeV fm	$\hbar c$	$197.326\,9631(49)$	MeV fm	2.5×10^{-8}
n.u. of mass:				
electron mass	m_e	$9.109\,382\,15(45) \times 10^{-31}$	kg	5.0×10^{-8}
n.u. of energy	$m_e c^2$	$8.187\,104\,38(41) \times 10^{-14}$	J	5.0×10^{-8}
in MeV		$0.510\,998\,910(13)$	MeV	2.5×10^{-8}
n.u. of momentum	$m_e c$	$2.730\,924\,06(14) \times 10^{-22}$	kg m s^{-1}	5.0×10^{-8}
in MeV/c		$0.510\,998\,910(13)$	MeV/c	2.5×10^{-8}
n.u. of length ($\hbar/m_e c$)	$\lambda\!\!\!^{-}_C$	$386.159\,264\,59(53) \times 10^{-15}$	m	1.4×10^{-9}
n.u. of time	$\hbar/m_e c^2$	$1.288\,088\,6570(18) \times 10^{-21}$	s	1.4×10^{-9}
		Atomic units (a.u.)		
a.u. of charge:				
elementary charge	e	$1.602\,176\,487(40) \times 10^{-19}$	C	2.5×10^{-8}
a.u. of mass:				
electron mass	m_e	$9.109\,382\,15(45) \times 10^{-31}$	kg	5.0×10^{-8}
a.u. of action:				
reduced Planck constant ($h/2\pi$)	$\hbar$	$1.054\,571\,628(53) \times 10^{-34}$	J s	5.0×10^{-8}
a.u. of length:				
Bohr radius (bohr) ($\alpha/4\pi R_\infty$)	a_0	$0.529\,177\,208\,59(36) \times 10^{-10}$	m	6.8×10^{-10}
a.u. of energy:				
Hartree energy (hartree) ($e^2/4\pi\epsilon_0 a_0 = 2R_\infty hc = \alpha^2 m_e c^2$)	E_h	$4.359\,743\,94(22) \times 10^{-18}$	J	5.0×10^{-8}
a.u. of time	$\hbar/E_h$	$2.418\,884\,326\,505(16) \times 10^{-17}$	s	6.6×10^{-12}
a.u. of force	E_h/a_0	$8.238\,722\,06(41) \times 10^{-8}$	N	5.0×10^{-8}
a.u. of velocity (αc)	$a_0 E_h/\hbar$	$2.187\,691\,2541(15) \times 10^{6}$	m s^{-1}	6.8×10^{-10}
a.u. of momentum	$\hbar/a_0$	$1.992\,851\,565(99) \times 10^{-24}$	kg m s^{-1}	5.0×10^{-8}
a.u. of current	$eE_h/\hbar$	$6.623\,617\,63(17) \times 10^{-3}$	A	2.5×10^{-8}
a.u. of charge density	e/a_0^3	$1.081\,202\,300(27) \times 10^{12}$	C m^{-3}	2.5×10^{-8}
a.u. of electric potential	E_h/e	$27.211\,383\,86(68)$	V	2.5×10^{-8}
a.u. of electric field	E_h/ea_0	$5.142\,206\,32(13) \times 10^{11}$	V m^{-1}	2.5×10^{-8}
a.u. of electric field gradient	E_h/ea_0^2	$9.717\,361\,66(24) \times 10^{21}$	V m^{-2}	2.5×10^{-8}
a.u. of electric dipole moment	ea_0	$8.478\,352\,81(21) \times 10^{-30}$	C m	2.5×10^{-8}
a.u. of electric quadrupole moment	ea_0^2	$4.486\,551\,07(11) \times 10^{-40}$	C m^2	2.5×10^{-8}
a.u. of electric polarizability	$e^2a_0^2/E_h$	$1.648\,777\,2536(34) \times 10^{-41}$	C^2 m^2 J^{-1}	2.1×10^{-9}
a.u. of 1$^{\mathrm{st}}$ hyperpolarizability	$e^3a_0^3/E_h^2$	$3.206\,361\,533(81) \times 10^{-53}$	C^3 m^3 J^{-2}	2.5×10^{-8}
a.u. of 2$^{\mathrm{nd}}$ hyperpolarizability	$e^4a_0^4/E_h^3$	$6.235\,380\,95(31) \times 10^{-65}$	C^4 m^4 J^{-3}	5.0×10^{-8}
a.u. of magnetic flux density	$\hbar/ea_0^2$	$2.350\,517\,382(59) \times 10^{5}$	T	2.5×10^{-8}
a.u. of magnetic				
dipole moment ($2\mu_B$)	$\hbar e/m_e$	$1.854\,801\,830(46) \times 10^{-23}$	J T^{-1}	2.5×10^{-8}
a.u. of magnetizability	$e^2a_0^2/m_e$	$7.891\,036\,433(27) \times 10^{-29}$	J T^{-2}	3.4×10^{-9}
a.u. of permittivity ($10^7/c^2$)	e^2/a_0E_h	$1.112\,650\,056\ldots \times 10^{-10}$	F m^{-1}	(exact)

TABLE 55 The values of some energy equivalents derived from the relations $E = mc^2 = hc/\lambda = h\nu = kT$, and based on the 2006 CODATA adjustment of the values of the constants; 1 eV = (e/C) J, 1 u = $m_\mathrm{u} = \frac{1}{12}m(^{12}\mathrm{C}) = 10^{-3}$ kg mol$^{-1}/N_\mathrm{A}$, and $E_\mathrm{h} = 2R_\infty hc = \alpha^2 m_\mathrm{e} c^2$ is the Hartree energy (hartree).

	Relevant unit			
	J	kg	m^{-1}	Hz
1 J	(1 J) = 1 J	$(1\ \mathrm{J})/c^2 =$ $1.112\,650\,056\ldots \times 10^{-17}$ kg	$(1\ \mathrm{J})/hc =$ $5.034\,117\,47(25) \times 10^{24}\ \mathrm{m}^{-1}$	$(1\ \mathrm{J})/h =$ $1.509\,190\,450(75) \times 10^{33}$ Hz
1 kg	$(1\ \mathrm{kg})c^2 =$ $8.987\,551\,787\ldots \times 10^{16}$ J	(1 kg) = 1 kg	$(1\ \mathrm{kg})c/h =$ $4.524\,439\,15(23) \times 10^{41}\ \mathrm{m}^{-1}$	$(1\ \mathrm{kg})c^2/h =$ $1.356\,392\,733(68) \times 10^{50}$ Hz
$1\ \mathrm{m}^{-1}$	$(1\ \mathrm{m}^{-1})hc =$ $1.986\,445\,501(99) \times 10^{-25}$ J	$(1\ \mathrm{m}^{-1})h/c =$ $2.210\,218\,70(11) \times 10^{-42}$ kg	$(1\ \mathrm{m}^{-1}) =$ $1\ \mathrm{m}^{-1}$	$(1\ \mathrm{m}^{-1})c =$ 299 792 458 Hz
1 Hz	$(1\ \mathrm{Hz})h =$ $6.626\,068\,96(33) \times 10^{-34}$ J	$(1\ \mathrm{Hz})h/c^2 =$ $7.372\,496\,00(37) \times 10^{-51}$ kg	$(1\ \mathrm{Hz})/c =$ $3.335\,640\,951\ldots \times 10^{-9}\ \mathrm{m}^{-1}$	(1 Hz) = 1 Hz
1 K	$(1\ \mathrm{K})k =$ $1.380\,6504(24) \times 10^{-23}$ J	$(1\ \mathrm{K})k/c^2 =$ $1.536\,1807(27) \times 10^{-40}$ kg	$(1\ \mathrm{K})k/hc =$ $69.503\,56(12)\ \mathrm{m}^{-1}$	$(1\ \mathrm{K})k/h =$ $2.083\,6644(36) \times 10^{10}$ Hz
1 eV	(1 eV) = $1.602\,176\,487(40) \times 10^{-19}$ J	$(1\ \mathrm{eV})/c^2 =$ $1.782\,661\,758(44) \times 10^{-36}$ kg	$(1\ \mathrm{eV})/hc =$ $8.065\,544\,65(20) \times 10^{5}\ \mathrm{m}^{-1}$	$(1\ \mathrm{eV})/h =$ $2.417\,989\,454(60) \times 10^{14}$ Hz
1 u	$(1\ \mathrm{u})c^2 =$ $1.492\,417\,830(74) \times 10^{-10}$ J	(1 u) = $1.660\,538\,782(83) \times 10^{-27}$ kg	$(1\ \mathrm{u})c/h =$ $7.513\,006\,671(11) \times 10^{14}\ \mathrm{m}^{-1}$	$(1\ \mathrm{u})c^2/h =$ $2.252\,342\,7369(32) \times 10^{23}$ Hz
$1\ E_\mathrm{h}$	$(1\ E_\mathrm{h}) =$ $4.359\,743\,94(22) \times 10^{-18}$ J	$(1\ E_\mathrm{h})/c^2 =$ $4.850\,869\,34(24) \times 10^{-35}$ kg	$(1\ E_\mathrm{h})/hc =$ $2.194\,746\,313\,705(15) \times 10^{7}\ \mathrm{m}^{-1}$	$(1\ E_\mathrm{h})/h =$ $6.579\,683\,920\,722(44) \times 10^{15}$ Hz

TABLE 56 The values of some energy equivalents derived from the relations $E = mc^2 = hc/\lambda = h\nu = kT$, and based on the 2006 CODATA adjustment of the values of the constants; 1 eV = (e/C) J, 1 u = $m_\mathrm{u} = \frac{1}{12}m(^{12}\mathrm{C}) = 10^{-3}$ kg mol$^{-1}/N_\mathrm{A}$, and $E_\mathrm{h} = 2R_\infty hc = \alpha^2 m_\mathrm{e}c^2$ is the Hartree energy (hartree).

	Relevant unit			
	K	eV	u	E_h
1 J	$(1\ \mathrm{J})/k =$ $7.242\,963(13) \times 10^{22}$ K	$(1\ \mathrm{J}) =$ $6.241\,509\,65(16) \times 10^{18}$ eV	$(1\ \mathrm{J})/c^2 =$ $6.700\,536\,41(33) \times 10^{9}$ u	$(1\ \mathrm{J}) =$ $2.293\,712\,69(11) \times 10^{17}$ E_h
1 kg	$(1\ \mathrm{kg})c^2/k =$ $6.509\,651(11) \times 10^{39}$ K	$(1\ \mathrm{kg})c^2 =$ $5.609\,589\,12(14) \times 10^{35}$ eV	$(1\ \mathrm{kg}) =$ $6.022\,141\,79(30) \times 10^{26}$ u	$(1\ \mathrm{kg})c^2 =$ $2.061\,486\,16(10) \times 10^{34}$ E_h
1 m^{-1}	$(1\ \mathrm{m}^{-1})hc/k =$ $1.438\,7752(25) \times 10^{-2}$ K	$(1\ \mathrm{m}^{-1})hc =$ $1.239\,841\,875(31) \times 10^{-6}$ eV	$(1\ \mathrm{m}^{-1})h/c =$ $1.331\,025\,0394(19) \times 10^{-15}$ u	$(1\ \mathrm{m}^{-1})hc =$ $4.556\,335\,252\,760(30) \times 10^{-8}$ E_h
1 Hz	$(1\ \mathrm{Hz})h/k =$ $4.799\,2374(84) \times 10^{-11}$ K	$(1\ \mathrm{Hz})h =$ $4.135\,667\,33(10) \times 10^{-15}$ eV	$(1\ \mathrm{Hz})h/c^2 =$ $4.439\,821\,6294(64) \times 10^{-24}$ u	$(1\ \mathrm{Hz})h =$ $1.519\,829\,846\,006(10) \times 10^{-16}$ E_h
1 K	$(1\ \mathrm{K}) =$ 1 K	$(1\ \mathrm{K})k =$ $8.617\,343(15) \times 10^{-5}$ eV	$(1\ \mathrm{K})k/c^2 =$ $9.251\,098(16) \times 10^{-14}$ u	$(1\ \mathrm{K})k =$ $3.166\,8153(55) \times 10^{-6}$ E_h
1 eV	$(1\ \mathrm{eV})/k =$ $1.160\,4505(20) \times 10^{4}$ K	$(1\ \mathrm{eV}) =$ 1 eV	$(1\ \mathrm{eV})/c^2 =$ $1.073\,544\,188(27) \times 10^{-9}$ u	$(1\ \mathrm{eV}) =$ $3.674\,932\,540(92) \times 10^{-2}$ E_h
1 u	$(1\ \mathrm{u})c^2/k =$ $1.080\,9527(19) \times 10^{13}$ K	$(1\ \mathrm{u})c^2 =$ $931.494\,028(23) \times 10^{6}$ eV	$(1\ \mathrm{u}) =$ 1 u	$(1\ \mathrm{u})c^2 =$ $3.423\,177\,7149(49) \times 10^{7}$ E_h
1 E_h	$(1\ E_\mathrm{h})/k =$ $3.157\,7465(55) \times 10^{5}$ K	$(1\ E_\mathrm{h}) =$ $27.211\,383\,86(68)$ eV	$(1\ E_\mathrm{h})/c^2 =$ $2.921\,262\,2986(42) \times 10^{-8}$ u	$(1\ E_\mathrm{h}) =$ 1 E_h